MAE 171A
Mechanical Engineering Laboratory

Fracture mechanics and viscoelastic response of polymers (time-dependent deformation)

Lecture Notes
Outline

- Theoretical cohesive strength of materials
- Stress intensity factor
- Griffith fracture theory for brittle fracture
  - Development of stress intensity factor
- Introduction to polymers
  - Mechanical properties
- Viscoelastic response of polymers
  - Creep and stress relaxation
  - Temperature effects
- Laboratory experiments
Some disasters due to material failure

Aloha Airlines 1988
Small section of roof ripped 12 cm
Resulting explosive decompression tore off the entire top half of the aircraft

Challenger 1986
O-ring seal in the right solid rocket booster failed and caused a flame
SS Schenectady

1944 brittle fracture
- 2500 Liberty ships built
- 700 experienced severe structural failure
- 145 broke into 2 pieces

Reasons:
- Flaws in welded joints
- Low fracture toughness materials used

(Elastic energy stored in the bulk $\sim \beta$)/(Energy to open a crack $\sim \beta$) $\sim l$.
It is easy to break large objects and very difficult to break a small ones!
Recent disaster

The New York Times (3/18, 2008) reports, "The Interstate 35W bridge over the Mississippi in Minneapolis collapsed after construction workers had put 99 tons of sand on the roadway directly over two of the bridge's weakest points," a new report from the National Transportation Safety Board (NTSB) revealed. According to a report "Stress at one of the two weakest points was 83 percent more than it could have handled." However, the load would not have been excessive for a well-designed bridge, according to experts."
Constructive applications of fracture

1. To break into small pieces large diameter radioactive metallic vessels for recycling after atomic power stations will be closed

2. Fragmentation of minerals in gold mining (median size of particles in attritor is leveled at about 1 micron and has very low dependence on specific energy input

3. Utilization of high quality steel from military equipment (e.g., tanks)
Development of theoretical strength of a material based on different parameters

⇒ Two ways to estimate strength
(1) Theoretical cohesive strength
   Force necessary to break atomic bonds
(2) Work of fracture to create new free surfaces
   → Area under the stress strain curve
First consider the theoretical cohesive strength of a material

- A material is held together by strong atomic bonds
- Consider a crack in the material
  - How much stress must be added to break the bond?
(1) Theoretical cohesive strength – force (stress)-displacement curve

\[ \sigma = a = \text{equilibrium spacing} \]
\[ \sigma_t = \text{theoretical tensile strength} \]
Consider that the cohesive strength of a material is the force required to separate two atoms (break bonds).

\[ \sigma_c = \text{cohesive strength} \]
\[ a_o = \text{equilibrium atomic spacing} \]
\[ \Rightarrow \text{no force on the atoms} \]

Simplified force vs. atom displacement relationship to describe cohesive strength (fracture strength)
(1) Theoretical cohesive strength

Assume \( \sigma = \sigma_c \sin \left( \frac{2\pi x}{\lambda} \right) \) and \( \sin \left( \frac{2\pi x}{\lambda} \right) \approx \frac{2\pi x}{\lambda} \) for small values of \( \frac{2\pi x}{\lambda} \),

thus \( \sigma = \frac{2\sigma_c \pi x}{\lambda} \)

The slope of the curve for small displacements

\[
\frac{d\sigma}{dx} = \frac{2\sigma_c \pi}{\lambda}
\]
(1) Theoretical cohesive strength

\[ \sigma = E \varepsilon \quad \text{and} \quad \varepsilon = \frac{x}{a_o} \]

where \( a_o \) is the lattice constant, \( x \)-displacement.

\[ \lambda \sim 2a_o \]

From other (continuum) point of view, Hook e’s law must also apply in this region:

\[ \sigma = E \frac{x}{a_o} \quad \text{then} \quad \frac{d\sigma}{dx} = \frac{E}{a_o} \], using equation from previous slide \( \frac{E}{a_o} = \frac{2\pi \sigma_c}{\lambda} \)

obtain \( \sigma_c = \frac{E}{\pi} \)
(2) Work of fracture

Another method to calculate the cohesive strength is to consider the work of fracture.

The area under the force displacement curve can be used to describe the work of fracture per atom.

\[
\text{Work} = \int_{0}^{\lambda/2} \sigma_c \sin \left( \frac{2\pi x}{\lambda} \right) dx
\]
(2) Work of fracture

Solving the integral gives:

\[ \text{Work} = \frac{\sigma_c \lambda}{\pi} = 2 \gamma_s \quad \text{where} \quad \gamma_s \text{ is the surface energy} \]

(2 new surfaces created)

Using also previous equation for \( \sigma_c \):

\[ \frac{E}{a_o} = \frac{2\pi \sigma_c}{\lambda} \]

get

\[ \sigma_c = \sqrt{\frac{E \gamma_s}{a_o}} \]

For most materials, values for \( \gamma_s \) and \( a_o \) give

\[ \sigma_c = E/10 \]
Theoretical vs. experimental values

<table>
<thead>
<tr>
<th>material</th>
<th>E (GPa)</th>
<th>UTS (GPa) predicted</th>
<th>UTS (GPa) experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>glass</td>
<td>70</td>
<td>7</td>
<td>0.006</td>
</tr>
<tr>
<td>aluminum oxide</td>
<td>400</td>
<td>40</td>
<td>2</td>
</tr>
<tr>
<td>stainless steel</td>
<td>200</td>
<td>20</td>
<td>0.9</td>
</tr>
<tr>
<td>aluminum</td>
<td>70</td>
<td>7</td>
<td>0.5</td>
</tr>
<tr>
<td>nylon</td>
<td>70</td>
<td>7</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Why are theoretical values orders of magnitude much greater than experimental values (except for freshly made glass fibers which strength approached the theoretical tensile strength of the order of one-tenth the elastic modulus)? The fiber strength was increased by polishing.

This is the problem Inglis and Griffith tackled.
Stress concentration factors
(Inglis, 1913)

Consider an elliptical crack in the center of a body subjected to an applied tensile stress $\sigma_a$

- $2c = \text{crack length}$
- $2b = \text{crack width}$
- $\sigma_{\text{max}} = \text{max stress at the end of major axis}$

Mathematically,

$$\sigma_{\text{max}} = \frac{b^2}{c}$$
Stress concentration factors

The plate is subjected to uniform in-plane tensile stresses perpendicular to the major axis of the elliptical notch. The maximum tensile stress $\sigma_{\text{max}}$ would occur at the ends of the major axis of the elliptical notch.

\[ \frac{\sigma_{\text{max}}}{\sigma_a} = 1 + \frac{2c}{b} \] (stress concentration factor for the notch)

The radius of curvature for elliptic crack $\rho = \frac{b^2}{c}$

\[ \sigma_{\text{max}} = \sigma_a \left( 1 + 2 \sqrt{\frac{c}{\rho}} \right) \]

Depends on the shape of the crack (relative value of $c/b$ or $c/\rho$)

- $c/b = 1$, $\sigma_{\text{max}} = 3\sigma_a$
- $c/b = 2$, $\sigma_{\text{max}} = 5\sigma_a$
- $c/b = 16$, $\sigma_{\text{max}} = 33\sigma_a$
Stress concentration factors

For sharp notches \((c >> \rho)\) \(\sigma_{\text{max}} = 2\sigma_a \sqrt{c/\rho}\)

Fracture stress should approach zero as the radius of curvature is reduced to zero. Experimental data show that the stress required to produce fracture actually approaches a constant. Thus the maximum principal stress criterion for failure has limiting validity. (R.P. Wei, Fracture Mechanics, Cambridge, Univ. Press., 2010).
Stress concentration around a crack
Summary of Inglis formulation

Showed that:

- Force applied to ends of an elastic plate would produce locally increased tensile stresses at the tip of a crack
  - may exceed the elastic limit of the material and lead to the propagation of the crack
- Increase in the length of the crack exaggerates the stress even more, such that the crack would continue to spread
- Small crack tip radii increase the stress at the crack tip
- The shape of the crack rather than the scale was important in determining the stress concentration
Problems with Inglis formulation

⇒ The stress concentration factor, $k_t = \frac{\sigma_{\text{max}}}{\sigma_a}$

◇ Has no dependence on the crack scale
  → Only on the ratio of $c/\rho$

⇒ Even if local failure was initiated it does not mean that crack will propagate because crack propagation requires energy

⇒ How can we approach a design problems so that we know the maximum stress a material can withstand, given a flaw size distribution?
Griffith fracture theory
(analysis of the equilibrium and stability of cracks)

⇒ Tensile strength of freshly drawn glass fibers was much greater than that of old or aged fibers

⇒ Tensile strength decreased with increasing length of fiber
  ◆ Volume effect
  ◆ Same as da Vinci found

⇒ Tensile strength varied widely from sample to sample


Alan Arnold Griffith 1893-1963
Griffith fracture theory

Griffith concluded that:

- Fibers are weakened by microscopic flaws on the surface or interior of the fiber
  - Mechanical behavior will be dependent on presence of many small scale cracks
- Analysis can be extended to more general case of any brittle material

- defect-free glass fiber
- surface flaws
- surface and internal flaws
Griffith theory of brittle fracture
(new approach based on analysis of stability of crack)

⇒ When a crack propagates
  ▪ There is a release of elastic strain energy
    ▪ Decrease in energy
  ▪ During the crack extension process
    ▪ New surfaces created at the faces of the crack
      ▪ Increase of energy

Griffith performed an energy balance (a defect would grow when the elastic energy released by the growth of the defect exceeded the energy required to form the crack surfaces)
Model of Griffith crack analysis

Plate (thickness has a unit length) extended and rigidly fixed, elastic energy is stored unloading at regions adjacent to a growing crack
Griffith theory of brittle fracture - plane strain condition

The decrease in elastic strain energy $U_{SE}$ for generalized plane stress, is given by:
(for plain strain the numerator is modified by $(1-v^2)$ which usually for simplicity is not included in the discussion)

$$U_{SE} = -\frac{\pi \sigma^2 c^2}{E}$$

Total energy $= U_T = U_s + U_{SE}$

Elastic strain energy given by area under $\sigma - \varepsilon$ curve

$Area = \sigma\varepsilon/2$
Griffith theory of brittle fracture

\[ U - U_0 = U_T = U_{SE} + U_s = -\frac{\pi \sigma^2 c^2}{E} + 4c\gamma_s \]

\( U(U_0) \) – potential energy of body with (without) crack,

The maximum energy (equil.con d.) occurs when \( \frac{dU_T}{dc} = 0 \)

The equilibrium is unstable

Taking the derivative and setting \( = 0 \) yields \( \sigma_c = \sqrt{\frac{2\gamma_s E}{\pi c}} \)

\( \sigma_c \) is the critical stress needed for the crack to propagate: the crack will grow if \( \sigma > \sigma_c \)
How does this compare with experimental values?

Griffith calculated $\gamma_s$ for glass knowing:

- $E = 6.2 \times 10^{10} \text{ N/m}^2$
- $c = 0.001 \text{ m}$
- $\sigma_c = 8.3 \times 10^6 \text{ N/m}^2$
  \[\Rightarrow \gamma_s = 1.75 \text{ J/m}^2\]

Griffith measured $\gamma_s$ as a function of $T$ (between 745 and 1110 \text{ C}) and extrapolated to RT
  \[\Rightarrow \gamma_s = 0.54 \text{ J/m}^2\]

This is in excellent agreement despite three times difference!

- Why? Because surface energies usually are measured within an order of magnitude
Fracture toughness

\[ \sigma_c = \sqrt{\frac{2E\gamma_s}{\pi c}} \]  then \[ \sigma_c \sqrt{c} = \sqrt{\frac{2E\gamma_s}{\pi}} = \text{material property} \]

\[ K \equiv Y\sigma\sqrt{c} = \text{stress intensity factor} \]

\[ K_{IC} = Y\sigma_c\sqrt{c} \] where \( K_{IC} \) = fracture toughness under plane strain conditions

\( Y = \text{geometrical constant} = \text{function of specimen and crack geometry} \)
Fracture toughness characterizes the ability of a material to resist brittle fracture when a crack is present.

$K_{IC}$ is a material property (like density, elastic modulus, etc.) and has units $\text{MPa} \sqrt{\text{m}}$. It cannot be calculated based on other mechanical properties and must be measured experimentally.

$I$ refers to Mode I loading
$c$ refers to ‘critical’ – where the material will fail if you know $K_{IC}$ and yield stress or TS, you can calculate the maximum flaw size that can be tolerated before material fails.

The geometrical constant $Y$ has many complex formulations.
Fracture modes

Mode I Tension, this fracture mode we study in 171A

Mode II In-plane shear sliding

Mode III Out-of-plane shear tearing
Stress field around Mode I tensile loading

\[ \sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \]

\[ \sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \]

\[ \tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \]
\( k_t, K \) and \( K_{IC} \)

⇒ The stress concentration factor, \( k_t \)
   \[ k_t = \frac{\sigma_{\text{tip}}}{\sigma_{\text{appl}}} \]
   - Ratio of stress at crack tip to applied nominal stress
   - Can be approximated as ratio of \( 2\sqrt{\frac{c}{\rho}} \)

⇒ The stress intensity factor, \( K_I \) in Mode I
   - It is a scale factor used to define the magnitude of crack-tip stress field

⇒ The fracture toughness, \( K_{IC} \)
   - A materials property, \( K_I = K_{IC} \) at the onset of crack growth. If the specimen is thickness \( B \) is much greater than the plastic zone size \( (B \geq 2.5 \frac{K_{IC}}{\sigma_y})^2 \) then abrupt fracture will occur when the crack-tip stress intensity factor reaches the plane strain fracture toughness \( K_{IC} \).
   - Subscript I refers to mode one, \( \text{plane strain} \) conditions

⇒ These \( k_t, K_I, \) and \( K_{IC} \) values should not be confused
Values for yield strength, elastic modulus and fracture toughness $K_{IC}$

<table>
<thead>
<tr>
<th>Material</th>
<th>Yield Str. (MPa)</th>
<th>$E$ (GPa)</th>
<th>$K_{IC}$ (MPam$^{1/2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4340 steel</td>
<td>1470</td>
<td>200</td>
<td>46</td>
</tr>
<tr>
<td>Maraging steel</td>
<td>1730</td>
<td>200</td>
<td>90</td>
</tr>
<tr>
<td>Ti-6Al-4V</td>
<td>900</td>
<td>114</td>
<td>57</td>
</tr>
<tr>
<td>2024-T3 Al alloy</td>
<td>385</td>
<td>72.4</td>
<td>26</td>
</tr>
<tr>
<td>7075-T6 Al alloy</td>
<td>450</td>
<td>71.7</td>
<td>24</td>
</tr>
</tbody>
</table>

- Fracture toughness is not proportional to yield strength or elastic modulus.
- It may increase or decrease with increasing yield strength.
Fracture control design philosophy
(R.W. Hertzberg, Deformation and fracture mechanics of engineering materials, 1989)

Fracture toughness $K_{IC}$, design stress ($\sigma$) and crack size ($a$) control the conditions for fracture. For example, the fracture condition for an infinitely large cracked plate is

$$K_{IC} = \sigma (\pi a)^{1/2}$$

where $K_{IC}$ is determined by selection of a suitable material, e.g., Al alloys used in aircraft industry due to their high strength and low density;

$\sigma$ is a design stress, e.g., determined by aircraft’s payload capacity;

$a$ is allowable flaw size or NDT flaw detection.

You must first decide what is most important about your component design: certain material properties, the design stress or the flaw size that must be tolerated for safe operation. Once any combination of two of these variables is defined, the third parameter is fixed.

For example, having fixed $K_{IC}$ and $\sigma$, the allowable flaw size is defined by above equation and beyond the control of the aircraft designers. This allowable flaw size that can be tolerated by the material under the applied stress must be larger than the size of “hidden” crack (e.g., diameter of the rivet head covering the hole, usual source of high stress concentration in aircraft components).
Polymers

- Poly (many) mer (unit)
  - A polymer unit (mer) is typically composed of an organic group consisting of mainly C, H, and O atoms
  - Strong atomic bonds (covalent) hold these atoms together in the unit
    - These units are connect together to form large macromolecules in long chains composed typically of > 10,000 mer units (degree of polymerization)

- Macromolecules can be held together with weaker hydrogen or van der Waals forces (thermoplastics)
  - Can be recycled
- Macromolecules can be held together by crosslinking (thermosets)
  - Cannot be recycled
- The polymers used in this lab are thermoplastics
Natural polymers

⇒ Spider silk fibers
  ▪ Made up of intertwined proteins
    ▪ Tensile strength ~ 1.6 GPa
      ▪ Strongest naturally occurring
      ▪ A strand long enough could circle globe

▪ Silk worm
  ▪ Polypeptide
    ▪ Synthetically called nylon

▪ Rubber
▪ Cellulose
▪ Chitin (polysaccharine)
  ▪ Found in crustaceans
    ⇒ Hard, insoluble, flexible
    ☞ Still can’t reproduce it in the lab
Structure of polymers

⇒ Polymers have an amorphous structure
   - Long macromolecules tangled with each other

⇒ Crystalline regions have orientation to the macromolecules
   - Orientation can result from tensile stress
     → Alignment of the long macromolecules

⇒ Most polymers consist of amorphous and crystalline regions

amorphous + crystalline
Thermoplastics - stress/strain curve

As T increases
- yield and fracture stress decrease
- elastic modulus at low strain decreases
Polypropylene

\[ \text{Propylene (gas at RT) has the molecular formula } C_3H_6 \]

During polymerization, the double \( C = C \) is broken, leaving an unattached bond on both \( C \) atoms, creating the mer unit

\[ \text{These units link up to form a long chain. Other chains are formed to form the solid polymer} \]
Polypropylene (PP)

- Molecular formula \((\text{C}_3\text{H}_6)_n\)
- Used for food packaging, textiles, reusable containers (Rubbermaid®)
- \(T_g = -20°C\)

Packaging for DVDs
Polymethyl methacrylate (PMMA)

⇒ Molecular formula
$(C_5O_2H_8)_n$

⇒ Trade names Lucite®, Plexiglas®

蒈 Used as an alternate to glass

⇒ Rigid, impact resistant

⇒ $T_g = 105^\circ C$
PMMA (‘acrylic’ glass)

Ithaa -
worlds first PMMA glass undersea restaurant.
5 m below sea level.

Hilton Maldives Resort & Spa
Polycarbonate (PC)

- Molecular formula \((\text{C}_{16}\text{H}_{14}\text{O}_3)_n\)
- Trade name Lexan®
- Used for glasses lens, DVDs, compact disks
  - Impact resistant - used for bullet-proof glass
- Stronger, more expensive than PMMA
- \(T_g = 150^\circ\text{C}\)

![Chemical structure of polycarbonate](image)

- Benzene ring \(\text{C}_6\text{H}_4\)
- \(n = \) degree of polymerization
Polycarbonate applications
Viscoelastic response of polymers

$E_c(t) = \frac{\sigma_0}{\varepsilon(t)}$

Time dependent strain

Load constant

$E_r(t) = \frac{\sigma(t)}{\varepsilon_0}$

Time dependent stress

Elongation constant
Viscoelastic response - mechanical analogs (elements have zero mass)

- Spring (purely elastic)
- Dashpot (purely viscous)
- Maxwell model
- Voight model
- Four element model
Spring and dashpot

Spring (elastic deformation)
\[ \sigma = E \varepsilon, \quad \varepsilon = \sigma / E \]
\[ \text{or } \gamma = \tau / G \]
\[ \rightarrow \gamma = \text{shear strain} \]
\[ \rightarrow \tau = \text{shear stress} \]
\[ \rightarrow G = \text{shear modulus} \]

Dashpot (viscous flow)
\[ \sigma = \eta d\varepsilon / dt; \quad d\varepsilon / dt = \sigma / \eta \]
\[ \text{or } d\gamma / dt = \tau / \eta \]
\[ \rightarrow \eta = \text{fluid/solid viscosity} \]
Maxwell model

Two element, spring (s) in series with a dashpot (d)

When stress is applied, it is uniform throughout the element

\[ \sigma_{\text{applied}} = \sigma_s = \sigma_d \]

Total strain, \( \varepsilon_t = \varepsilon_s + \varepsilon_d \)
Maxwell model

⇒ Then \( \sigma_s = E_s \varepsilon_s = \sigma_d = \eta (d\varepsilon_d /dt) \)
⇒ \( \varepsilon_s = \sigma_s /E \) and \( \varepsilon_d = \int \sigma_d /\eta dt \), \( \sigma_s = \sigma_d = \sigma \)

\[
\varepsilon_T = \frac{\sigma}{E_s} + \int \frac{\sigma}{\eta} dt
\]

\[
\varepsilon(t) = \frac{\sigma}{E_s} + \frac{\sigma}{\eta} t
\]

\[
\varepsilon = \frac{\sigma}{E} + \frac{\sigma}{\eta}
\]
Characteristics of the Maxwell model

⇒ For stress relaxation
  Sample at $\sigma_o$ held between 2 fixed plates
    → Allowed to relax
  $\varepsilon_t = \varepsilon_o$
    → Strain rate is kept 0

⇒ Then
\[ \dot{\varepsilon} = 0 = \frac{\sigma}{\eta} + \frac{1}{E} \frac{d\sigma}{dt} \]

\[ \sigma(t) = \sigma_o \exp(-Et/\eta) = \sigma_o \exp(-t/\tau) \]

→ Relaxation time ($\tau$) defined as $\eta/E$
→ $t >> \tau$, viscous component only
→ $t << \tau$, elastic component only

Stress decreases as a function of increasing time
Voight model

⇒ Two element system, spring and dashpot in parallel

⇒ When stress is applied, \( \varepsilon_t = \varepsilon_s = \varepsilon_d = \varepsilon \)

⇒ And \( \sigma_{\text{applied}} = \sigma_s + \sigma_d \)

⇒ Then

\[
\sigma_T(t) = E_s \varepsilon + \eta \frac{d\varepsilon}{dt}
\]
Characteristics of the Voight model

For creep

Load is kept constant on the sample

\[ \sigma(t) = \sigma_o = E\varepsilon + \eta \frac{d\varepsilon}{dt} \]

Then

\[ \varepsilon(t) = \frac{\sigma_o}{E} \left(1 - e^{-t/\tau}\right) \]

Strain increases to a constant value as a function of increasing time
Stress relaxation, three different models

Models of linear viscoelasticity: (a) Maxwell, (b) Voigt, (c) standard linear solid.

Relaxation functions of (a) Maxwell, (b) Voigt, and (c) standard linear solid.
In the case of stress relaxation, the strain is a constant $\varepsilon_0$

$$\sigma(t) = \varepsilon_0 \left( k_e + k_1 e^{-\frac{t}{\tau}} \right)$$
Four element model for creep

\[ \sigma > 0: \varepsilon(t) = \frac{\sigma_0}{E_1} + \frac{\sigma_0}{\eta_1} t + \frac{\sigma_0}{E_2} (1 - e^{-t/\tau_2}), \tau_2 = \frac{\eta_2}{E_2}; \]

\[ \sigma = 0: \varepsilon(t) = \varepsilon_2(t = t_0) e^{-(t-t_0)/\tau_2} + \frac{\sigma}{\eta_1} t_0. \]

First two terms in the top equation correspond to Maxwell model and third to Voight model.
Strain \( \varepsilon(t) = \) elastic + viscous + viscoelastic.
We need four equations to estimate 4 parameters in four elements model. For example,

$E_1$ can be found from the jumps of strain at $t=0$ and at $t=t_0$ based on the equation below (these jumps of strains are equal to each other):

$$\varepsilon(t=0) = \frac{\sigma}{E_1}$$

Second equation (to find $\eta_1$) can be based on the measured strain limit at $t>>t_0$:

$$\varepsilon(t>>t_0) = t_0 \frac{\sigma}{\eta_1}$$
or it can be based on the measured slope of a straight line at $t>>\tau_2$, and $t_0>t'>t$:

$$\dot{\varepsilon}(t_0>t'>t') = \frac{\sigma}{\eta_1}$$

Third equation (to find $\eta_2$) can be based on the measured slope at the vicinity of $t=0$:

$$\dot{\varepsilon}(t=0) = \frac{\sigma}{\eta_1} + \frac{\sigma}{\eta_2}$$

Fourth equation (to estimate $E_2$, if $t>>\tau_1$) can be based on the measured value of strain at $t=t_0$ (before the drop of the stress)

$$\varepsilon(t_0) \approx \frac{\sigma}{E_1} + \frac{\sigma}{E_2} + \frac{\sigma}{\eta_1} t_0$$
Glass transition temperature, $T_g$

$\Rightarrow T < T_g$

*Molecules have restricted motion*

*Material behaves in a brittle, glassy manner*

$\Rightarrow T > T_g$

*Polymer can deform elastically or plastically before fracture*
Elastic modulus as a function of temperature

Onset of short-range molecular segmental motion

Negligible molecule segmental motion

Complete molecular motion

Liquid flow occurs

Elastic modulus as a function of time

Relaxation modulus, $E_r$

Log $E$ (Pa)

Log $E$ (Pa)
Effect of molecular weight on elastic modulus as a function of temperature

\[ \log E \text{ vs } T \]

increasing MW
Laboratory objectives

You are given compact test samples

- 3 different thermoplastic polymers
- Different thicknesses
- Different crack geometries

You will

- Calculate $K_{lc}$
- Perform stress relaxation and creep experiments
  - Determine viscosity and relaxation time
  - Plot the creep modulus
- Determine relaxation times for various materials
- Identify the effects of temperature on $K_{lc}$
ASTM Standards

Mechanical tests in industrial environment must follow ASTM Standards. For example,


Both Standards are available on the website for 171A.
Requirements for sample sizes


Fracture toughness of material reflects its yield strength and its thickness, thus the design of specimens to properly measure fracture toughness (typically not known beforehand) is not straightforward (R.P. Wei, 2010).
Fracture toughness of material reflects its yield strength and its thickness, thus the design of specimens to properly measure fracture toughness (typically not known beforehand) is not straightforward.

\[ K = \frac{P}{BW^{1/2}} f\left(\frac{a_0}{W}\right) \]

- \( P = \) applied load
- \( a_0 = \) crack length
- \( W = \) specimen width
- \( B = \) specimen thickness
Specimen and crack geometry

\[
f\left(\frac{a}{W}\right) = \frac{2 + a/W}{(1 - a/W)^{3/2}} \left[ 0.886 + 4.64 \left(\frac{a}{W}\right) - 13.32 \left(\frac{a}{W}\right)^2 \\
+ 14.72 \left(\frac{a}{W}\right)^3 - 5.6 \left(\frac{a}{W}\right)^4 \right]
\]
Plane stress (thin sheet, (a)) and plane strain (thick plate, (b)) conditions

Materials and parameters in Compact Tension Configuration:
Room T (week 1)

Polymer specimen
• polycarbonate
• PMMA
• polypropylene

Specimens
• various thickness
• various crack lengths
• various crack radii
• constant width
• constant length
Interpretation of results

1. Draw a straight line AB
2. Draw a second line AB’ with a compliance 5% greater than that of line AB
3. If maximum load that the specimen was able to sustain, \( P_{\text{max}} \), falls within lines AB and AB’ use \( P_{\text{max}} \) to calculate \( K_Q \)
4. If \( P_{\text{max}} \) falls outside line AB and line AB’ then use the intersection of line AB’ and the load curve as \( P_Q \), furthermore if \( P_{\text{max}} / P_Q < 1.1 \) use \( P_Q \) in the calculation of \( K_Q \)
5. However if \( P_{\text{max}} / P_Q > 1.1 \) the test is invalid

**NOTE** 1—\( C \) is the inverse slope of line AB.

**FIG. 5 Determination of \( C \) and \( P_Q \)**
Did you get a valid value for $K_{IC}$?

The following size criteria must be satisfied to accept value of $K_Q$ (the conditional or trial $K_{IC}$) as $K_{IC}$

$$B, a, (W-a) > 2.5 \left( \frac{K_Q}{\sigma_y} \right)^2,$$

Where:

$\sigma_y$ is the yield stress of the material in the condition of testing. *Otherwise the test is not a valid $K_{IC}$ test.*

The criteria require that $B$ must be sufficient to ensure plane strain and that $(W-a)$ be sufficient to avoid excessive plasticity in the ligament.
How to use crack-tip radius and crack length data

⇒ From the work to fracture

\[ \sigma_c = \sqrt{E \gamma_s / a_o} \]

⇒ And the stress intensity equation

\[ \sigma_{\text{max}} = 2\sigma_{\text{appl}} \sqrt{a / \rho} \]

⇒ Assume that \( \sigma_c = \sigma_{\text{max}} \)

⇒ Then

\[ \sigma_{\text{appl}} = \sqrt{E \gamma / a_o} / \sqrt{4(a / \rho)} = \sqrt{E \gamma / 4a_o} \sqrt{\rho / a} = A \left( \sqrt{\frac{\rho}{a}} \right) \]

measured
calculate for known \( \rho, c \)
measured
Deformation of polymers (week 2)

⇒ Samples will be tested in compression
  ⇩ Barrel shaped configuration

⇒ In one case, the platens are fixed
  ⇩ Stress relaxation, Maxwell model
    → Stress is recorded, strain constant

⇒ In the other case, the measured load is fixed
  ⇩ Creep, Voight model or better Four element model
    → Strain is recorded, stress constant
Model behavior under suddenly applied load

(1) Model can be applied to interpret experimental data starting from the point (see vertical arrow) where machine applied constant load (approximately at $t=70$ sec, for convenience in the graph the sign of load was changed);

(2) After load was removed (approximately at $t=650$ sec, see another vertical arrow) strains should be calculated based on manual measurements of sample size.
1. The test started when strain reach the constant value, (see vertical arrow) (in this example approximately at $t=70$ sec). For convenience in the graphs the signs of load and strain were changed;  
2. Based on the behavior of your sample you need to decide which model to select to interpret the data: Maxwell model, (if final stress is assumed to relax to zero) or standard linear solid model (three elements or Kelvin model) if the stress assumed a constant value.
Temperature effects on $K_{lc}$ (week 3)

⇒ You will use the data from the first week (20°C)
⇒ You will perform the same measurements while the samples are in an ice bath (0°C)
⇒ You will perform the same measurements while the samples are heated by a heat lamp