1. One page of notes/formula sheet allowed
2. The number of points (Total=50) are indicated next to the question

**Question 1**

(a) Show that the body centered cubic (bcc) crystal lattice has two atoms/unit cell and that the face centered cubic (fcc) crystal lattice has four atoms/unit cell. Also give one example each of an element/material that has the bcc and fcc crystal structure (4 points)

\[
\begin{align*}
\text{bcc} & \rightarrow (1 \text{ atom at center}) + (8 \text{ atoms at corners}) \times \frac{1}{8} = 2 \\
\text{fcc} & \rightarrow (6 \text{ atoms on faces}) \times \frac{1}{2} + (8 \text{ atoms at corners}) \times \frac{1}{8} = 4
\end{align*}
\]

Examples: bcc crystal structure $\rightarrow$ Fe  
fcc crystal structure $\rightarrow$ Al, Cu, Ni

(b) Does the surface energy of a single crystal depend on the crystallographic orientation, with respect to the surface? If so, for an fcc crystal is the surface energy higher for the (100) plane or the (111) plane? Why? (4 points)

As the atomic packing is different for various crystallographic planes, the number of unsatisfied bonds (which determine the surface energy) also change from plane to plane.

In fcc, the (100) is more loosely packed compared to (111) $\rightarrow$ see HW1, problem(k)  
$\Rightarrow$ higher surface energy for (100)

PRINCIPLE: denser packing $\rightarrow$ more number of neighbors and less number of unsatisfied bonds  
$\Rightarrow$ lower surface energy.
(c) The slip system for a face centered cubic (fcc) crystal is given as \{111\}<1\bar{1}0>.

In this context, define the terms (i) slip plane, and (ii) slip direction and sketch them both on a diagram. Also, write down all the possible slip planes and slip directions? (6 points)

**Slip plane**: Most densely packed crystallographic plane along which dislocation motion, leading to slip, occurs.

**Slip direction**: A particular direction in the slip plane, with highest atom packing density.

There are 12 slip systems in all:

\[
\begin{align*}
&\{111\} [1\bar{1}0], & &\{1\bar{1}1\} [11\bar{0}], & &\{1\bar{1}1\} [101], & &\{1\bar{1}1\} [01\bar{1}] \\
&[1\bar{1}0], & &[01\bar{1}], & &[11\bar{0}], & &[011] \\
&[1\bar{1}1], & &[10\bar{1}], & &[0\bar{1}1], & &[1\bar{1}1]
\end{align*}
\]

**Question 2**

(a) Briefly discuss, with reference to one/two applications why materials should be case hardened, say through carburization? (3 points)

"Case" → Carbon rich outer region

Case hardening improves the hardness & fatigue resistance of material.

It is typically done by diffusing excess carbon into the outer surface layer.

**Applications**: Steel gears, engine components, parts subject to high "self-drilling" screws, pressure & sharp impacts.

(b) A plate of iron is exposed to a carburizing (carbon-rich) atmosphere on one side and a decarburizing (carbon-deficient) atmosphere on the other side at 700 °C. If a condition of steady state is achieved, calculate the diffusion flux of carbon through the plate if the concentrations of carbon at positions of 5 mm and 10 mm beneath the carburizing surface are 1.2 kg/m³ and 0.8 kg/m³, respectively. Assume a diffusion coefficient of \(3 \times 10^{-12} \text{ m}^2/\text{s}\) at 700 °C (5 points)

\[
\begin{align*}
J &= -D_A \frac{C_A - C_B}{x_A - x_B} \\
&= -\left(3 \times 10^{-12} \frac{\text{m}^2}{\text{sec}}\right) \frac{(1.2 - 0.8) \text{kg/m}^3}{(5 \times 10^{-3} - 10^{-2}) \text{m}} \\
&= 2.4 \times 10^{-10} \text{ kg/m}^2\text{.sec}
\end{align*}
\]
(c) How does the diffusion of atoms through a lattice change with temperature? (2 points)

The diffusion increases exponentially with temperature, as the mobility of atoms is enhanced.

**Question 3**

(a) What is the difference between an engineering stress-engineering strain curve and a true stress-true strain curve? Which is easier to experimentally determine? (2 points)

\[
\text{Engineering strain } (\varepsilon) = \frac{\text{Applied force}}{\text{"Original" cross-sectional area}} \\
\text{True strain } (\varepsilon) = \frac{\text{Applied force}}{\text{"Instantaneous" cross-sectional area}}
\]

\[
\text{Engineering strain } (\varepsilon) = \frac{\Delta L}{L_0} \\
\text{True strain } (\varepsilon) = \ln \left( \frac{L_{\text{new}}}{L_0} \right)
\]

The engineering strain-engineering stress curve is easier to experimentally determine.

(b) Define (and indicate on an engineering stress-strain curve) the following quantities:
   (i) Elastic region, (ii) Plastic region, (iii) Ductility, and (iv) Fracture stress (6 points)

- **Elastic region**: Here the deformation \( \varepsilon \) is non-permanent — when applied load is released, the piece returns to original shape.
- **Plastic region**: In this region, the deformation is permanent/irreversible.
- **Ductility**: Measure of the degree of plastic deformation that has been sustained at fracture. It is defined by the area under the stress-strain curve.
- **Fracture Stress**: Point at which fracture occurs.
(c) Comparing a brittle solid (say, a ceramic) to a ductile metal, which material possesses a greater toughness? Why? (2 points)

The ductile metal has greater toughness
(Toughness is the ability of a material to absorb energy upon fracture and includes both strength and ductility)

(d) Show that the area under the elastic part of the stress-strain curve is proportional to the energy stored in the material (Assume an elastic modulus of E and that a stress s is applied) Why is this quantity termed resilience? (4 points)

\[ \text{Area} = \frac{1}{2} \cdot s \cdot e \cdot \left( \frac{N}{m^2/m} \right) = \frac{\text{Joules}}{m^3} \]

(e) Define hardness of a material. How is it related to the engineering stress-strain curve?

Compare the hardness of a brittle ceramic (say, porcelain) to a ductile metal (say, Aluminum). (2 points)

The hardness represents a material's resistance to localized plastic deformation. It cannot be directly determined from the engineering stress-strain curve.

A brittle ceramic is much harder compared to a ductile metal as plastic deformation cannot be significantly sustained in the former.

**Question 4** Explain why there is an increased stress near the edge of a crack? Would the stress concentration be higher for a sharp crack or a blunt crack? (4 points)

An increased stress is due to an abrupt change in cross-sectional area which can support applied loads.

\[ \text{Stress concentration scales as } \sqrt{\frac{r}{a}} \]

where \( r \) is the radius of curvature of the crack

A sharp crack \( \rightarrow \) small \( r \) \( \rightarrow \) high stress concentration

A blunt crack \( \rightarrow \) large \( r \) \( \rightarrow \) smaller stress concentration.
Question 5 (6 points)
(a) Briefly explain the difference between ductile and brittle fracture?

\[ \begin{array}{ll}
\text{Brittle fracture} & \text{Ductile fracture} \\
\text{Negligible plastic deformation} & \text{Substantial plastic deformation} \\
\text{Rapid crack propagation} & \text{Slow crack propagation} \\
\text{Shiny/bright fracture surface} & \text{Dull/fibrous fracture surface}
\end{array} \]

(b) Give one example each of a material that fails in a ductile mode and one that fails through brittle fracture

Ductile mode: Metals/Alloys
Brittle fracture mode: Ceramics/glass.

(c) Define "fatigue of materials" and give a practical example of failure by fatigue.

Fatigue is a form of failure that occurs in structures subjected to dynamic (flexural) stresses. Failure occurs at a stress level considerably lower than the tensile/yield strength for a static load.

- Fatigue occurs in mechanical structures subject to flexural/dynamic stresses, e.g., bridges, aircraft wings, rotating elements such as car axles.