An ideal ramjet is operated at 50,000 ft and Mach 3. The diffuser and nozzle are assembled to be isentropic, and the combustion is to be modeled as an ideal heat interaction at constant Mach number with constant total pressure. The cross-sectional area and Mach number for certain engine stations are given. The total temperature leaving the combustor $T_r$ is 4000°R.

a) Determine the mass flow rate of air through the engine (lbm/s)

We do the usual

$$ \dot{m} = \rho_1 A_1 V_1 = \frac{P_1 A_1}{RT_1} M_1 \sqrt{\gamma R g_c T_1} = P_1 A_1 M_1 \sqrt{\frac{\gamma R g_c}{RT_1}} $$

$$ T_1 = \frac{\theta}{T_{std}} = 0.7519 \times 518.7 \text{°R} = 390.0 \text{°R} $$

$$ P_1 = \frac{\delta P_{std}}{T_{std}} = 0.1151 \times 2116.2 \frac{\text{lb}}{\text{ft}^2} = 243.57 \frac{\text{lb}}{\text{ft}^2} = 1.6914 \text{ psia} $$

$$ \dot{m} = 243.57 \frac{\text{lb}}{\text{ft}^2} \left(4.235 \frac{\text{ft}^2}{3}\right) \sqrt{\frac{1.4 \times 32.174 \frac{\text{lbm ft}}{\text{lb}} 390.0 \text{°R}}{53.35 \frac{\text{lbm ft}}{\text{lbm °R}}}} $$

$$ \dot{m} = 143.98 \text{ lbm/s} $$

b) Complete the table with flow areas, static pressures, static temperatures, and velocities.

Start by determining total temperature and pressure

$$ T_{t1} = T_1 \left(1 + \frac{\gamma - 1}{2} M_1^2\right) = 390.0 \text{°R} \left(1 + \frac{1.4-1}{2} 3^2\right) = 1092.0 \text{°R} $$

$$ P_{t1} = P_1 \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}} = 243.57 \frac{\text{lb}}{\text{ft}^2} \left(1 + \frac{1.4-1}{2} 3^2\right)^{\frac{1.4}{1.4-1}} = 8946.99 \frac{\text{lb}}{\text{ft}^2} = 62.13 \text{ psia} $$

The total pressure will remain 8, 946.99 $\frac{\text{lb}}{\text{ft}^2}$ for all stations, and the total temperature will be 1092.0°R for stations 1, 2, and 3, and 4000°R for stations 4, 5, and 6.

We can find velocity by

$$ V_1 = M_1 \sqrt{\gamma R g_c T_1} = 3 \sqrt{1.4 \left(53.35 \frac{\text{ft lb}}{\text{lbm °R}}\right) \left(32.174 \frac{\text{lbm ft}}{\text{lb}}\right) 390.0 \text{°R}} = 2904.27 \text{ ft/s} $$

So we've filled in station 1 of the table

<table>
<thead>
<tr>
<th>Station</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area ft$^2$</td>
<td>4.235</td>
</tr>
<tr>
<td>Mach</td>
<td>3</td>
</tr>
<tr>
<td>P psia</td>
<td>1.691</td>
</tr>
<tr>
<td>T °R</td>
<td>390.</td>
</tr>
<tr>
<td>fps in V</td>
<td>2904</td>
</tr>
</tbody>
</table>
We can use the isentropic tables to find $A^*$. For $M=3$, $A/A^* = 4.235$. Since $A=4.235$, $A^* = 1$. Since $M=1$ at station 2, $A_2 = 1 \text{ ft}^2$. Also from the tables, we get

$$
\frac{T_2}{T_2} = 1.2 = \frac{T_1}{T_2}
$$

$$
T_2 = \frac{T_1}{1.2} = \frac{1092.0 \text{ °R}}{1.2} = 910 \text{ °R}
$$

$$
\frac{P_2}{P_2} = 1.893 = \frac{P_1}{P_2}
$$

$$
P_2 = \frac{62.13 \text{ psia}}{1.893} = 32.82 \text{ psia}
$$

And, for velocity

$$
V_2 = M_2 \sqrt{\gamma R g_c T_2} = 1 \sqrt{1.4 \left(53.35 \frac{\text{ft lbf}}{\text{lbm °R}}\right) \left(32.174 \frac{\text{lbm ft}}{\text{lbm lbf s}^2}\right) 910 \text{ °R}} = 1478.78 \text{ ft} / \text{s}
$$

And the table is now

<table>
<thead>
<tr>
<th>Station</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area ft$^2$</td>
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<td>1</td>
<td>3.9103</td>
</tr>
<tr>
<td>Mach</td>
<td>3</td>
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<td>0.15</td>
</tr>
<tr>
<td>P psia</td>
<td>1.691</td>
<td>32.82</td>
<td>61.16</td>
</tr>
<tr>
<td>$T \text{ °R}$</td>
<td>390</td>
<td>910</td>
<td>1087.1</td>
</tr>
<tr>
<td>fps in V</td>
<td>2904</td>
<td>1479</td>
<td>242.44</td>
</tr>
</tbody>
</table>

Again using the isentropic tables for $M=0.15$:

$$
\frac{A}{A^*} = 3.9103
$$

$$
\frac{T_2}{T_2} = \frac{T_1}{1.0045} = \frac{T_1}{T_3}
$$

$$
T_3 = \frac{T_1}{1.0045} = \frac{1092.0 \text{ °R}}{1.0045} = 1087.1 \text{ °R}
$$

$$
\frac{P_3}{P_3} = \frac{P_1}{1.0115} = \frac{P_3}{P_3}
$$

$$
P_3 = \frac{62.13 \text{ psia}}{1.0115} = 61.16 \text{ psia}
$$

$$
V_3 = M_3 \sqrt{\gamma R g_c T_3} = .15 \sqrt{1.4 \left(53.35 \frac{\text{ft lbf}}{\text{lbm °R}}\right) \left(32.174 \frac{\text{lbm ft}}{\text{lbm lbf s}^2}\right) 1087.1 \text{ °R}} = 242.44 \text{ ft} / \text{s}
$$

<table>
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<tr>
<th>Station</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area ft$^2$</td>
<td>4.235</td>
<td>1</td>
<td>3.9103</td>
</tr>
<tr>
<td>Mach</td>
<td>3</td>
<td>1</td>
<td>0.15</td>
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<tr>
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<td>1.691</td>
<td>32.82</td>
<td>61.16</td>
</tr>
<tr>
<td>$T \text{ °R}$</td>
<td>390</td>
<td>910</td>
<td>1087.1</td>
</tr>
<tr>
<td>fps in V</td>
<td>2904</td>
<td>1479</td>
<td>242.44</td>
</tr>
</tbody>
</table>
In going from 3 to 4, the total pressure stays the same but the total temperature increases. The pressure at 3 will be the same as the pressure at 4 since the Mach number and total pressure are the same.

\[ P_4 = 61.16 \text{ psia} \]
\[ \frac{T_{4t}}{T_4} = 1.0045 \]
\[ T_4 = \frac{T_{4t}}{1.0045} = \frac{4000^\circ \text{R}}{1.0045} = 3982.1^\circ \text{R} \]
\[ V_4 = M_4 \sqrt{\gamma \, R \, g_c \, T_4} = 0.15 \sqrt{1.4 \left( 53.35 \frac{\text{ft lbf}}{\text{lbm}^2 \text{R}} \right) \left( 32.174 \frac{\text{lbm ft}}{\text{lbm}^2 \text{s}^2} \right) 3982.1^\circ \text{R}} = 464.01 \text{ ft/s} \]

Now, we can return to mass flow rate

\[ \dot{m} = P_4 A_4 M_4 \sqrt{\frac{\gamma \, g_c}{RT_4}} \]

Chugging on, we note that from station 4, we could find the new \( A^* \)

\[ A^* = A_5 = \frac{A_4}{3.9103} = \frac{7.485 \text{ ft}^2}{3.9103} = 1.914 \text{ ft}^2 \]

Then the usual

\[ \frac{T_{5t}}{T_5} = 1.2 = \frac{T_{4t}}{T_4} \]
\[ T_5 = \frac{T_{4t}}{1.2} = \frac{4000^\circ \text{R}}{1.2} = 3333.3^\circ \text{R} \]
\[ \frac{P_{5t}}{P_5} = 1.893 = \frac{P_{4t}}{P_4} \]
\[ P_5 = \frac{62.13 \text{ psia}}{1.893} = 32.82 \text{ psia} \]
\[ V_5 = M_5 \sqrt{\gamma \, R \, g_c \, T_5} = 1 \sqrt{1.4 \left( 53.35 \frac{\text{ft lbf}}{\text{lbm}^2 \text{R}} \right) \left( 32.174 \frac{\text{lbm ft}}{\text{lbm}^2 \text{s}^2} \right) 3333.3^\circ \text{R}} = 2830.24 \text{ ft/s} \]
And, lastly

\[ A_6 = \frac{A}{A_6} = 4.235 \]
\[ A^* \times 4.235 \times 1.914 \text{ ft}^2 = 8.106 \text{ ft}^2 \]
\[ \frac{T_{sl}}{T_{sl}} = 2.8 = \frac{T_{sl}}{T_{sl}} \]
\[ \frac{T_{sl}}{2.8} = \frac{400 \, ^\circ R}{2.8} = 1428.57 \, ^\circ R \]
\[ \frac{P_{sl}}{P_{sl}} = 36.73 = \frac{P_{sl}}{P_{sl}} \]
\[ P_6 = \frac{62.13 \, \text{psia}}{36.73} = 1.691 \, \text{psia} \]

\[ V_6 = M_6 \sqrt{\gamma R g_c T_{sl}} = 3 \sqrt{1.4 \left( 53.35 \frac{\text{ft lbf}}{\text{lbm} \, ^{\circ} \text{R}} \right) \left( 32.174 \frac{\text{lbm ft}}{\text{lbf s}^2} \right) 1428.57 \, ^\circ R} = 5558.48 \, \text{ft/s} \]

Such that, finally,

<table>
<thead>
<tr>
<th>Station</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tr>
<td>Area ft²</td>
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<td>3.9103</td>
<td>7.485</td>
<td>1.914</td>
<td>8.106</td>
</tr>
<tr>
<td>Mach</td>
<td>3</td>
<td>1</td>
<td>0.15</td>
<td>0.15</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>P psia</td>
<td>1.691</td>
<td>32.82</td>
<td>61.16</td>
<td>61.16</td>
<td>32.82</td>
<td>1.691</td>
</tr>
<tr>
<td>T °R</td>
<td>390.0</td>
<td>910.0</td>
<td>1087.1</td>
<td>3982.1</td>
<td>3333.3</td>
<td>1428.6</td>
</tr>
<tr>
<td>fps in V</td>
<td>2904</td>
<td>1479</td>
<td>242.44</td>
<td>464.01</td>
<td>2830.24</td>
<td>5558.48</td>
</tr>
</tbody>
</table>

c) Find the thrust (magnitude and direction) of the diffuser, combustor, and nozzle.

The forces are

\[ F_{\text{diffuser}} = \frac{m}{g_c} (V_3 - V_1) + (P_3 - P_1) A_3 \]
\[ F_{\text{diffuser}} = \frac{143.98 \, \text{lbm/s}}{32.174 \, \text{lbm ft/lbf s}^2} (242.44 \, \text{ft/s} - 2904 \, \text{ft/s}) + (61.16 \, \text{psia} - 1.691 \, \text{psia}) \frac{144 \, \text{in}^2}{\text{ft}^2} 3.9103 \, \text{ft}^2 \]
\[ F_{\text{diffuser}} = 21575.4 \, \text{lbf} \]

\[ F_{\text{combustor}} = \frac{m}{g_c} (V_4 - V_3) + (P_4 - P_1) A_4 - (P_3 - P_1) A_3 \]
\[ F_{\text{combustor}} = \frac{143.98 \, \text{lbm/s}}{32.174 \, \text{lbm ft/lbf s}^2} (464.01 \, \text{ft/s} - 242.44 \, \text{ft/s}) + (61.16 \, \text{psia} - 1.691 \, \text{psia}) \frac{144 \, \text{in}^2}{\text{ft}^2} 7.485 \, \text{ft}^2 - (61.16 \, \text{psia} - 1.691 \, \text{psia}) \frac{144 \, \text{in}^2}{\text{ft}^2} 3.9103 \, \text{ft}^2 \]
\[ F_{\text{combustor}} = 31603.6 \, \text{lbf} \]

\[ F_{\text{nozzle}} = \frac{m}{g_c} (V_6 - V_4) - (P_4 - P_1) A_4 \]
\[ F_{\text{nozzle}} = \frac{143.98 \, \text{lbm/s}}{32.174 \, \text{lbm ft/lbf s}^2} (5558.48 \, \text{ft/s} - 464.01 \, \text{ft/s}) - (61.16 \, \text{psia} - 1.691 \, \text{psia}) \frac{144 \, \text{in}^2}{\text{ft}^2} 7.485 \, \text{ft}^2 \]
\[ F_{\text{nozzle}} = -41300.1 \, \text{lbf} \]

The nozzle therefore has a thrust to the right (a drag)
d) Find the thrust of the ramjet

This is done by summing all the components

\[ F_{\text{ramjet}} = F_{\text{diffuser}} + F_{\text{combustor}} + F_{\text{nozzle}} \]
\[ F_{\text{ramjet}} = 21\,575.4 \text{ lbf} + 31\,603.6 \text{ lbf} + (-41\,300.1) \text{ lbf} \]
\[ F_{\text{ramjet}} = 11\,878.9 \text{ lbf} \]
2.38 Air at 20 kPa, 260K, and Mach 3 passes through a normal shock. Determine
a) Total temperature and pressure upstream of shock.

Start with the relation from class

\[ \frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} M^2 \]

\[ T_0 = T_1 \left(1 + \frac{\gamma - 1}{2} M^2\right) = 260 \, K \left(1 + \frac{1.4 - 1}{2} \times 3^2\right) \]

\[ T_0 = 728 \, K \]

Now use the pressure relation presented in class

\[ \frac{p_0}{p_1} = \left(\frac{T_0}{T_1}\right)^{\frac{\gamma}{\gamma - 1}} \]

\[ p_0 = p_1 \left(\frac{T_0}{T_1}\right)^{\frac{\gamma}{\gamma - 1}} = 20 \, kPa \left(\frac{728 \, K}{260 \, K}\right)^{1.4 \over 1.4 - 1} \]

\[ p_0 = 734.65 \, kPa \]

b) Total temperature and pressure downstream of shock.

Since the flow through the shock is adiabatic, the total temperature is a constant

\[ T_{02} = T_{01} = 728 \, K \]

The pressure downstream of the shock can be found using

\[ \frac{p_{02}}{p_{01}} = \left[\frac{\gamma+1}{\gamma} M_1^2 \left(\frac{T_0}{T_1}\right)\right]^{\gamma - 1} \left[\frac{1+\frac{\gamma - 1}{2} M_1^2}{\frac{\gamma + 1}{\gamma}}\right]^{\frac{\gamma}{\gamma - 1}} = 0.3283 \]

\[ p_{02} = 0.3283 \times p_{01} = 0.3283 \times 734.65 \, kPa \]

\[ p_{02} = 241.22 \, kPa \]

c) Static temperature and pressure downstream of the shock.

Start by finding downstream Mach number.

\[ M_2 = \sqrt{\frac{M_1^2 + \frac{2}{\gamma - 1}}{\frac{2\gamma}{\gamma - 1} M_1^2 - 1}} = \sqrt{\frac{3^2 + \frac{2}{1.4 - 1}}{\frac{2 \times 1.4}{1.4 - 1} \times 3 - 1}} = 0.4752 \]

Back to

\[ \frac{T_{02}}{T_2} = 1 + \frac{\gamma - 1}{2} M^2 = 1 + \frac{1.4 - 1}{2} (0.4752)^2 = 1.0452 \]

\[ T_2 = \frac{T_{02}}{1.0452} = \frac{728 \, K}{1.0452} \]

\[ T_2 = 696.5 \, K \]

and again,
\[ \frac{P_2}{P_{02}} = \left( \frac{T_2}{T_{02}} \right)^{\gamma - 1} \]

\[ P_2 = P_{02} \left( \frac{T_2}{T_{02}} \right)^{\gamma - 1} = 241.22 \text{ kPa} \left( \frac{696.5 \text{ K}}{728 \text{ K}} \right)^{1.4 - 1} \]

\[ P_2 = 206.67 \text{ kPa} \]
3.11 A rocket nozzle has an ideal thrust coefficient $C_{Fi}$ of 1.5, a chamber pressure $P_c$ of 100 atm, and a throat area $A_t$ of 0.15 m$^2$. Determine the ideal thrust $F_i$.

Use equation 3.43

$$C_{Fi} = \frac{F_i}{P_c A_t}$$

$$F_i = C_{Fi} P_c A_t = (1.5) \left( 100 \ \text{atm} \times \frac{101325 \ \text{Pa}}{\text{atm}} \right) (0.15 \ m^2)$$

$$F_i = 2,279,812.5 \ N$$
5.3 Calculate the variation with $T_t$ of exit Mach number, exit velocity, specific thrust, fuel/air ratio, and thrust specific fuel consumption of an ideal turbojet engine for compressor pressure ratios of 10 and 20 at a flight Mach number of 2 and $T_0 = 217$ K. Perform calculations at $T_t$ values of 2400, 2200, 2000, and 1800K. Use $h_{PR} = 42800$ kJ/kg, $c_p = 1.004$ kJ/(kg K), and $\gamma = 1.4$.

For most of this, we're using equations 5.32. To start, let's find $\tau_r$

$$\tau_r = \frac{T_0}{T_0} = 1 + \frac{\gamma - 1}{2} M_0^2 = 1 + \frac{1.4 - 1}{2} 2^2 = 1.8$$

next, $a_0$

$$a_0 = \sqrt{\gamma R g_c T_0} = \sqrt{1.4 \left( \frac{287}{kJK} \right) (1) (217 K)} = 295.28 \text{ m/s}$$

Note that $\pi_c = \frac{P_c}{P_c}$. For $\tau_c$

$$\tau_c = (\pi_c)^{\frac{\gamma - 1}{\gamma}}$$

For $\pi_c = 10$, $\tau_c = 1.9307$

For $\pi_c = 20$, $\tau_c = 2.3535$

First for the table is $\tau_\lambda$

$$\tau_\lambda = \frac{T_t}{T_0}$$

For $T_t = 1800, 2000, 2200, 2400$ K, $\tau_\lambda = 8.2949, 9.2166, 10.1382, 11.0600$

next,

$$\tau_t = 1 - \frac{\tau_r}{\tau_c} (\tau_c - 1)$$

For $\pi_c = 10$ and $T_t = 1800, 2000, 2200, 2400$ K, $\tau_t = 0.7980, 0.8182, 0.8348, 0.8485$

For $\pi_c = 20$ and $T_t = 1800, 2000, 2200, 2400$ K, $\tau_t = 0.7063, 0.7357, 0.7597, 0.7797$

now, for $M_9$ we use equation 5.23

$$M_9 = \sqrt{\frac{2}{\gamma - 1} (\tau_r \tau_c \tau_t - 1)}$$

For $\pi_c = 10$ and $T_t = 1800, 2000, 2200, 2400$ K, $M_9 = 2.9776, 3.0360, 3.0831, 3.1215$

For $\pi_c = 20$ and $T_t = 1800, 2000, 2200, 2400$ K, $M_9 = 3.1560, 3.2532, 3.3304, 3.3934$

back to 5.32g

$$V_9 = a_0 \sqrt{\frac{2}{\gamma - 1} \frac{\tau_t}{\tau_r \tau_c} (\tau_r \tau_c \tau_t - 1)} = a_0 M_9 \sqrt{\frac{\tau_t}{\tau_r \tau_c}}$$

For $\pi_c = 10$ and $T_t = 1800, 2000, 2200, 2400$ K, $V_9$ (in m/s) = 1358.35, 1460.20, 1554.91, 1644.30

For $\pi_c = 20$ and $T_t = 1800, 2000, 2200, 2400$ K, $V_9$ (in m/s) = 1304.02, 1416.89, 1521.29, 1619.02
\[ \frac{F}{m_0} = \frac{a_0}{g_c} \left( \frac{V_a}{a_0} - M_0 \right) \]

For \( \pi_c = 10 \) and \( T_{14} = 1800, 2000, 2200, 2400 \) \( K \), \( \frac{F}{m_0} \) (in N/(kg/s)) = 767.79, 869.64, 964.35, 1053.74

For \( \pi_c = 20 \) and \( T_{14} = 1800, 2000, 2200, 2400 \) \( K \), \( \frac{F}{m_0} \) (in N/(kg/s)) = 713.46, 826.33, 930.73, 1028.46

next, 5.32i

\[ f = \frac{c_p T_0}{h_{PR}} \left( \tau_L - \tau_r \tau_c \right) = \frac{1.064 \frac{\mu L}{kg K}}{42800 \frac{\mu L}{kg K}} (\tau_L - \tau_r \tau_c) = 0.005009 (\tau_L - \tau_r \tau_c) \]

For \( \pi_c = 10 \) and \( T_{14} = 1800, 2000, 2200, 2400 \) \( K \), \( f = 0.0245, 0.0292, 0.0339, 0.0386 \)

For \( \pi_c = 20 \) and \( T_{14} = 1800, 2000, 2200, 2400 \) \( K \), \( f = 0.0207, 0.0254, 0.0300, 0.0347 \)

and, finally, we get \( S \) from 5.32j

\[ S = \frac{f}{\pi_c} \]

For \( \pi_c = 10 \) and \( T_{14} = 1800, 2000, 2200, 2400 \) \( K \),
\( S \) (in (kg/s)/N) = 3.1910 \( \times \) 10\(^{-5}\), 3.3577 \( \times \) 10\(^{-5}\), 3.5153 \( \times \) 10\(^{-5}\), 3.6631 \( \times \) 10\(^{-5}\)

For \( \pi_c = 20 \) and \( T_{14} = 1800, 2000, 2200, 2400 \) \( K \),
\( S \) (in (kg/s)/N) = 2.9014 \( \times \) 10\(^{-5}\), 3.0738 \( \times \) 10\(^{-5}\), 3.2233 \( \times \) 10\(^{-5}\), 3.3740 \( \times \) 10\(^{-5}\)

<table>
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<tr>
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<th>( T_{14} (K) )</th>
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<td>10.1382</td>
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<tr>
<td>( \tau_L )</td>
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<td>0.8348</td>
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<tr>
<td>( M_0 )</td>
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<td>3.0831</td>
<td>3.1215</td>
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<tr>
<td>( \tau_L )</td>
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<td>0.7357</td>
<td>0.7597</td>
<td>0.7797</td>
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<tr>
<td>( M_0 )</td>
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<td>3.2532</td>
<td>3.3304</td>
<td>3.3934</td>
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<tr>
<td>( V_c ) (m/s)</td>
<td>1304.02</td>
<td>1416.89</td>
<td>1521.29</td>
<td>1619.02</td>
<td></td>
</tr>
<tr>
<td>( \frac{F}{m_0} ) (N/(kg/s))</td>
<td>713.46</td>
<td>826.33</td>
<td>930.73</td>
<td>1028.46</td>
<td></td>
</tr>
<tr>
<td>( f )</td>
<td>0.0207</td>
<td>0.0254</td>
<td>0.03</td>
<td>0.0347</td>
<td></td>
</tr>
<tr>
<td>( S ) (in (mg/s)/N)</td>
<td>29.014</td>
<td>30.738</td>
<td>32.233</td>
<td>33.74</td>
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</tbody>
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5.4 Show that the thermal efficiency for an ideal turbojet engine is given by equation 5.22.

Thermal efficiency is
\[
\eta_T = \frac{\text{change in kinetic energy}}{\text{thermal energy in}} = \frac{1}{2} M \left( \frac{v_f^2 - v_i^2}{h_3 - h_1} \right) = \frac{h_{10} - h_0 - (h_{22} - h_0)}{h_{24} - h_3}
\]

Since KE=\((1/2)MV^2\) and TE=M\(\Delta h\). Also, \(h_0 - h = \frac{1}{2} V^2\).

Recall that we can define \(h_{1x} = h_{i(x-1)} + q + w\), where \(q\) is heat added and \(w\) is work done on the system. Since the only place where heat is added or work is done in an ideal turbojet is over the combustor, \(h_{i0} = h_{i1} = h_{i2} = h_{i3}\), and \(h_{i4} = h_{i5} = \ldots = h_{i9}\). Therefore,
\[
h_{i4} - h_{i5} = 0 = h_{i3} - h_{i2}
\]

Using \(h_{i5} = h_{i9}\)
\[
h_{i4} - h_{i9} = h_{i3} - h_{i2}
\]

Rearranging
\[
h_{i9} - h_{i2} = h_{i4} - h_{i3}
\]

Then
\[
\eta_T = \frac{h_{10} - h_0 - (h_{22} - h_0)}{h_{24} - h_3} = \frac{h_{24} - h_{22} - h_0 + h_0}{h_{24} - h_3} = \frac{h_{24} - h_9 - (h_{33} - h_0)}{h_{24} - h_3}
\]

for a constant pressure process,
\[
\eta_T = \frac{c_p(T_{i4} - T_{i0}) - c_p(T_{i3} - T_0)}{c_p(T_{i4} - T_{i0})} = \frac{(T_{i4} - T_{i0})(T_{i3} - T_0)}{(T_{i4} - T_{i3})} = \frac{T_0 \left( \frac{T_{i4}}{T_0} - 1 \right) - T_0 \left( \frac{T_{i3}}{T_0} - 1 \right)}{T_0 \left( \frac{T_{i4}}{T_0} - 1 \right)}
\]

For an ideal turbojet, there is no stagnation pressure loss. Also, \(P_0 = P_0\) since it is assumed that the exit of the turbojet sees ambient pressure.
\[
\frac{T_{i0}}{T_0} = \left( \frac{P_{i0}}{P_0} \right)^{\frac{\gamma - 1}{\gamma}} = \left( \frac{P_{i3}}{P_0} \right)^{\frac{\gamma - 1}{\gamma}} = \frac{T_{i0}}{T_0} = \frac{T_{i3}}{T_0} = \tau_c
\]

also, \(T_{i0} = T_{i4}\) since, ideally, there is nothing to change the stagnation temperature after the combustor
\[
\frac{T_{i0}}{T_0} = \frac{T_{i4}}{T_0} = \frac{T_{i3}}{T_0}
\]

thus,
\[
\eta_T = \frac{T_0 \left( \frac{T_{i4}}{T_0} - 1 \right) - T_0 \left( \frac{T_{i3}}{T_0} - 1 \right)}{T_0 \left( \frac{T_{i4}}{T_0} - 1 \right)} = \frac{T_{i3} - T_{i0}}{T_{i4} - T_{i0}} \frac{T_{i0}}{T_{i3}} = \frac{T_{i3} - T_{i0}}{T_{i4} - T_{i0}} = \left( \frac{T_{i3}}{T_{i0}} \right) - 1 \frac{T_{i0}}{T_{i3}} = 1 - \frac{T_{i0}}{T_{i3}}
\]

\[
\eta_T = 1 - \frac{T_{i0}}{T_{i3}} = 1 - \frac{1}{\tau_c T_{i0}}
\]
5.10 Compute the performance of three ideal turbofan engines with an ideal turbojet engine at two flight conditions. The first flight condition (case 1) is at Mach number 0.9 and 40,000ft. The second (case 2) is at Mach 2.6 and 60,000ft. The following are given: \( \pi_c = 20, \ T_{4a} = 3000 \degree R, \ c_p = 0.24 \text{ Btu/(lbm \degree R)}, \ \gamma = 1.4, \ h_{PR} = 18400 \text{ Btu/lbm}. \)

Start with \( \tau_c \)

\[
\tau_c = (\pi_c)^{\frac{\gamma-1}{\gamma}} = (20)^{\frac{1.4-1}{1.4}} = 2.3535
\]

Case 1: Using Appendix A, at 40,000ft and M=0.9

\[
T = \frac{T_{T_{std}}}{T_{T_{std}}} = 0.7519 \quad T_{T_{std}} = 0.7519 (518.69 \degree R) = 390.0 \degree R
\]

\[
a = a_{std} \sqrt{0.7519} = 967.7 \text{ ft/s}
\]

\[
\tau_r = \frac{T_0}{T_0} = 1 + \frac{\gamma-1}{2} M_0^2 = 1 + \frac{1.4-1}{2} \cdot 0.9^2 = 1.162
\]

\[
\tau_{\lambda} = \frac{T_{4a}}{T_0} = \frac{3000 \degree R}{390 \degree R} = 7.6923
\]

\[
f = \frac{c_p T_0}{h_{PR}} (\tau_{\lambda} - \tau_r \tau_c) = \frac{0.24 \text{ Btu/lbm \degree R}}{18400 \text{ Btu/lbm}} \frac{390 \degree R}{390 \degree R} (7.6923 - (1.162)(2.3535)) = 0.02483
\]

Case 2: Using Appendix A, at 60,000ft and M=2.6

\[
T = \frac{T_{T_{std}}}{T_{T_{std}}} = 0.7519 \quad T_{T_{std}} = 0.7519 (518.69 \degree R) = 390.0 \degree R
\]

\[
a = a_{std} \sqrt{0.7519} = 967.7 \text{ ft/s}
\]

\[
\tau_r = \frac{T_0}{T_0} = 1 + \frac{\gamma-1}{2} M_0^2 = 1 + \frac{1.4-1}{2} \cdot 2.6^2 = 2.352
\]

\[
\tau_{\lambda} = \frac{T_{4a}}{T_0} = \frac{3000 \degree R}{390 \degree R} = 7.6923
\]

\[
f = \frac{c_p T_0}{h_{PR}} (\tau_{\lambda} - \tau_r \tau_c) = \frac{0.24 \text{ Btu/lbm \degree R}}{18400 \text{ Btu/lbm}} \frac{390 \degree R}{390 \degree R} (7.6923 - (2.352)(2.3535)) = 0.01098
\]

Let's start with the turbofan, case 1:

\[
\tau_c = 2.3535, \ \tau_{\lambda} = 7.6923, \ \tau_r = 1.162, \ f = 0.02483
\]

\[
\tau_f = (\pi_f)^{\gamma-1/\gamma} = (4)^{4/1.4} = 1.4860
\]

\[
V_{9/a_0} = \sqrt{\frac{2}{\gamma-1} \left( \tau_{\lambda} - \tau_r \tau_c - 1 + a(\tau_f - 1) \right) - \frac{\tau_1}{\tau_r \tau_c}} = 3.703
\]

\[
V_{19/a_0} = \sqrt{\frac{2}{\gamma-1} (\tau_r \tau_f - 1)} = 1.906
\]

\[
F/m_0 = \frac{a_0}{g_c} \frac{1}{1+a} \left[ V_9 - M_0 + \alpha \left( V_{19/a_0} - M_0 \right) \right] = \frac{967.7 \text{ ft/s}}{32.174 \text{ ft/s}} \frac{1}{2} \left[ 5.606 \right] = 84.3 \text{ lbf/s lbm} = 826.7 \frac{N \cdot \text{s}}{\text{kg}}
\]

\[
f = 0.02483
\]

\[
S = \frac{f}{(1+a)(\frac{f}{m_0})} = \frac{0.02483}{2 \times 826.7 \frac{mg}{Ns}} = 15.02 \frac{mg}{Ns}
\]
Next, turbofan, case 2:

\[ \tau_c = 2.3535, \quad \tau_\lambda = 7.6923, \quad \tau_r = 2.352, \]
\[ \tau_f = (\pi_f)^{(y-1)/\gamma} = (4)^{4/1.4} = 1.4860 \]

\[ \frac{V_g}{a_0} = \sqrt{\frac{2}{y-1} \left[ \tau_\lambda - \tau_f \left( \tau_c - 1 + \alpha(\tau_f - 1) \right) \right]} = 3.143 \]
\[ \frac{V_{\theta g}}{a_0} = \sqrt{\frac{2}{y-1} (\tau_r \tau_f - 1)} = 3.532 \]

\[ \frac{F}{m_0} = \frac{a_0}{g_c} \frac{1}{1+\alpha} \left[ V_g - M_0 + \alpha \left( \frac{V_{\theta g}}{a_0} - M_0 \right) \right] = \frac{967.7 \text{ ft/s}}{32.174 \text{ ft/s}} \left( \frac{1}{2} \cdot 1.864 \right) = 28.03 \text{ lbf/s lbm} \frac{9.8067 N/(kg/s)}{\text{lbf/(lbm/s)}} = 274.9 \text{ N/s kg} \]

\( f = 0.01098 \)
\[ S = \frac{f}{(1+\alpha) \left( \frac{V_0}{a_0} \right)} = \frac{0.01098 \cdot 274.9}{3.143} = 19.97 \text{ mg N/s} \]

Case 1, \( \alpha^* \)

\[ \tau_c = 2.3535, \quad \tau_\lambda = 7.6923, \quad \tau_r = 1.162, \quad f = 0.02483 \]
\[ \alpha^* = \frac{1}{\tau_\lambda (\tau_r - 1)} \left[ \frac{\tau_\lambda}{\tau_r - 1} - \frac{\tau_\lambda}{\tau_r \tau_c} - \frac{1}{2} \left( \sqrt{\tau_r \tau_f - 1} + \sqrt{\tau_r - 1} \right) \right] = 5.158 \]
\[ \frac{V_g}{a_0} = \sqrt{\frac{2}{y-1} \left[ \tau_\lambda - \tau_r \left( \tau_c - 1 + \alpha(\tau_f - 1) \right) \right]} = 1.403 \]
\[ \frac{V_{\theta g}}{a_0} = \sqrt{\frac{2}{y-1} (\tau_r \tau_f - 1)} = 1.906 \]

\[ \frac{F}{m_0} = \frac{a_0}{g_c} \frac{1}{1+\alpha} \left[ V_g - M_0 + \alpha \left( \frac{V_{\theta g}}{a_0} - M_0 \right) \right] = \frac{967.7 \text{ ft/s}}{32.174 \text{ ft/s}} \left( \frac{1}{2} \cdot 6.195 \right) = 30.3 \text{ lbf/s lbm} \frac{9.8067 N/(kg/s)}{\text{lbf/(lbm/s)}} = 296.73 \text{ N/s kg} \]

\( f = 0.02483 \)
\[ S = \frac{f}{(1+\alpha) \left( \frac{V_0}{a_0} \right)} = \frac{0.02483 \cdot 296.73}{6.158 \cdot 296.73} = 13.59 \text{ mg N/s} \]

Case 2, \( \alpha^* \)

\[ \tau_c = 2.3535, \quad \tau_\lambda = 7.6923, \quad \tau_r = 2.352 \]
\[ \tau_f = (\pi_f)^{(y-1)/\gamma} = (4)^{4/1.4} = 1.4860 \]
\[ \alpha^* = \frac{1}{(\tau_f - 1)} \left[ \tau_s - \tau_r \left( \tau_c - 1 \right) - \frac{\tau_1}{\tau_r} \tau_c - \frac{1}{4} \left( \sqrt{\tau_r \tau_f - 1} + \sqrt{\tau_r - 1} \right)^2 \right] = 1.084 \]

\[ \frac{V_f}{a_0} = \sqrt{\frac{2}{\gamma - 1} \left( \tau_s - \tau_r \left( \tau_c - 1 \right) - \frac{\tau_1}{\tau_r} \tau_c \right) - \frac{1}{\tau_r \tau_f}} = 3.066 \]

\[ \frac{V_f}{a_0} = \sqrt{\frac{2}{\gamma - 1} \left( \tau_r \tau_f - 1 \right)} = 3.532 \]

\[ \frac{F}{m_0} = \frac{a_0}{g_c} \frac{1}{1 + \alpha} \left[ \frac{V_f}{a_0} - M_0 + \alpha \left( \frac{V_f}{a_0} - M_0 \right) \right] \]

\[ \frac{967.7 \text{ ft/s}}{32.174 \text{ lbf/ft}^2} \text{ lbf/s} \frac{1}{2} \left( 1.9423 \right) = 28.03 \text{ lbf/s} \frac{9.8067 \text{ N/(kg/s)}}{\text{ lbf/(lbfm/s)}} = 274.9 \text{ N/s} \]

\[ f = 0.01098 \]

\[ S = \frac{f}{(1 + \alpha) \left( \frac{F}{m_0} \right)} = \frac{0.02483}{2 \times 274.9 \frac{N}{kg}} = 19.17 \text{ mg} \frac{N}{s} \]

**Case 1, \( \pi_f^* \)**

\[ \tau_c = 2.3535, \ \tau_s = 7.6923, \ \tau_r = 1.162, \ f = 0.02483 \]

\[ \tau_f^* = \frac{\tau_s - \tau_r \left( \tau_c - 1 \right) - \frac{\tau_1}{\tau_r} \tau_c + \alpha \left( \tau_f - 1 \right)}{\tau_r (1 + \alpha)} = 2.3532 \]

\[ \pi_f^* = \left( \frac{\tau_f^*}{\gamma - 1} \right)^{\gamma/(\gamma - 1)} = 19.989 \]

\[ \frac{V_f}{a_0} = \sqrt{\frac{2}{\gamma - 1} \left( \tau_s - \tau_r \left( \tau_c - 1 \right) - \frac{\tau_1}{\tau_r} \tau_c \right) - \frac{1}{\tau_r \tau_f}} = 2.945 \]

\[ \frac{V_f}{a_0} = \sqrt{\frac{2}{\gamma - 1} \left( \tau_r \tau_f - 1 \right)} = 2.945 \]

\[ \frac{F}{m_0} = \frac{a_0}{g_c} \frac{1}{1 + \alpha} \left[ \frac{V_f}{a_0} - M_0 + \alpha \left( \frac{V_f}{a_0} - M_0 \right) \right] \]

\[ \frac{967.7 \text{ ft/s}}{32.174 \text{ lbf/ft}^2} \text{ lbf/s} \frac{1}{2} \left( 4.09 \right) = 61.5 \text{ lbf/s} \frac{9.8067 \text{ N/(kg/s)}}{\text{ lbf/(lbfm/s)}} = 603.2 \text{ N/s} \]

\[ f = 0.02483 \]

\[ S = \frac{f}{(1 + \alpha) \left( \frac{F}{m_0} \right)} = \frac{0.02483}{2 \times 603.2 \frac{N}{kg}} = 20.58 \text{ mg} \frac{N}{s} \]

**Case 2, \( \pi_f^* \)**

\[ \tau_c = 2.3535, \ \tau_s = 7.6923, \ \tau_r = 2.352 \]

\[ \tau_f^* = \frac{\tau_s - \tau_r \left( \tau_c - 1 \right) - \frac{\tau_1}{\tau_r} \tau_c + \alpha \left( \tau_f - 1 \right)}{\tau_r (1 + \alpha)} = 1.3849 \]

\[ \pi_f^* = \left( \frac{\tau_f^*}{\gamma - 1} \right)^{\gamma/(\gamma - 1)} = 3.126 \]

\[ \frac{V_f}{a_0} = \sqrt{\frac{2}{\gamma - 1} \left( \tau_s - \tau_r \left( \tau_c - 1 \right) - \frac{\tau_1}{\tau_r} \tau_c \right) - \frac{1}{\tau_r \tau_f}} = 3.327 \]

\[ \frac{V_f}{a_0} = \sqrt{\frac{2}{\gamma - 1} \left( \tau_r \tau_f - 1 \right)} = 3.360 \]

\[ \frac{F}{m_0} = \frac{a_0}{g_c} \frac{1}{1 + \alpha} \left[ \frac{V_f}{a_0} - M_0 + \alpha \left( \frac{V_f}{a_0} - M_0 \right) \right] \]

\[ \frac{967.7 \text{ ft/s}}{32.174 \text{ lbf/ft}^2} \text{ lbf/s} \frac{1}{2} \left( 1.5195 \right) = 22.85 \text{ lbf/s} \frac{9.8067 \text{ N/(kg/s)}}{\text{ lbf/(lbfm/s)}} = 224.1 \text{ N/s} \]

\[ f = 0.01098 \]

\[ S = \frac{f}{(1 + \alpha) \left( \frac{F}{m_0} \right)} = \frac{0.01098}{2 \times 224.1 \frac{N}{kg}} = 24.5 \text{ mg} \frac{N}{s} \]

Turbojet, Case 1

Printed by Mathematica for Students
\[ \tau_c = 2.3535, \quad \tau_\alpha = 7.6923, \quad \tau_r = 1.162, \quad f = 0.02483 \]
\[ \tau_f = 1 - \frac{\tau_r}{\tau_\alpha} (\tau_c - 1) = 0.7955 \]
\[ \frac{V_9}{a_0} = \frac{2}{\gamma - 1} \frac{\tau_\alpha}{\tau_r \tau_c} (\tau_r \tau_c \tau_f - 1) = 4.066 \]
\[ \frac{F}{m_0} = \frac{a_0}{g_c} \left( \frac{V_9}{a_0} - M_0 \right) = \frac{967.7 \text{ ft/s}}{32.174 \text{ lbm/s}^2} \times [3.166] = 95.22 \text{ lbm/lbm} = 933.8 \text{ N/s kg} \]
\[ f = 0.02483 \]
\[ S = \frac{f}{a_0} = \frac{0.02483}{933.8 \text{ N/s kg}} = 26.6 \text{ mg N/s} \]

**Turbojet, Case 2**

\[ \tau_c = 2.3535, \quad \tau_\alpha = 7.6923, \quad \tau_r = 2.352 \]
\[ \tau_f = 1 - \frac{\tau_r}{\tau_\alpha} (\tau_c - 1) = 0.5862 \]
\[ \frac{V_9}{a_0} = \frac{2}{\gamma - 1} \frac{\tau_\alpha}{\tau_r \tau_c} (\tau_r \tau_c \tau_f - 1) = 3.949 \]
\[ \frac{F}{m_0} = \frac{a_0}{g_c} \left( \frac{V_9}{a_0} - M_0 \right) = \frac{967.7 \text{ ft/s}}{32.174 \text{ lbm/s}^2} \times [1.349] = 40.57 \text{ lbm/lbm} = 397.9 \text{ N/s kg} \]
\[ f = 0.01098 \]
\[ S = \frac{f}{a_0} = \frac{0.01098}{397.9 \text{ N/s kg}} = 27.6 \text{ mg N/s} \]

<table>
<thead>
<tr>
<th>Engine</th>
<th>( \alpha )</th>
<th>( n_f )</th>
<th>( V_9/a_0 )</th>
<th>( V_19/a_0 )</th>
<th>( \frac{F}{m_0} \text{ (N/s kg)} )</th>
<th>( f )</th>
<th>( \frac{S \text{ (mg N/s)}}{\text{kg}} )</th>
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<td><strong>Turbofan</strong></td>
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