CHAPTER 2

Introduction to Conduction
Recall that conduction refers to the transport of energy in a medium due to a temperature gradient, and the physical mechanism is one of random atomic or molecular activity. In Chapter 1 we learned that conduction heat transfer is governed by Fourier's law and that use of the law to determine the heat flux depends on knowledge of the manner in which temperature varies within the medium (the temperature distribution). By way of introduction, we restricted our attention to simplified conditions (one-dimensional, steady-state conduction in a plane wall) for which the temperature distribution is readily deduced to be linear. However, Fourier's law is applicable to transient, multidimensional conduction in complex geometries for which the nature of the temperature distribution is not evident.

The objectives of this chapter are twofold. First, we wish to develop a deeper understanding of Fourier's law. What are its origins? What form does it take for different geometries? How does its proportionality constant (the thermal conductivity) depend on the physical nature of the medium? Our second objective is to develop, from basic principles, the general equation, termed the heat equation, which governs the temperature distribution in a medium. It is the solution to this equation that provides knowledge of the temperature distribution, which may then be used with Fourier's law to determine the heat flux.

2.1
The Conduction Rate Equation

Although the conduction rate equation, Fourier's law, was introduced in Section 1.2, it is now appropriate to consider its origin. Fourier's law is phenomenological; that is, it is developed from observed phenomena rather than being derived from first principles. Hence, we view the rate equation as a generalization based on much experimental evidence. For example, consider the steady-state conduction experiment of Figure 2.1. A cylindrical rod of known material is insulated on its lateral surface, while its end faces are maintained at different temperatures, with \( T_1 > T_2 \). The temperature difference causes conduction heat transfer in the positive \( x \) direction. We are able to measure the heat transfer rate \( q_x \), and we seek to determine how \( q_x \) depends on the following variables: \( \Delta T \), the temperature difference; \( \Delta x \), the rod length; and \( A \), the cross-sectional area.

We might imagine first holding \( \Delta T \) and \( \Delta x \) constant and varying \( A \). If we do so, we find that \( q_x \) is directly proportional to \( A \). Similarly, holding \( \Delta T \) and \( A \) constant, we observe that \( q_x \) varies inversely with \( \Delta x \). Finally, holding \( A \) and \( \Delta x \) constant, we find that \( q_x \) is directly proportional to \( \Delta T \). The collective effect is then

\[
q_x \propto A \frac{\Delta T}{\Delta x}
\]

**Figure 2.1**
Steady-state heat conduction experiment.
In changing the material (e.g., from a metal to a plastic), we would find that the above proportionality remains valid. However, we would also find that, for equal values of \( A \), \( \Delta x \), and \( \Delta T \), the value of \( q_x \) would be smaller for the plastic than for the metal. This suggests that the proportionality may be converted to an equality by introducing a coefficient that is a measure of the material behavior. Hence, we write

\[
q_x = kA \frac{\Delta T}{\Delta x}
\]

where \( k \), the thermal conductivity (\( \text{W/m \cdot K} \)), is an important property of the material. Evaluating this expression in the limit as \( \Delta x \to 0 \), we obtain for the heat rate

\[
q_x = -kA \frac{dT}{dx}
\]

or for the heat flux

\[
q_x'' = \frac{q_x}{A} = -k \frac{dT}{dx}
\]

(2.1)

Recall that the minus sign is necessary because heat is always transferred in the direction of decreasing temperature.

Fourier’s law, as written in Equation 2.2, implies that the heat flux is a directional quantity. In particular, the direction of \( q_x'' \) is normal to the cross-sectional area \( A \). Or, more generally, the direction of heat flow will always be normal to a surface of constant temperature, called an isothermal surface. Figure 2.2 illustrates the direction of heat flow \( q_x'' \) in a plane wall for which the temperature gradient \( \frac{dT}{dx} \) is negative. From Equation 2.2, it follows that \( q_x'' \) is positive. Note that the isothermal surfaces are planes normal to the \( x \) direction.

Recognizing that the heat flux is a vector quantity, we can write a more general statement of the conduction rate equation (Fourier’s law) as follows:

\[
q^n = -k \nabla T = -k \left( \frac{\partial T}{\partial x} \hat{i} + \frac{\partial T}{\partial y} \hat{j} + \frac{\partial T}{\partial z} \hat{k} \right)
\]

(2.3)

where \( \nabla \) is the three-dimensional del operator and \( T(x, y, z) \) is the scalar temperature field. It is implicit in Equation 2.3 that the heat flux vector is in a direction perpendicular to the isothermal surfaces. An alternative form of Fourier’s law is therefore

\[
q^n_n = -k \frac{\partial T}{\partial n}
\]

(2.4)

where \( q^n_n \) is the heat flux in a direction \( n \), which is normal to an isotherm, as shown for the two-dimensional case in Figure 2.3. The heat transfer is sustained by a temperature gradient along \( n \). Note also that the heat flux vector can be resolved into components such that, in Cartesian coordinates, the general expression for \( q^n \) is

\[
q^n = iq_x^n + jq_y^n + kq_z^n
\]

(2.5)
where, from Equation 2.3, it follows that

\[ q_x = -k \frac{\partial T}{\partial x}, \quad q_y = -k \frac{\partial T}{\partial y}, \quad q_z = -k \frac{\partial T}{\partial z} \]  \quad (2.6)

Each of these expressions relates the heat flux **across a surface** to the temperature gradient in a direction perpendicular to the surface. It is also implicit in Equation 2.3 that the medium in which the conduction occurs is isotropic. For such a medium the value of the thermal conductivity is independent of the coordinate direction.

Fourier's law is the cornerstone of conduction heat transfer, and its key features are summarized as follows. It is **not** an expression that may be derived from first principles; it is instead a generalization based on experimental evidence. It is an expression that **defines** an important material property, the thermal conductivity. In addition, Fourier's law is a vector expression indicating that the heat flux is normal to an isotherm and in the direction of decreasing temperature. Finally, note that Fourier's law applies for all matter, regardless of its state (solid, liquid, or gas).

### 2.2

**The Thermal Properties of Matter**

To use Fourier's law, the thermal conductivity of the material must be known. This property, which is referred to as a **transport property**, provides an indication of the rate at which energy is transferred by the diffusion process. It depends on the physical structure of matter, atomic and molecular, which is related to the state of the matter. In this section we consider various forms of matter, identifying important aspects of their behavior and presenting typical property values.

#### 2.2.1 Thermal Conductivity

From Fourier's law, Equation 2.6, the thermal conductivity associated with conduction in the \( x \)-direction is defined as

\[ k_x = -\frac{q_x}{(\partial T/\partial x)} \]

Similar definitions are associated with thermal conductivities in the \( y \)- and \( z \)-directions \((k_y, k_z)\), but for an isotropic material the thermal conductivity is independent of the direction of transfer, \( k_y = k_z = k_x = k \).

From the foregoing equation, it follows that, for a prescribed temperature gradient, the conduction heat flux increases with increasing thermal conductivity. In general, the thermal conductivity of a solid is larger than that of a liquid, which is larger than that of a gas. As illustrated in Figure 2.4, the thermal conductivity of a solid may be more than four orders of magnitude larger than that of a gas. This trend is due largely to differences in intermolecular spacing for the two states.

*The Solid State* In the modern view of materials, a solid may be comprised of free electrons and atoms bound in a periodic arrangement called the lattice.
Accordingly, transport of thermal energy may be due to two effects: the migration of free electrons and lattice vibrational waves. When viewed as a particle-like phenomenon, the lattice vibration quanta are termed phonons. In pure metals, the electron contribution to conduction heat transfer dominates, while in nonconductors and semiconductors, the phonon contribution is dominant.

Kinetic theory yields the following expression for the thermal conductivity [1]:

\[ k = \frac{1}{3} C \bar{c} \lambda_{mfp} \]  

(2.7)

For conducting materials such as metals, \( C = C_e \) is the electron specific heat per unit volume, \( \bar{c} \) is the mean electron velocity, and \( \lambda_{mfp} = \lambda_e \) is the electron mean free path, which is defined as the average distance traveled by an electron before it collides with either an imperfection in the material or with a phonon. In nonconducting solids, \( C = C_{ph} \) is the phonon specific heat, \( \bar{c} \) is the average speed of sound, and \( \lambda_{mfp} = \lambda_{ph} \) is the phonon mean free path, which again is determined by collisions with imperfections or other phonons. In all cases, the thermal conductivity increases as the mean free path of the energy carriers (electrons or phonons) is increased.

When electrons and phonons carry thermal energy leading to conduction heat transfer in a solid, the thermal conductivity may be expressed as

\[ k = k_e + k_{ph} \]  

(2.8)

To a first approximation, \( k_e \) is inversely proportional to the electrical resistivity, \( \rho_e \). For pure metals, which are of low \( \rho_e \), \( k_e \) is much larger than \( k_{ph} \). In contrast, for alloys, which are of substantially larger \( \rho_e \), the contribution of \( k_{ph} \) to \( k \) is no longer negligible. For nonmetallic solids, \( k \) is determined primarily by \( k_{ph} \), which increases as the frequency of interactions between the atoms and the lattice decreases. The regularity of the lattice arrangement has an important effect on \( k_{ph} \) with crystalline (well-ordered) materials like quartz having a higher thermal conductivity than amorphous materials like glass. In fact, for crystalline, nonmetallic solids such as diamond and beryllium oxide, \( k_{ph} \) can be quite large, exceeding values of \( k \) associated with good conductors, such as aluminum.
The temperature dependence of \( k \) is shown in Figure 2.5 for representative metallic and nonmetallic solids. Values for selected materials of technical importance are also provided in Table A.1 (metallic solids) and Tables A.2 and A.3 (nonmetallic solids). More detailed treatments of thermal conductivity are available in the literature [2].

**The Solid State: Micro- and Nanoscale Effects** In the preceding discussion, the bulk thermal conductivity is described, and the thermal conductivity values listed in Tables A.1 through A.3 are appropriate for use when the physical dimensions of the material of interest are relatively large. This is the case in many commonplace engineering problems. However, in several areas of technology, such as microelectronics, the material’s characteristic dimensions can be on the order of micrometers or nanometers, in which case care must be taken to account for the possible modifications of \( k \) that can occur as the physical dimensions become small.

Cross sections of films of the same material having thicknesses \( L_1 \) and \( L_2 \) are shown in Figure 2.6. Electrons or phonons that are associated with conduction of thermal energy are also shown qualitatively. Note that the physical boundaries of the film act to scatter the energy carriers and redirect their propagation. For large \( L/L_{ref} \) (Figure 2.6a), the effect of the boundaries on reducing the average energy carrier path length is minor, and conduction heat transfer occurs as described for bulk materials. However, as the film becomes thinner, the physical boundaries of the material can decrease the average net distance traveled by the energy carriers, as
shown in Figure 2.6b. Moreover, electrons and phonons moving in the thin $y$-direction (representing conduction in the $y$-direction) are affected by the boundaries to a more significant degree than energy carriers moving in the $x$-direction. As such, for films characterized by small $L \lambda_{mfp}$, we find that $k_y < k_x < k_z$, where $k$ is the bulk thermal conductivity of the film material.

For $L \lambda_{mfp} \geq 1$, predicted values of $k_x$ and $k_y$ may be estimated to within 20% from the following expression [1]:

\[ k_x/k = 1 - 2\lambda_{mfp}/(3\pi L) \quad (2.9a) \]
\[ k_y/k = 1 - \lambda_{mfp}/(3L) \quad (2.9b) \]

Equations 2.9a, b reveal that the values of $k_x$ and $k_y$ are within approximately 5% of the bulk thermal conductivity if $L \lambda_{mfp} > 7$ (for $k_x$) and $L \lambda_{mfp} > 4.5$ (for $k_y$). Values of the mean free path as well as critical film thicknesses below which microscale effects must be considered, $L_{crit}$, are included in Table 2.1 for several materials at $T = 300$ K. For films with $\lambda_{mfp} < L < L_{crit}$, $k_x$ and $k_y$ are reduced from the bulk value as indicated in Equations 2.9a, b. No general guidelines exist for predicting values of the thermal conductivities for $L \lambda_{mfp} < 1$. Note that, in solids, the value of $\lambda_{mfp}$ decreases as the temperature increases.

In addition to scattering from physical boundaries, as in the case of Figure 2.6b, energy carriers may be redirected by chemical dopants embedded within a material or by grain boundaries that separate individual clusters of material in otherwise homogeneous matter. Nanostructured materials are chemically identical to their conventional counterparts but are thermally processed to provide very small grain sizes in the final material and, from the heat transfer perspective, significantly increase the scattering and reflection of energy carriers at the grain boundaries.

<table>
<thead>
<tr>
<th>Table 2.1 Mean free path and critical film thickness for various materials at $T \approx 300$ K [3,4]</th>
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</thead>
<tbody>
<tr>
<td>Material</td>
</tr>
<tr>
<td>-------------------------------</td>
</tr>
<tr>
<td>Aluminum oxide</td>
</tr>
<tr>
<td>Diamond (IIa)</td>
</tr>
<tr>
<td>Gallium arsenide</td>
</tr>
<tr>
<td>Gold</td>
</tr>
<tr>
<td>Silicon</td>
</tr>
<tr>
<td>Silicon dioxide</td>
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<tr>
<td>Yttria-stabilized zirconia</td>
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</table>
Measured values of the thermal conductivity of a bulk, nanostructured yttria-stabilized zirconia material are shown in Figure 2.7. This particular ceramic is widely used for insulation purposes in high-temperature combustion devices such as gas turbine engines. Conduction is dominated by phonon transfer, and the mean free path of the phonon energy carriers is, from Table 2.1, \( \lambda_{\text{mfp}} = 25 \text{ nm} \) at 300 K. As the grain sizes are reduced to characteristic dimensions less than 25 nm (and more grain boundaries are introduced in the material per unit volume), significant reduction of the thermal conductivity occurs. Extrapolation of the results of Figure 2.7 to higher temperatures is not recommended, since the mean free path decreases with increasing temperature (\( \lambda_{\text{mfp}} \approx 4 \text{ nm} \) at \( T = 1525 \text{ K} \)) and grains of the material may coalesce, merge, and enlarge at elevated temperatures. Therefore, \( L/\lambda_{\text{mfp}} \) becomes larger at high temperatures, and reduction of \( k \) due to nanoscale effects becomes less pronounced.

**The Fluid State** The fluid state includes both liquids and gases. Since the intermolecular spacing is much larger and the motion of the molecules is more random for the fluid state than for the solid state, thermal energy transport is less effective. The thermal conductivity of gases and liquids is therefore generally smaller than that of solids.

The effect of temperature, pressure, and chemical species on the thermal conductivity of a gas may be explained in terms of the kinetic theory of gases [5]. From this theory it is known that the thermal conductivity is directly proportional to the density of the gas, the mean molecular speed \( \bar{c} \), and the mean free path \( \lambda_{\text{mfp}} \), which is the average distance traveled by an energy carrier (a molecule) before experiencing a collision.

\[
k = \frac{1}{3} \bar{c} \rho \bar{c} \lambda_{\text{mfp}}
\]  \hspace{1cm} (2.10)

Because \( \bar{c} \) increases with increasing temperature and decreasing molecular weight, the thermal conductivity of a gas increases with increasing temperature and decreasing
molecular weight. These trends are shown in Figure 2.8. However, because \( \rho \) and \( \lambda_{\text{eff}} \) are directly and inversely proportional to the gas pressure, respectively, the thermal conductivity is independent of pressure except in extreme cases as, for example, when conditions approach that of a perfect vacuum. Therefore, the assumption that \( k \) is independent of gas pressure for large volumes of gas is appropriate for the pressures of interest in this text. Accordingly, although the values of \( k \) presented in Table A.4 pertain to atmospheric pressure or the saturation pressure corresponding to the prescribed temperature, they may be used over a much wider pressure range.

Molecular conditions associated with the liquid state are more difficult to describe, and physical mechanisms for explaining the thermal conductivity are not well understood [6]. The thermal conductivity of nonmetallic liquids generally decreases with increasing temperature. As shown in Figure 2.9, water, glycerine, and engine oil are notable exceptions. The thermal conductivity of liquids is usually insensitive to pressure except near the critical point. Also, it is generally true that thermal conductivity decreases with increasing molecular weight. Values of the thermal conductivity are often tabulated as a function of temperature for the saturated state of the liquid. Tables A.5 and A.6 present such data for several common liquids.

Liquid metals are commonly used in high heat flux applications, such as occur in nuclear power plants. The thermal conductivity of such liquids is given in Table A.7. Note that the values are much larger than those of the nonmetallic liquids [7].

**The Fluid State: Micro- and Nanoscale Effects**  As in the solid state, the bulk thermal conductivity may be modified as the characteristic dimensions of the
system become small, in particular for small values of $L\lambda_{mfp}$. As for the situation shown in Figure 2.6b, the mean free path of the molecules becomes restricted when the fluid is, for example, contained in an enclosure of small physical dimension.

**Insulation Systems** Thermal insulations consist of low thermal conductivity materials combined to achieve an even lower system thermal conductivity. In conventional *fiber-, powder-, and flake*-type insulations, the solid material is finely dispersed throughout an air space. Such systems are characterized by an effective thermal conductivity, which depends on the thermal conductivity and surface radiative properties of the solid material, as well as the nature and volumetric fraction of the air or void space. A special parameter of the system is its bulk density (solid mass/total volume), which depends strongly on the manner in which the material is packed.

If small voids or hollow spaces are formed by bonding or fusing portions of the solid material, a rigid matrix is created. When these spaces are sealed from each other, the system is referred to as a *cellular* insulation. Examples of such rigid insulations are *foamed* systems, particularly those made from plastic and glass materials. Reflective insulations are composed of multilayered, parallel, thin sheets or foils of high reflectivity, which are spaced to reflect radiant energy back to its source. The spacing between the foils is designed to restrict the motion of air, and in high-performance insulations, the space is evacuated. In all types of insulation, evacuation of the air in the void space will reduce the effective thermal conductivity of the system.

It is important to recognize that heat transfer through any of these insulation systems may include several modes: conduction through the solid materials; conduction
or convection through the air in the void spaces; and radiation exchange between the surfaces of the solid matrix. The effective thermal conductivity accounts for all of these processes, and values for selected insulation systems are summarized in Table A.3. Additional background information and data are available in the literature [8, 9].

As with thin films, micro- and nanoscale effects can influence the effective thermal conductivity of insulating materials. The value of \( k \) for a nanostructured silica aerogel material that is composed of approximately 5\% by volume solid material and 95\% by volume air that is trapped within pores of \( L = 20 \) nm is shown in Figure 2.10. Note that at \( T \approx 300 \) K, the mean free path for air at atmospheric pressure is approximately 80 nm. As the gas pressure is reduced, \( \lambda_{mfp} \) would increase for an unconfined gas, but the molecular motion of the trapped air is restricted by the walls of the small pores and \( k \) is reduced to extremely small values relative to the thermal conductivities of conventional matter reported in Figure 2.4.

### 2.2.2 Other Relevant Properties

In our analysis of heat transfer problems, it will be necessary to use several properties of matter. These properties are generally referred to as thermophysical properties and include two distinct categories, transport and thermodynamic properties. The transport properties include the diffusion rate coefficients such as \( k \), the thermal conductivity (for heat transfer), and \( \nu \), the kinematic viscosity (for momentum transfer). Thermodynamic properties, on the other hand, pertain to the equilibrium state of a system. Density (\( \rho \)) and specific heat (\( c_p \)) are two such properties used extensively in thermodynamic analysis. The product \( \rho c_p \) (J/m\(^2\) · K), commonly termed the volumetric heat capacity, measures the ability of a material to store thermal energy. Because substances of large density are typically characterized by small specific heats, many solids and liquids, which are very good energy storage media, have comparable heat capacities (\( \rho c_p > 1 \) MJ/m\(^3\) · K). Because of their very small densities, however, gases are poorly suited for thermal energy storage (\( \rho c_p \approx 1\) kJ/m\(^3\) · K). Densities and specific heats are provided in the tables of Appendix A for a wide range of solids, liquids, and gases.
In heat transfer analysis, the ratio of the thermal conductivity to the heat capacity is an important property termed the thermal diffusivity \( \alpha \), which has units of \( \text{m}^2/\text{s} \):

\[
\alpha = \frac{k}{\rho c_p}
\]

It measures the ability of a material to conduct thermal energy relative to its ability to store thermal energy. Materials of large \( \alpha \) will respond quickly to changes in their thermal environment, while materials of small \( \alpha \) will respond more sluggishly, taking longer to reach a new equilibrium condition.

The accuracy of engineering calculations depends on the accuracy with which the thermophysical properties are known [11–13]. Numerous examples could be cited of flaws in equipment and process design or failure to meet performance specifications that were attributable to misinformation associated with the selection of key property values used in the initial system analysis. Selection of reliable property data is an integral part of any careful engineering analysis. The casual use of data that have not been well characterized or evaluated, as may be found in some literature or handbooks, is to be avoided. Recommended data values for many thermophysical properties can be obtained from Reference 14. This reference, available in most institutional libraries, was prepared by the Thermophysical Properties Research Center (TPRC) at Purdue University.

**Example 2.1**

The thermal diffusivity \( \alpha \) is the controlling transport property for transient conduction. Using appropriate values of \( k \), \( \rho \), and \( c_p \) from Appendix A, calculate \( \alpha \) for the following materials at the prescribed temperatures: pure aluminum, 300 and 700 K; silicon carbide, 1000 K; paraffin, 300 K.

**Solution**

**Known:** Definition of the thermal diffusivity \( \alpha \).

**Find:** Numerical values of \( \alpha \) for selected materials and temperatures.

**Properties:** Table A.1, pure aluminum (300 K):

\[
\begin{align*}
\rho &= 2702 \text{ kg/m}^3 \\
\rho c_p &= 903 \text{ J/kg \cdot K} \\
k &= 237 \text{ W/m \cdot K}
\end{align*}
\]

\[
\alpha = \frac{k}{\rho c_p} = \frac{237 \text{ W/m \cdot K}}{2702 \text{ kg/m}^3 \times 903 \text{ J/kg \cdot K}} = 97.1 \times 10^{-6} \text{ m}^2/\text{s}
\]

Table A.1, pure aluminum (700 K):

\[
\begin{align*}
\rho &= 2702 \text{ kg/m}^3 & \text{at 300 K} \\
c_p &= 1090 \text{ J/kg \cdot K} & \text{at 700 K (by linear interpolation)} \\
k &= 225 \text{ W/m \cdot K} & \text{at 700 K (by linear interpolation)}
\end{align*}
\]
Hence

\[ \alpha = \frac{k}{\rho c_p} = \frac{225 \text{ W/m} \cdot \text{K}}{2702 \text{ kg/m}^3 \times 1090 \text{ J/kg} \cdot \text{K}} = 76 \times 10^{-6} \text{ m}^2/\text{s} \]

Table A.2, silicon carbide (1000 K):

\[ \begin{align*}
\rho &= 3160 \text{ kg/m}^3 \\
c_p &= 1195 \text{ J/kg} \cdot \text{K} \\
k &= 87 \text{ W/m} \cdot \text{K}
\end{align*} \text{ at 300 K} \]

\[ \alpha = \frac{87 \text{ W/m} \cdot \text{K}}{3160 \text{ kg/m}^3 \times 1195 \text{ J/kg} \cdot \text{K}} = 23 \times 10^{-6} \text{ m}^2/\text{s} \]

Table A.3, paraffin (300 K):

\[ \begin{align*}
\rho &= 900 \text{ kg/m}^3 \\
c_p &= 2890 \text{ J/kg} \cdot \text{K} \\
k &= 0.24 \text{ W/m} \cdot \text{K}
\end{align*} \text{ at 1000 K} \]

\[ \alpha = \frac{k}{\rho c_p} = \frac{0.24 \text{ W/m} \cdot \text{K}}{900 \text{ kg/m}^3 \times 2890 \text{ J/kg} \cdot \text{K}} = 9.2 \times 10^{-8} \text{ m}^2/\text{s} \]

**Comments:**

1. Note temperature dependence of the thermophysical properties of aluminum and silicon carbide. For example, for silicon carbide, \( \alpha(1000 \text{ K}) \approx 0.1 \times \alpha(300 \text{ K}) \); hence properties of this material have a strong temperature dependence.

2. The physical interpretation of \( \alpha \) is that it provides a measure of heat transport \( (k) \) relative to energy storage \( (\rho c_p) \). In general, metallic solids have higher \( \alpha \), while nonmetallics (e.g., paraffin) have lower values of \( \alpha \).

3. Linear interpolation of property values is generally acceptable for engineering calculations.

4. Use of the low-temperature (300 K) density at higher temperatures ignores thermal expansion effects but is also acceptable for engineering calculations.

5. The \textit{IHT} software provides a library of thermophysical properties for selected solids, liquids, and gases that can be accessed from the toolbar button, \textit{Properties}. The properties are expressed as intrinsic functions, as shown for the thermal conductivity of atmospheric air,

\[ k = k_{\text{ infiltration}} (\text{"Air"}, T) \quad \text{// Thermal conductivity, W/m \cdot K} \]

and are based on values tabulated in Appendix A, with temperatures in kelvin units. You can create your own functions to represent property values or other input data by using the \textit{User-Defined Function} feature in \textit{IHT}, as described in the \textit{Help} section.
2.3
The Heat Diffusion Equation

A major objective in a conduction analysis is to determine the temperature field in a medium resulting from conditions imposed on its boundaries. That is, we wish to know the temperature distribution, which represents how temperature varies with position in the medium. Once this distribution is known, the conduction heat flux at any point in the medium or on its surface may be computed from Fourier's law. Other important quantities of interest may also be determined. For a solid, knowledge of the temperature distribution could be used to ascertain structural integrity through determination of thermal stresses, expansions, and deflections. The temperature distribution could also be used to optimize the thickness of an insulating material or to determine the compatibility of special coatings or adhesives used with the material.

We now consider the manner in which the temperature distribution can be determined. The approach follows the methodology described in Section 1.3.3 of applying the energy conservation requirement. In this case, we define a differential control volume, identify the relevant energy transfer processes, and introduce the appropriate rate equations. The result is a differential equation whose solution, for prescribed boundary conditions, provides the temperature distribution in the medium.

Consider a homogeneous medium within which there is no bulk motion (advection) and the temperature distribution \( T(x, y, z) \) is expressed in Cartesian coordinates. Following the methodology of applying conservation of energy (Section 1.3.3), we first define an infinitesimally small (differential) control volume, \( dx \cdot dy \cdot dz \), as shown in Figure 2.11. Choosing to formulate the first law at an instant of time, the second step is to consider the energy processes that are relevant to this control volume. If there are temperature gradients, conduction heat transfer will occur across

**Figure 2.11** Differential control volume, \( dx \cdot dy \cdot dz \), for conduction analysis in Cartesian coordinates.
each of the control surfaces. The conduction heat rates perpendicular to each of the control surfaces at the $x$, $y$, and $z$ coordinate locations are indicated by the terms $q_x$, $q_y$, and $q_z$, respectively. The conduction heat rates at the opposite surfaces can then be expressed as a Taylor series expansion where, neglecting higher order terms,

\begin{align}
q_{x+dx} &= q_x + \frac{\partial q_x}{\partial x} \, dx \\
q_{y+dy} &= q_y + \frac{\partial q_y}{\partial y} \, dy \\
q_{z+dz} &= q_z + \frac{\partial q_z}{\partial z} \, dz
\end{align}

(2.11a, 2.11b, 2.11c)

In words, Equation 2.11a simply states that the $x$ component of the heat transfer rate at $x + dx$ is equal to the value of this component at $x$ plus the amount by which it changes with respect to $x$ times $dx$.

Within the medium there may also be an energy source term associated with the rate of thermal energy generation. This term is represented as

$$\dot{E}_s = \dot{q} \, dx \, dy \, dz$$

(2.12)

where $\dot{q}$ is the rate at which energy is generated per unit volume of the medium (W/m$^3$). In addition, there may occur changes in the amount of the internal thermal energy stored by the material in the control volume. If the material is not experiencing a change in phase, latent energy effects are not pertinent, and the energy storage term may be expressed as

$$\dot{E}_s = \rho c_p \frac{\partial T}{\partial t} \, dx \, dy \, dz$$

(2.13)

where $\rho c_p \partial T / \partial t$ is the time rate of change of the sensible (thermal) energy of the medium per unit volume.

Once again it is important to note that the terms $\dot{E}_s$ and $\dot{E}_a$ represent different physical processes. The energy generation term $\dot{E}_s$ is a manifestation of some energy conversion process involving thermal energy on one hand and some other form of energy, such as chemical, electrical, or nuclear, on the other. The term is positive (a source) if thermal energy is being generated in the material at the expense of some other energy form; it is negative (a sink) if thermal energy is being consumed. In contrast, the energy storage term $\dot{E}_a$ refers to the rate of change of thermal energy stored by the matter.

The last step in the methodology outlined in Section 1.3.3 is to express conservation of energy using the foregoing rate equations. On a rate basis, the general form of the conservation of energy requirement is

$$\dot{E}_{in} + \dot{E}_s - \dot{E}_{out} = \dot{E}_{st}$$

(1.11c)

Hence, recognizing that the conduction rates constitute the energy inflow, $\dot{E}_{in}$, and outflow, $\dot{E}_{out}$, and substituting Equations 2.12 and 2.13, we obtain

$$q_x + q_y + q_z + \dot{q} \, dx \, dy \, dz - q_{x+dx} - q_{y+dy} - q_{z+dz} = \rho c_p \frac{\partial T}{\partial t} \, dx \, dy \, dz$$

(2.14)
Chapter 2 • Introduction to Conduction

Substituting from Equations 2.11, it follows that

\[-\frac{\partial q_x}{\partial x} dx - \frac{\partial q_y}{\partial y} dy - \frac{\partial q_z}{\partial z} dz + \dot{q} \; dx \; dy \; dz = \rho c_p \frac{\partial T}{\partial t} \; dx \; dy \; dz \]  \hfill (2.15)

The conduction heat rates may be evaluated from Fourier's law,

\[q_x = -k \; dy \; dz \frac{\partial T}{\partial x} \] \hfill (2.16a)

\[q_y = -k \; dx \; dz \frac{\partial T}{\partial y} \] \hfill (2.16b)

\[q_z = -k \; dx \; dy \frac{\partial T}{\partial z} \] \hfill (2.16c)

where each heat flux component of Equation 2.6 has been multiplied by the appropriate control surface (differential) area to obtain the heat transfer rate. Substituting Equations 2.16 into Equation 2.15 and dividing out the dimensions of the control volume \((dx \; dy \; dz)\), we obtain

\[\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t} \]  \hfill (2.17)

Equation 2.17 is the general form, in Cartesian coordinates, of the heat diffusion equation. This equation, often referred to as the heat equation, provides the basic tool for heat conduction analysis. From its solution, we can obtain the temperature distribution \(T(x, y, z)\) as a function of time. The apparent complexity of this expression should not obscure the fact that it describes an important physical condition, that is, conservation of energy. You should have a clear understanding of the physical significance of each term appearing in the equation. For example, the term \(\frac{\partial(k\dot{T}/\partial x)}{\partial x}\) is related to the net conduction heat flux into the control volume for the \(x\)-coordinate direction. That is, multiplying by \(dx\),

\[\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) dx = q_x^n - q_x^{n+1} \]  \hfill (2.18)

with similar expressions applying for the fluxes in the \(y\) and \(z\) directions. In words, the heat equation, Equation 2.17, therefore states that at any point in the medium the net rate of energy transfer by conduction into a unit volume plus the volumetric rate of thermal energy generation must equal the rate of change of thermal energy stored within the volume.

It is often possible to work with simplified versions of Equation 2.17. For example, if the thermal conductivity is constant, the heat equation is

\[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \]  \hfill (2.19)

where \(\alpha = kl/pc_p\) is the thermal diffusivity. Additional simplifications of the general form of the heat equation are often possible. For example, under steady-state
conditions, there can be no change in the amount of energy storage; hence Equation 2.17 reduces to

\[
\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = 0
\]  

(2.20)

Moreover, if the heat transfer is one-dimensional (e.g., in the x direction) and there is no energy generation, Equation 2.20 reduces to

\[
\frac{d}{dx} \left( k \frac{dT}{dx} \right) = 0
\]  

(2.21)

The important implication of this result is that under steady-state, one-dimensional conditions with no energy generation, the heat flux is a constant in the direction of transfer \((dq''/dx = 0)\).

The heat equation may also be expressed in cylindrical and spherical coordinates. The differential control volumes for these two coordinate systems are shown in Figures 2.12 and 2.13.

**Cylindrical Coordinates** When the del operator \(\nabla\) of Equation 2.3 is expressed in cylindrical coordinates, the general form of the heat flux vector, and hence of Fourier's law, is

\[
q'' = -k\nabla T = -k \left( i \frac{\partial T}{\partial r} + j \frac{1}{r} \frac{\partial T}{\partial \phi} + k \frac{\partial T}{\partial z} \right)
\]  

(2.22)

where

\[
q''_r = -k \frac{\partial T}{\partial r} \quad q''_\phi = \frac{k}{r} \frac{\partial T}{\partial \phi} \quad q''_z = -k \frac{\partial T}{\partial z}
\]  

(2.23)

**Figure 2.12** Differential control volume, \(dr \cdot r \, d\phi \cdot dz\), for conduction analysis in cylindrical coordinates \((r, \phi, z)\).
are heat flux components in the radial, circumferential, and axial directions, respectively. Applying an energy balance to the differential control volume of Figure 2.12, the following general form of the heat equation is obtained:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_v \frac{\partial T}{\partial t} \quad (2.24)
\]

**Spherical Coordinates** In spherical coordinates the general form of the heat flux vector and Fourier’s law is

\[
\mathbf{q}^s = -k \nabla T = -k \left( \mathbf{i} \frac{\partial T}{\partial r} + \mathbf{j} \frac{1}{r} \frac{\partial T}{\partial \theta} + \mathbf{k} \frac{1}{r \sin \theta} \frac{\partial T}{\partial \phi} \right) \quad (2.25)
\]

where

\[
q_r = -k \frac{\partial T}{\partial r} \quad q_\theta = -k \frac{\partial T}{r \sin \theta} \quad q_\phi = -k \frac{\partial T}{r \sin \theta \sin \phi} \quad (2.26)
\]

are heat flux components in the radial, polar, and azimuthal directions, respectively. Applying an energy balance to the differential control volume of Figure 2.13, the following general form of the heat equation is obtained:

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( kr^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( k \sin \theta \frac{\partial T}{\partial \theta} \right) + \dot{q} = \rho c_v \frac{\partial T}{\partial t} \quad (2.27)
\]

Since it is important that you be able to apply conservation principles to differential control volumes, you should attempt to derive Equation 2.24 or 2.27 (see Problems 2.35 and 2.36). Note that the temperature gradient in Fourier’s law must have units of K/m. Hence, when evaluating the gradient for an angular coordinate,
it must be expressed in terms of the differential change in arc length. For example, the heat flux component in the circumferential direction of a cylindrical coordinate system is \( q_\phi^* = -(klr)(\partial T/\partial \phi) \), and not \( q_\phi^* = -k(\partial T/\partial \phi) \).

**Example 2.2**

The temperature distribution across a wall 1 m thick at a certain instant of time is given as

\[
T(x) = a + bx + cx^2
\]

where \( T \) is in degrees Celsius and \( x \) is in meters, while \( a = 900^\circ \text{C}, b = -300^\circ \text{C/m}, \) and \( c = -50^\circ \text{C/m}^2 \). A uniform heat generation, \( q = 1000 \text{ W/m}^3 \), is present in the wall of area 10 m² having the properties \( \rho = 1600 \text{ kg/m}^3, k = 40 \text{ W/m·K}, \) and \( c_p = 4 \text{ kJ/kg·K} \).

1. Determine the rate of heat transfer entering the wall \((x = 0)\) and leaving the wall \((x = 1 \text{ m})\).
2. Determine the rate of change of energy storage in the wall.
3. Determine the time rate of temperature change at \( x = 0, 0.25, \) and \( 0.5 \text{ m} \).

**Solution**

**Known:** Temperature distribution \( T(x) \) at an instant of time \( t \) in a one-dimensional wall with uniform heat generation.

**Find:**
1. Heat rates entering, \( q_{in} (x = 0) \), and leaving, \( q_{out} (x = 1 \text{ m}), \) the wall.
2. Rate of change of energy storage in the wall, \( \dot{E}_{st} \).
3. Time rate of temperature change at \( x = 0, 0.25, \) and \( 0.5 \text{ m} \).

**Schematic:**

![Diagram of heat distribution in a wall with labeled variables such as temperature \( T(x) \), heat rates \( q_{in}, q_{out} \), and wall thickness \( L = 1 \text{ m} \).]
Assumptions:
1. One-dimensional conduction in the x direction.
2. Isotropic medium with constant properties.
3. Uniform internal heat generation, \( q \) (W/m³).

Analysis:
1. Recall that once the temperature distribution is known for a medium, it is a simple matter to determine the conduction heat transfer rate at any point in the medium, or at its surfaces, by using Fourier's law. Hence the desired heat rates may be determined by using the prescribed temperature distribution with Equation 2.1. Accordingly,

\[
q_m = q_s(0) = -kA \frac{\partial T}{\partial x} \bigg|_{x=0} = -kA(b + 2cx) \bigg|_{x=0} = -bkA = 300^\circ C/m \times 40 W/m \cdot K \times 10 m^2 = 120 kW
\]

Similarly,

\[
q_{out} = q_s(L) = -kA \frac{\partial T}{\partial x} \bigg|_{x=L} = -kA(b + 2cx) \bigg|_{x=L} = -[- -300^\circ C/m

+ 2(-50^\circ C/m²) \times 1 m] \times 40 W/m \cdot K \times 10 m^2 = 160 kW
\]

2. The rate of change of energy storage in the wall \( \dot{E}_w \) may be determined by applying an overall energy balance to the wall. Using Equation 1.11c for a control volume about the wall,

\[
\dot{E}_w + \dot{E}_g - \dot{E}_{out} = \dot{E}_{in}
\]

where \( \dot{E}_g = qAL \), it follows that

\[
\dot{E}_w = \dot{E}_m + \dot{E}_g - \dot{E}_{out} = q_m + qAL - q_{out}
\]

\[
\dot{E}_m = 120 kW + 1000 W/m³ \times 10 m² \times 1 m - 160 kW
\]

\[
\dot{E}_w = -30 kW
\]

3. The time rate of change of the temperature at any point in the medium may be determined from the heat equation, Equation 2.19, rewritten as

\[
\frac{\partial T}{\partial t} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{\rho c_p}
\]

From the prescribed temperature distribution, it follows that

\[
\frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right)
\]

\[
= \frac{\partial}{\partial x} (b + 2cx) = 2c = 2(-50^\circ C/m²) = -100^\circ C/m²
\]
Note that this derivative is independent of position in the medium. Hence the time rate of temperature change is also independent of position and is given by

$$\frac{\partial T}{\partial t} = \frac{40 \text{ W/m} \cdot \text{K}}{1600 \text{ kg/m}^3 \times 4 \text{ kJ/kg} \cdot \text{K}} \times (-100^\circ \text{C} / \text{m}^2) + \frac{1000 \text{ W/m}^3}{1600 \text{ kg/m}^3 \times 4 \text{ kJ/kg} \cdot \text{K}}$$

$$= -6.25 \times 10^{-4} \text{C/s} + 1.56 \times 10^{-4} \text{C/s}$$

$$= -4.69 \times 10^{-4} \text{C/s}$$

Comments:
1. From the above result it is evident that the temperature at every point within the wall is decreasing with time.
2. Fourier's law can always be used to compute the conduction heat rate from knowledge of the temperature distribution, even for unsteady conditions with internal heat generation.

Microscale Effects  For most practical situations, the heat diffusion equations generated in this text may be used with confidence. However, these equations are based on use of Fourier's law to describe the conduction effects, which does not account for the finite speed at which thermal information is propagated within the medium by the various energy carriers. The consequences of the finite propagation speed may be neglected if the heat transfer events of interest occur over a sufficiently long time scale, $\Delta t$, such that

$$\frac{\lambda_{mp}}{c\Delta t} \ll 1$$  \hspace{1cm} (2.28)

The heat diffusion equations of this text are likewise invalid for problems where boundary scattering must be explicitly considered. For example, the temperature distribution within the thin film of Figure 2.6b cannot be determined by applying the foregoing heat diffusion equations. Additional discussion of micro- and nanoscale heat transfer applications and analysis methods is available in the literature [1, 15].

2.4 Boundary and Initial Conditions

To determine the temperature distribution in a medium, it is necessary to solve the appropriate form of the heat equation. However, such a solution depends on the physical conditions existing at the boundaries of the medium and, if the situation is time dependent, on conditions existing in the medium at some initial time. With regard to the boundary conditions, there are several common possibilities that are
Table 2.2 Boundary conditions for the heat diffusion equation at the surface \((x = 0)\)

<table>
<thead>
<tr>
<th>Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Constant surface temperature (T(0, t) = T_r) (2.29)</td>
</tr>
</tbody>
</table>

![Diagram of constant surface temperature](image)

| 2. | Constant surface heat flux (a) Finite heat flux \(-k \frac{\partial T}{\partial x} \big|_{x=0} = q_s^-\) (2.30) |

![Diagram of constant surface heat flux](image)

| (b) Adiabatic or insulated surface \(\frac{\partial T}{\partial x} \big|_{x=0} = 0\) (2.31) |

![Diagram of adiabatic or insulated surface](image)

| 3. | Convection surface condition \(-k \frac{\partial T}{\partial x} \big|_{x=0} = h[T_w - T(0, t)]\) (2.32) |

![Diagram of convection surface condition](image)

simply expressed in mathematical form. Because the heat equation is second order in the spatial coordinates, two boundary conditions must be expressed for each coordinate needed to describe the system. Because the equation is first order in time, however, only one condition, termed the initial condition, must be specified.

Three kinds of boundary conditions commonly encountered in heat transfer are summarized in Table 2.2. The conditions are specified at the surface \(x = 0\) for a one-dimensional system. Heat transfer is in the positive \(x\) direction with the temperature distribution, which may be time dependent, designated as \(T(x, t)\). The first condition corresponds to a situation for which the surface is maintained at a fixed temperature \(T_r\). It is commonly termed a Dirichlet condition, or a boundary condition of the first kind. It is closely approximated, for example, when the surface is in contact with a melting solid or a boiling liquid. In both cases there is heat transfer at the surface, while the surface remains at the temperature of the phase change process. The second condition corresponds to the existence of a fixed or constant heat flux \(q_s^-\) at the surface. This heat flux is related to the temperature gradient at the surface by Fourier's law. Equation 2.6, which may be expressed as

\[ q_s^- (0) = -k \frac{\partial T}{\partial x} \big| _{x=0} = q_s^- \]

It is termed a Neumann condition, or a boundary condition of the second kind, and may be realized by bonding a thin film electric heater to the surface. A special case of this condition corresponds to the perfectly insulated, or adiabatic, surface for
which \( \partial T/\partial x \vert_{x=0} = 0 \). The boundary condition of the third kind corresponds to the existence of convection heating (or cooling) at the surface and is obtained from the surface energy balance discussed in Section 1.3.2.

**Example 2.3**

A long copper bar of rectangular cross section, whose width \( w \) is much greater than its thickness \( L \), is maintained in contact with a heat sink at its lower surface, and the temperature throughout the bar is approximately equal to that of the sink, \( T_0 \). Suddenly, an electric current is passed through the bar and an airstream of temperature \( T_a \) is passed over the top surface, while the bottom surface continues to be maintained at \( T_0 \). Obtain the differential equation and the boundary and initial conditions that could be solved to determine the temperature as a function of position and time in the bar.

**Solution**

**Known:** Copper bar initially in thermal equilibrium with a heat sink is suddenly heated by passage of an electric current.

**Find:** Differential equation and boundary and initial conditions needed to determine temperature as a function of position and time within the bar.

**Schematic:**

![Schematic Diagram](image)

**Assumptions:**
1. Since the bar is long and \( w \gg L \), end and side effects are negligible and heat transfer within the bar is primarily one dimensional in the \( x \) direction.
2. Uniform volumetric heat generation, \( \dot{q} \).
3. Constant properties.

**Analysis:** The temperature distribution is governed by the heat equation (2.17), which, for the one-dimensional and constant property conditions of the present problem, reduces to

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} \frac{1}{\alpha} \frac{\partial T}{\partial t} = 0
\]

(1)
where the temperature is a function of position and time, \( T(x, t) \). Since this differential equation is second order in the spatial coordinate \( x \) and first order in time \( t \), there must be two boundary conditions for the \( x \) direction and one condition, termed the initial condition, for time. The boundary condition at the bottom surface corresponds to case 1 of Table 2.2. In particular, since the temperature of this surface is maintained at a value, \( T_o \), which is fixed with time, it follows that

\[
T(0, t) = T_o
\]  

(2)

The convection surface condition, case 3 of Table 2.2, is appropriate for the top surface. Hence

\[
-k \frac{\partial T}{\partial x}
\bigg|_{x=L} = h[T(L, t) - T_m]
\]

(3)

The initial condition is inferred from recognition that, before the change in conditions, the bar is at a uniform temperature \( T_o \). Hence

\[
T(x, 0) = T_o
\]

(4)

If \( T_o, T_m, \dot{q}, \) and \( h \) are known, Equations 1 through 4 may be solved to obtain the time-varying temperature distribution \( T(x, t) \) following imposition of the electric current.

Comments:

1. The heat sink at \( x = 0 \) could be maintained by exposing the surface to an ice bath or by attaching it to a cold plate. A cold plate contains coolant channels machined in a solid of large thermal conductivity (usually copper). By circulating a liquid (usually water) through the channels, the plate, and hence the surface to which it is attached, may be maintained at a nearly uniform temperature.

2. The temperature of the top surface \( T(L, t) \) will change with time. This temperature is an unknown and may be obtained after finding \( T(x, t) \).

3. We may use our physical intuition to sketch temperature distributions in the bar at selected times from the beginning to the end of the transient process. If we assume that \( T_m > T_o \), and that the electric current is sufficiently large to heat the bar to temperatures in excess of \( T_m \), the following distributions would correspond to the initial condition \( (t = 0) \), the final (steady-state) condition \( (t \to \infty) \), and two intermediate times.

![Temperature Distribution Diagram](image)

Note how the distributions comply with the initial and boundary conditions. What is a special feature of the distribution labeled (b)?
4. Our intuition may also be used to infer the manner in which the heat flux varies with time at the surfaces \( x = 0, L \) of the bar. On \( q_x' - t \) coordinates, the transient variations are as follows.

![Graph showing variations in heat flux over time](image)

Convince yourself that the foregoing variations are consistent with the temperature distributions of Comment 3. For \( t \to \infty \), how are \( q_x'(0) \) and \( q_x'(L) \) related to the volumetric rate of energy generation?

### 2.5 Summary

The primary objectives of this chapter were to improve your understanding of the conduction rate equation (Fourier's law) and to familiarize you with the heat equation. You may test your understanding of related concepts by addressing the following questions.

- In the general formulation of Fourier's law (applicable to any geometry), what are the vector and scalar quantities? Why is there a minus sign on the right-hand side of the equation?
- What is an isothermal surface? What can be said about the heat flux at any location on this surface?
- What form does Fourier's law take for each of the orthogonal directions of Cartesian, cylindrical, and spherical coordinate systems? In each case, what are the units of the temperature gradient? Can you write each equation from memory?
- An important property of matter is defined by Fourier's law. What is it? What is its physical significance? What are its units?
- What is an isotropic material?
- Why is the thermal conductivity of a solid generally larger than that of a liquid? Why is the thermal conductivity of a liquid larger than that of a gas?
- Why is the thermal conductivity of an electrically conducting solid generally larger than that of a nonconductor? Why are materials such as beryllium oxide, diamond, and silicon carbide (see Table A.2) exceptions to this rule?
- Is the effective thermal conductivity of an insulation system a true manifestation of the efficacy with which heat is transferred through the system by conduction alone?
- Why does the thermal conductivity of a gas increase with increasing temperature? Why is it approximately independent of pressure?
- What is the physical significance of the thermal diffusivity? How is it defined and what are its units?