HW #1
1.2, 1.7, 1.14, 2.3, 2.6

1.2 Develop the following analytical expressions for a turbojet engine:

a) When \( \dot{m}_f \ll \dot{m}_o, P_e = P_a, \) and \( \phi_{\text{inlet}} = \phi_{\text{noz}} = 0, \) then the installed thrust is given by:

\[
T = \frac{\dot{m}_o}{g_e} (V_e - V_0)
\]

From equation 1.5,

\[
F = \frac{(\dot{m}_o + \dot{m}_f) V_e - \dot{m}_o V_0}{g_e} + (P_e - P_0) A_e
\]

when we apply \( \dot{m}_f \ll \dot{m}_o \) and \( P_e = P_a, \) we get

\[
F = \frac{\dot{m}_o}{g_e} V_e - \dot{m}_o V_0
\]

From equation 1.9,

\[
T = F(1 - \phi_{\text{inlet}} - \phi_{\text{noz}})
\]

but, \( \phi_{\text{inlet}} = \phi_{\text{noz}} = 0, \) so \( T = F. \) Thus,

\[
T = \frac{\dot{m}_o}{g_e} (V_e - V_0)
\]

b) By the same conditions, show

\[
\text{TSFC} = \frac{T_g/\dot{m}_o + 2 V_0}{2 \eta_T h_{PR}}
\]

From equation 1.20,

\[
\text{TSFC} = \frac{V_0}{\eta_p \eta_T h_{PR}}
\]

and from equation 1.16,

\[
\eta_p = \frac{2}{V_e/V_0 + 1}
\]

from part (a),

\[
T = \frac{\dot{m}_o}{g_e} (V_e - V_0)
\]

\[
\frac{T g_e}{\dot{m}_o} = V_e - V_0
\]

\[
\frac{T g_e}{\dot{m}_o V_0} = \frac{V_e}{V_0} - 1
\]

\[
\frac{V_e}{V_0} = \frac{T g_e}{\dot{m}_o V_0} + 1
\]
plugging this into 1.16

\[
\eta_P = \frac{2}{\frac{T_g}{m_0} + \frac{2}{V_0} + 1 + 1}
\]

\[
\eta_P \left( \frac{T_g}{m_0} + 2 \right) = 2
\]

\[
\eta_P \left( \frac{T_g}{m_0} + 2 V_0 \right) = 2
\]

\[
\eta_P = \frac{2}{\frac{T_g}{m_0} + 2 V_0}
\]

\[
\frac{V_0}{\eta_P} = \frac{T_g}{m_0} + 2 V_0
\]

and, finally, this goes into equation 1.20 to get

\[
\text{TSFC} = \left( \frac{T_g}{m_0} + 2 V_0 \right)
\]

\[\frac{2}{\eta_T \eta_{PR}}\]

c) For \( V_0 = 0 \) and 500 ft/s, plot the preceding equation for TSFC [in (lbm/h)/lbf] vs specific thrust \( T/m_0 \) [in lbf/(lbm/sec)] for values of specific thrust from 0 to 120. Use \( \eta_T = 0.4 \) and \( \eta_{PR} = 18,400 \) Btu/lbm.

It is very easy to mess up the units of this problem. Note that the input variable, \( T/m_0 \), is in lbf/(lbm/sec), but the output variable is in (lbm/hour)/lbf. You must multiply by the number of seconds in an hour to output the correct units. Also, you must use the conversion 1btu=778.16 ft·lbf.

You can set up an equation of the form \( \text{TSFC} = \left( 3600 \text{ s/h} \right) \left( \frac{X \text{ lbf}}{\text{lbm} \text{ s}} \right)^2 \left( \frac{0.4}{18,400 \text{ Btu/lbm}} \right) \), where \( X = \frac{T}{m_0} \) is the input variable.

Here's Matlab code you can use to make the plot:

\[ X=0:0.1:120; \]
\[ \text{TSFC0}=3600*(X*32.174+2*0)/(2*0.4*18400*778.16); \]
\[ \text{TSFC500}=3600*(X*32.174+2*500)/(2*0.4*18400*778.16); \]
\[ \text{plot}(X,\text{TSFC0},X,\text{TSFC500}) \]
The blue line is the $V_0 = 0$ line and the green line is the $V_0 = 500 \text{ ft/s}$ line.

**d) explain the trends**

You're on your own here. Say something about how TSFC is always higher when the inlet velocity is higher, and that TSFC increases linearly with specific thrust.
1.7 The JT9D high-bypass-ratio turbofan engine with \( V_0 = 0, \ P_0 = 14.696 \text{ psia}, \ T_0 = 518.7 \degree \text{R}, \) and \( \dot{m}_C = 247 \text{ lbm/s}, \ \dot{m}_B = 1248 \text{ lbm/s}, \ V_{Ce} = 1190 \text{ ft/s}, \ V_{Be} = 885 \text{ ft/s}, \ \dot{m}_f = 15750 \text{ lbm/hr}. \) Estimate the following assuming \( P_0 = P_e:\)

a) Thrust

From Problem 1.5, we learn that thrust for a bypass engine is equal to the sum of the thrust from the core and the bypass stream.

\[
\begin{align*}
F &= F_C + F_B \\
F_C &= \frac{1}{g_c} \left( (\dot{m}_C + \dot{m}_f) V_{Ce} - \dot{m}_C V_0 \right) \\
F_B &= \frac{\dot{m}_B}{g_c} (V_{Be} - V_0)
\end{align*}
\]

Putting all these together, we get an equation for the thrust for a bypass engine

\[
F = \frac{1}{g_c} \left( (\dot{m}_C + \dot{m}_f) V_{Ce} - \dot{m}_C V_0 + \dot{m}_B V_{Be} - \dot{m}_B V_0 \right)
\]

We have values for all the variables on the right hand side, so we can calculate thrust, but again be very careful with units.

\[
F = \frac{1}{32.174} \left[ \left( 247 \frac{\text{lbm}}{\text{s}} + 15750 \frac{\text{lbm}}{\text{hour}} \frac{\text{hour}}{3600 \text{s}} \right) \left( 1190 \frac{\text{ft}}{\text{s}} - 247 \frac{\text{lbm}}{\text{s}} \right) + 1248 \frac{\text{lbm}}{\text{s}} \left( 885 \frac{\text{ft}}{\text{s}} - 1248 \frac{\text{lbm}}{\text{s}} \right) \right]
\]

\[
F = 43626 \text{ lbf}
\]

b) Thermal efficiency, \( \eta_T, \) with \( h_{PR} \approx 18,400 \frac{\text{Btu}}{\text{lbm}}. \)

Equation 1.13 tells us that for a single inlet and single exhaust

\[
\eta_T = \frac{\dot{W}_{\text{out}}}{\dot{Q}_{\text{in}}}
\]

\[
\dot{W}_{\text{out, single exhaust}} = \frac{1}{2 g_c} \left[ (\dot{m}_o + \dot{m}_f) V_o^2 - \dot{m}_o V_o^2 \right]
\]

\[
\dot{Q}_{\text{in}} = \dot{m}_f h_{PR}
\]

here, though, we have two inlets and two exhausts, therefore

\[
\dot{W}_{\text{out, with bypass}} = \dot{W}_{\text{Cout}} + \dot{W}_{\text{Bout}}
\]

\[
\dot{W}_{\text{out}} = \frac{1}{2 g_c} \left[ (\dot{m}_C + \dot{m}_f) V_{Ce}^2 - \dot{m}_C V_0^2 \right] + \frac{1}{2 g_c} \left[ (\dot{m}_B) V_{Be}^2 - \dot{m}_B V_0^2 \right]
\]

but \( V_0 = 0, \) so

\[
\dot{W}_{\text{out}} = \frac{1}{2 g_c} \left[ (\dot{m}_C + \dot{m}_f) V_{Ce}^2 + \dot{m}_B V_{Be}^2 \right]
\]

inserting this into the first equation gives us the result that

\[
\eta_T = \frac{\left[ (\dot{m}_C + \dot{m}_f) V_{Ce}^2 + \dot{m}_B V_{Be}^2 \right]}{2 g_c \dot{m}_f h_{PR}}
\]
and now we can put in values

\[
\eta_T = \frac{\left(247 \ \frac{\text{lbm}}{\text{s}} + 15750 \ \frac{\text{lbm}}{\text{hr}} \frac{1 \text{ hr}}{3600 \text{ s}}\right)\left(1190 \ \frac{\text{ft}}{\text{s}}\right)^2 + 1248 \ \frac{\text{lbm}}{\text{s}} \left(885 \ \frac{\text{ft}}{\text{s}}\right)^2}{2 \left(32.174 \ \frac{\text{lbm}}{\text{hr} \frac{1 \text{ hr}}{3600 \text{ s}}}\right)\left(15750 \ \frac{\text{lbm}}{\text{hr}} \frac{1 \text{ hr}}{3600 \text{ s}}\right)\left(18,400 \ \frac{\text{Btu}}{\text{lbm}} \frac{778.16 \text{ Btu}}{\text{lb}}\right)}
\]

\[
\eta_T = 0.3308 \quad \eta_T = 33.08\%
\]

c) Propulsive efficiency, \(\eta_P\), and uninstalled thrust specific fuel consumption, \(S\)

As defined in equation 1.14,

\[
\eta_P = \frac{T}{W_{\text{out}}}
\]

And since \(V_0 = 0\),

\[
\eta_P = 0
\]

From equation 1.10,

\[
S = \frac{m_f}{F}
\]

\[
S = \frac{15750 \ \frac{\text{lbm}}{\text{hr}}}{43,626 \ \text{lb} \ \text{ft}} = 0.361 \ \frac{\text{lbm/hr}}{\text{lb} \ \text{ft}} = 1.0028 \cdot 10^{-4} \ \frac{\text{lbm/s}}{\text{lb} \ \text{ft}}
\]
1.14 An aircraft with wind area 800 ft$^2$ in level flight at maximum $C_L / C_D$. $C_{D0} = 0.02$, $K_2 = 0$, $K_1 = 0.2$, find 

a) The maximum $C_L / C_D$ and corresponding $C_L$ and $C_D$

Equation 1.48 is used here

$$\left( \frac{C_L}{C_D} \right)^* = \frac{1}{2 \sqrt{C_{D0}K_1 + K_2}}$$

$$\left( \frac{C_L}{C_D} \right)^* = \frac{1}{2 \sqrt{(0.02)(0.2) + 0}}$$

$$\left( \frac{C_L}{C_D} \right)^* = 7.906$$

Also, equation 1.47 says that

$$C_L^* = \sqrt{C_{D0} / K_1}$$

$$C_L^* = \sqrt{0.02 / 0.2}$$

$$C_L^* = 0.3162$$

now we can find $C_D^*$

$$\frac{0.3162}{C_D^*} = 7.906$$

$$C_D^* = 0.04$$

b) The flight altitude and drag for aircraft weight of 45,000 lbf and Mach 0.8. Use eqns 1.29 and 1.30b.

Equations 1.29 and 1.30b are

$$L = n W = C_L q S_w$$

$$q = \frac{\gamma}{2} PM_0^2 = \frac{\gamma}{2} P_{\text{ref}} M_0^2$$

Solving for $\delta$ and combining the two equations,

$$\delta = \frac{2q}{\gamma P_{\text{ref}} M_0^2}$$

$$\delta = \frac{2(\frac{sW}{C_L S_w})}{\gamma P_{\text{ref}} M_0^2}$$

$$\delta = \frac{2n W}{\gamma P_{\text{ref}} M_0^2 C_L S_w}$$

Now we can plug in values

$$\delta = \frac{2(1)(45000 \text{ lbf})}{1.4(14.7 \frac{\text{ lbf}}{\text{ft}^2}) \left( \frac{144 \text{ in}^2}{\text{ ft}^2} \right) (0.8)^2 (0.3162)(800 \text{ ft}^2)}$$

$$\delta = 0.1876$$
Use Appendix A to see that this corresponds to an altitude of about 39,800 ft.

Next, equation 1.31 gives us drag

\[ D = C_D q S_w \]

\[ D = C_D \frac{n W}{C_L} \]

\[ D = 0.04 \left(1 \frac{(45,000 \text{ lbf})}{0.3162}\right) \]

\[ D = 5692.6 \text{ lbf} \]

c) Flight altitude and drag for an aircraft of weight 35,000 lbf and Mach 0.8

Analysis is nearly identical to part b, with only a change in weight.

\[ \delta = \frac{2 n W}{\gamma P_{ref} M_0^2 C_L S_w} \]

\[ \delta = \frac{2 \times 14.7 \left(144 \text{ in}^2 \right)^{0.8} \left(800 \text{ ft}^2\right)}{1.4 \left(14.7 \text{ lbf} / \text{in}^2 \right)^{0.8} \left(800 \text{ ft}^2\right)} \]

\[ \delta = 0.1459 \]

Again, use Appendix A to see that this corresponds to an altitude of about 45,000 ft.

Also,

\[ D = C_D \frac{n W}{C_L} \]

\[ D = 0.04 \left(1 \frac{(35,000 \text{ lbf})}{0.3162}\right) \]

\[ D = 4427.6 \text{ lbf} \]

d) Range for an installed engine TSFC rate of 0.8 (lbm/hr)/lbf, if the 10,000-lbf difference in aircraft weight between parts b and c is due only to fuel consumption.

First start with equation 1.43 for range factor

\[ RF = \frac{C_L}{C_D} \frac{V}{\text{TSFC}} \frac{g_c}{g_0} \]

We need to calculate velocity in ft/s rather than Mach. Use appendix A and the note that speed of sound \(a=\text{a}_{\text{std}} \sqrt{\theta}\).

\[ V = a M \]

\[ V = 1116 \frac{\text{ft}}{\text{s}} \sqrt{0.7519} \times 0.8 \]

\[ V = 774 \frac{\text{ft}}{\text{s}} \]
Now we can go back to RF

\[
RF = \frac{0.3162}{0.04} \cdot \frac{774 \text{ ft}}{774 \text{ hr}} \cdot \frac{32.174 \text{ lbm} \cdot \text{ft}}{32.174 \text{ lbm} \cdot \text{ft}^{-2} \cdot \text{hr}^{-2}}
\]

\[
RF = 27533.115 \text{ ft}\n\]

\[
RF = 4528.5 \text{ nm}\n\]

Next, we use equation 1.45a to find the range, \(s\)

\[
\frac{W_f}{W_i} = \exp\left(-\frac{s}{RF}\right)
\]

\[
s = RF \ln\left(\frac{W_f}{W_i}\right)
\]

\[
s = 4528.5 \text{ nm} \ln\left(\frac{45000 \text{ lbf}}{35000 \text{ lbf}}\right)
\]

\[
s = 1138 \text{ nm}\]

\[
= 1309.6 \text{ mi}\n\]
2.3 Consider the flow shown in Figure P2.2. It has radius $r_0$, velocity $V_1$, and pressure $P_1$. The fluid leaves with a velocity

$$V_2 = V_{\text{max}} \left[ 1 - \left( \frac{r}{r_0} \right)^2 \right]$$

with uniform pressure $P_2$. Use the conservation of mass and momentum equations to show that the force necessary to hold the pipe in place can be written as

$$F = \pi r_0^2 \left( P_1 - P_2 + \frac{\rho V_2^2}{3g_c} \right)$$

Start with equation 2.20

$$\sum F_\sigma = \frac{1}{g_c} \left( \frac{dM}{dt} + \dot{M}_\text{out} - \dot{M}_\text{in} \right)$$

by assuming conservation of mass (since there is no sink or source), $\frac{dM}{dt} = 0$.

$$\sum F_\sigma = \frac{1}{g_c} (\dot{M}_\text{out} - \dot{M}_\text{in})$$

The sum of the forces is

$$\sum F_\sigma = -F + (P_1 - P_2) A$$

Where $F$ is the force required to keep the pipe in place and $A$ is the area of the pipe.

We can combine these two equations to get

$$\sum F_\sigma = \frac{1}{g_c} (\dot{M}_\text{out} - \dot{M}_\text{in}) = -F + (P_1 - P_2) A$$

$$F = \frac{1}{g_c} (\dot{M}_\text{in} - \dot{M}_\text{out}) + (P_1 - P_2) A$$

These equations are wrong.

However, the first part of this equation does not make sense. Force should not be proportional to $\dot{M}_\text{in} - \dot{M}_\text{out}$ because that would imply that if $\dot{M}_\text{in} > \dot{M}_\text{out}$, the force would be positive. However, $\dot{M}_\text{in} > \dot{M}_\text{out}$ implies that air is moving from back to front, which should decrease the force required to keep the pipe steady. This means that we have defined our sum of forces to be in a different direction in each of the equations.

Instead, the equation should read:

$$F = \frac{1}{g_c} (\dot{M}_\text{out} - \dot{M}_\text{in}) + (P_1 - P_2) A$$
Momentum flux can be obtained by integrating

\[
\dot{M}_{\text{out}} = \dot{M}_2 = \int_{m_2} V_2 \, d\dot{m}
\]

\[
\dot{M}_2 = \int_{0}^{r_0} \rho \, V_2 \, 2 \pi \, r \, dr
\]

\[
\dot{M}_2 = \int_{0}^{r_0} \rho \, V_{\text{max}}^2 \left(1 - \left(\frac{r}{r_0}\right)^4\right)^2 \, 2 \pi \, r \, dr
\]

\[
\dot{M}_2 = 2 \pi \, \rho \, V_{\text{max}}^2 \int_{0}^{r_0} \left(1 - \left(\frac{r}{r_0}\right)^4\right) \, r \, dr
\]

\[
\dot{M}_2 = 2 \pi \, \rho \, V_{\text{max}}^2 \int_{0}^{r_0} \left(r - 2 \frac{r^3}{4r_0^3} + \frac{r^6}{6r_0^6}\right) \, dr
\]

\[
\dot{M}_2 = 2 \pi \, \rho \, V_{\text{max}}^2 \left[\frac{r_0^2}{2} - 2 \frac{r^3}{4r_0^3} + \frac{r^6}{6r_0^6}\right]
\]

\[
\dot{M}_2 = \frac{\pi \, \rho \, V_{\text{max}}^2 \, r_0^2}{3}
\]

Similarly,

\[
\dot{M}_{\text{in}} = \dot{M}_1 = \int_{m_1} V_1 \, d\dot{m}
\]

\[
\dot{M}_1 = \int_{0}^{r_0} \rho \, V_1 \, 2 \pi \, r \, dr
\]

\[
\dot{M}_1 = \rho \, V_1 \, 2 \pi \left[\int_{0}^{r_0} \right] \, dr
\]

\[
\dot{M}_1 = \rho \, V_1 \, 2 \pi \, r_0^2
\]

And plugging into the equation for force above

\[
F = \frac{1}{g_c} \left(\dot{M}_{\text{out}} - \dot{M}_{\text{in}}\right) + (P_1 - P_2) \, A
\]

\[
F = \pi \, r_0^2 \left(P_1 - P_2 - \frac{\rho}{g_c} \left(\frac{V_{\text{max}}^2}{3} - V_1^2\right)\right)
\]

Conservation of mass can be used to find \(V_{\text{max}}\)

\[
\dot{m}_1 = \dot{m}_2
\]

\[
\pi \, r_0^2 \, \rho \, V_1 = \rho \int_{0}^{r_0} V_{\text{max}} \left\{1 - \left(\frac{r}{r_0}\right)^2\right\} \, 2 \pi \, r \, dr
\]

\[
V_1 = \frac{2 \, V_{\text{max}}}{r_0^2} \int_{0}^{r_0} \left(1 - \left(\frac{r}{r_0}\right)^2\right) \, dr
\]

\[
V_1 = \frac{2 \, V_{\text{max}}}{r_0^2} \left[\frac{r^2}{2} - \frac{r^4}{4r_0^4}\right]_{0}^{r_0}
\]

\[
V_1 = \frac{2 \, V_{\text{max}}}{r_0^2} \left[\frac{r_0^2}{2} - \frac{r_0^4}{4r_0^4}\right]
\]

\[
V_1 = \frac{V_{\text{max}}}{2}
\]

\[
V_{\text{max}} = 2 \, V_1
\]
And we arrive at

\[ F = \pi r_0^2 \left\{ P_1 - P_2 + \frac{\rho}{\gamma} \left( \frac{(2V_1)^2}{3} - V_1^2 \right) \right\} \]

\[ F = \pi r_0^2 \left\{ P_1 - P_2 + \frac{\rho V_1^2}{g_c} \left( \frac{4}{3} - 1 \right) \right\} \]

\[ F = \pi r_0^2 \left\{ P_1 - P_2 + \frac{\rho V_1^2}{g_c} \left( \frac{4}{3} - 1 \right) \right\} \]
2.6 Using Figure P2.5, with 1500 lbm/s of air at 60°F and 14.7 psia entering the engine at a velocity of 450 ft/s and that 1250 lbm/s of bypass air leaves the engine at 60° to the horizontal, at a velocity of 890 ft/s and pressure of 14.7 psia. The remaining 250 lbm/s leaves the engine core at a velocity of 1200 ft/s and pressure of 14.7 psia. Determine the force on the strut, $F_x$. Assume an ambient pressure of 14.7 psia.

By symmetry, there is no net force in the y-direction. The momentum equation in the x-direction is

$$
\sum F_x = F_x = \frac{1}{\rho c} (\dot{M}_{x\text{ core out}} + \dot{M}_{x\text{ fan out}} - \dot{M}_{x\text{ in}})
$$

$$
\dot{M}_{x\text{ in}} = (1500 \text{ lbm/s})(450 \text{ ft/s}) = 675000 \text{ ft-lbm/s}^2
$$

$$
\dot{M}_{x\text{ core out}} = (250 \text{ lbm/s})(1200 \text{ ft/s}) = 300000 \text{ ft-lbm/s}^2
$$

$$
\dot{M}_{x\text{ fan out}} = (-1250 \text{ lbm/s})(890 \text{ ft/s}) \cos(60°) = -556250 \text{ ft-lbm/s}^2
$$

Therefore, the force is

$$
F_x = \frac{1}{32.174 \text{ lbm ft}/\text{lbm s}^2} \left[ 300000 \text{ ft-lbm/s}^2 + (-556250 \text{ ft-lbm/s}^2) - 675000 \text{ ft-lbm/s}^2 \right]
$$

$$
F_x = -28,945 \text{ lbf}
$$

This means a force of 28,945 lbf slowing the plane.