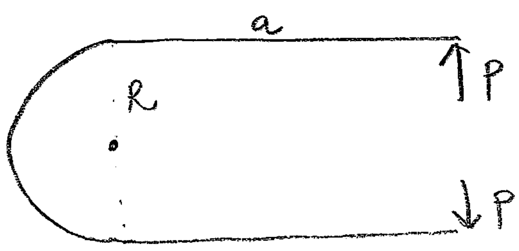
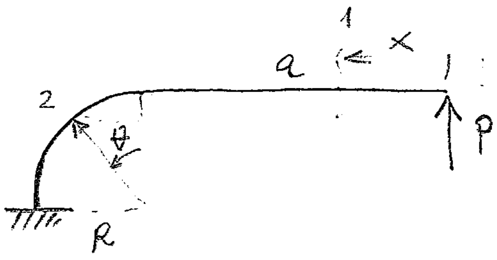
Hw #4 : Solutions

10.10)



$\Rightarrow$



$$\delta = \frac{\partial U}{\partial P}$$

$$U = U_1 + U_2 ,$$

$$U_1 = \int_0^a \frac{M_1^2 \, dx}{2EI} , \quad M_1 = P \cdot x \quad ; \quad \frac{\partial U_1}{\partial P} = \int_0^a \frac{M_1 \frac{\partial M_1}{\partial P}}{EI} dx$$

$$\frac{\partial U_1}{\partial P} = \int_0^a \frac{Px \cdot x}{EI} dx = \frac{Pa^3}{3EI}$$

$$U_2 = \int \frac{M_2^2 R d\theta}{2EI} , \quad M_2 = P(a + R\sin\theta) , \quad \frac{\partial M_2}{\partial P} = a + R\sin\theta$$

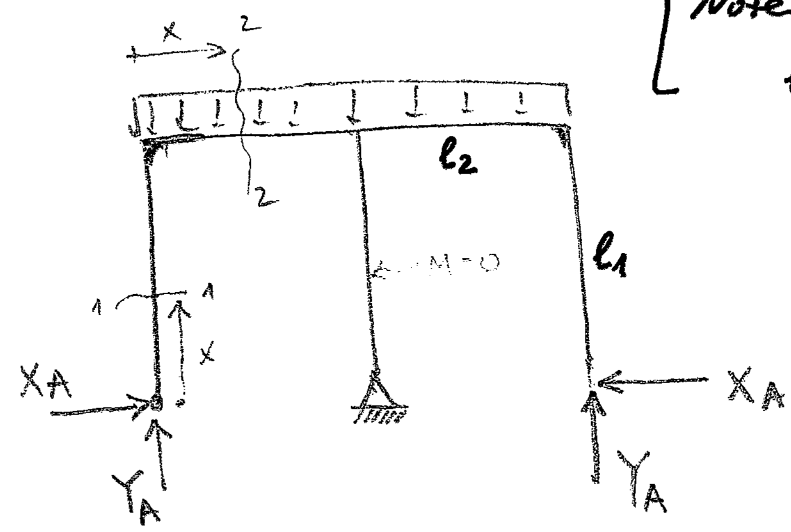
$$\frac{\partial U_2}{\partial P} = \int_0^{\pi/2} \frac{M_2 \frac{\partial M_2}{\partial P}}{EI} R d\theta = \frac{PR}{EI} \int_0^{\pi/2} (a + R\sin\theta)^2 d\theta$$

$$= \frac{PR}{EI} \int_0^{\pi/2} (a^2 + 2aR\sin\theta + R^2 \sin^2\theta) d\theta = \frac{PR}{EI} \left(a^2 \frac{\pi}{2} + 2aR + R^2 \frac{\pi}{4} \right)$$

$$\delta = \frac{\partial U_1}{\partial P} + \frac{\partial U_2}{\partial P} = \frac{Pa^3}{3EI} + \frac{Pa^3}{4EI} \left[2\pi \frac{R}{a} + 8 \left(\frac{R}{a}\right)^2 + \pi \left(\frac{R}{a}\right)^3 \right]$$

$$\Delta = 2\delta = \frac{Pa^3}{6EI} \left[4 + 6\pi \frac{R}{a} + 24 \left(\frac{R}{a}\right)^2 + 3\pi \left(\frac{R}{a}\right)^3 \right].$$

10.18)



[Note : My l_1 & l_2 are l_2 & l_1 in the book.]

$2Y_A + Y_B = 2pl_1$

$Y_B = 2pl_2 - 2Y_A$.

$$\frac{\partial U}{\partial X_A} = 0 \quad , \quad \frac{\partial U}{\partial Y_A} = 0$$

$M_1 = -X_A \cdot x$

$M_2 = Y_A x - X_A l_1 - \frac{px^2}{2}$

$$\frac{1}{2} U = \int_0^{l_1} \frac{M_1^2 dx}{2E_1 I_1} + \int_0^{l_2} \frac{M_2^2 dx}{2E_2 I_2} ,$$

$$\int_0^{l_1} \frac{M_1 \frac{\partial M_1}{\partial X_A}}{E_1 I_1} dx + \int_0^{l_2} \frac{M_2 \frac{\partial M_2}{\partial X_A}}{E_2 I_2} dx = 0$$

$$\int_0^{l_1} \frac{M_1 \frac{\partial M_1}{\partial Y_A}}{E_1 I_1} dx + \int_0^{l_2} \frac{M_2 \frac{\partial M_2}{\partial Y_A}}{E_2 I_2} dx = 0$$

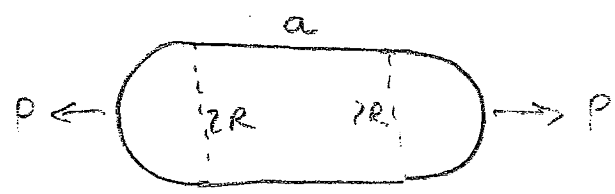
$$\left.\begin{array}{l} \displaystyle\int_0^{l_1} \frac{(-X_A x)(-x)}{E_1 I_1} dx + \int_0^{l_2} \frac{(Y_A x - X_A l_1 - \frac{1}{2}px^2)(-l_1)}{E_2 I_2} dx = 0 \\ \displaystyle\int_0^{l_1} \frac{(-X_A x) \cdot 0}{E_1 I_1} dx + \int_0^{l_2} \frac{(Y_A x - X_A l_1 - \frac{px^2}{2}) x}{E_2 I_2} dx = 0 \end{array}\right\}$$

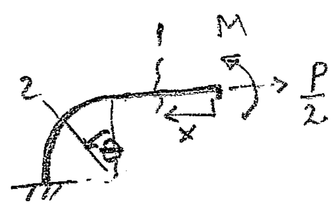
$$\therefore \quad \left.\begin{array}{l} 3k Y_A l_1 - 2(3k+1) X_A l_2 = k p l_1^2 \\ 8Y_A - 12 X_A l_2 = 3pl_1^2 \end{array}\right\} \Rightarrow \quad X_A = \frac{k}{4(3k+4)} \frac{pl_1^2}{l_2}$$

$$Y_A = \frac{3(k+1)}{2(3k+4)} pl_1 ,$$

$$k = \frac{E_2 I_2}{E_1 I_1} \frac{l_1}{l_2}$$

10.19)





$$\frac{\partial U}{\partial M} = 0$$

$$U = \int_0^a \frac{M_1^2 \, dx}{2EI} + \int_0^{\pi/2} \frac{M_2^2 \, R \, d\theta}{2EI}$$

$$M_1 = M \quad , \quad M_2 = M - \frac{P}{2} R (1 - \cos\theta)$$

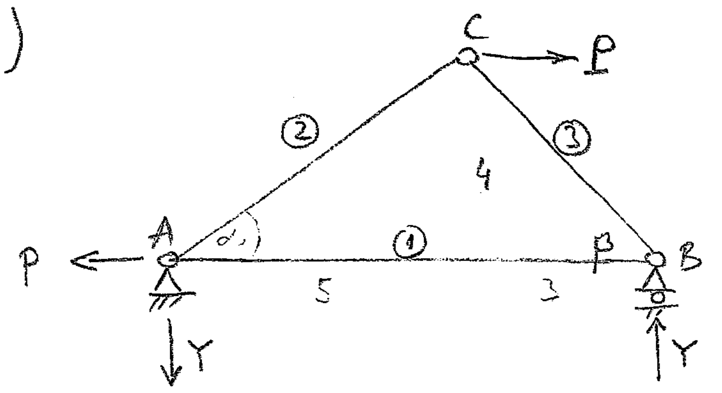
$$\frac{\partial M_1}{\partial M} = 1 \, , \qquad \frac{\partial M_2}{\partial M} = 1$$

$$\frac{\partial U}{\partial M} = \int_0^a \frac{M_1 \frac{\partial M_1}{\partial M} dx}{EI} + \int_0^{\pi/2} \frac{M_2 \frac{\partial M_2}{\partial M}}{EI} R \, d\theta = 0$$

$$\therefore \quad M = \frac{PR^2}{2} \, \frac{\pi - 2}{a + \pi R}$$

$M_{max} = M.$ $\qquad\qquad M(\theta = \pi/2) = M - \frac{PR}{2}.$

10.22)



$$Y \cdot 8 = P \cdot 4$$

$$Y = P/2$$

$$\tan\alpha = \frac{4}{5} = 0.8 \quad , \alpha = 38.66^\circ$$

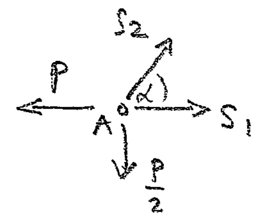
$$\sin\alpha = 0.625$$

$$\cos\alpha = 0.781$$

$$\beta = 53.13$$

$$\sin\beta = 0.8$$

$$u_c = \sum_{i=1}^{3} \frac{S_i \bar{S}_i}{EA} l_i$$



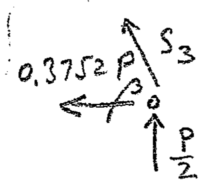
$$S_2 \sin\alpha = \frac{P}{2}$$

$$S_2 = 0.8P$$

$$S_1 = 0.3752P$$

$$u_c = \frac{S_1\bar{S}_1 l_1 + S_2\bar{S}_2 l_2 + S_3\bar{S}_3 l_3}{EI}$$

$$EI\,u_c = 0.3752P \cdot 0.3752 \cdot 8 + 0.8P \cdot 0.8 \cdot 6.4 + 0.625P \cdot 0.625 \cdot 5$$

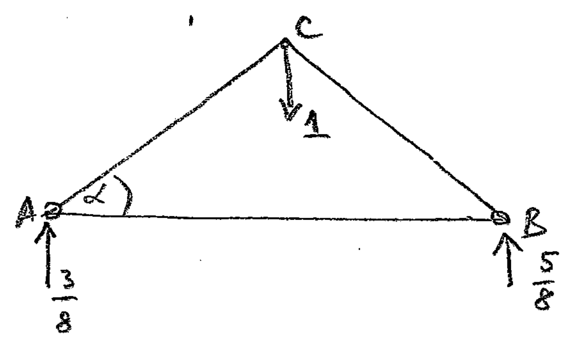


$$S_3 \sin\beta + \frac{P}{2} = 0$$

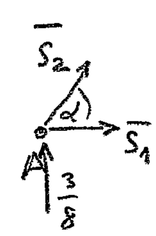
$$S_3 = -0.625P$$

$$u_c = \frac{7.175\,P \cdot m}{EI}$$

$$v_c = \sum_{i=1}^{3} \frac{S_i \bar{S}_i}{EI} l_i$$



$$EI\,v_c = 0.3752P \cdot 0.4686 \cdot 8$$
$$+ 0.8P \cdot (-0.6) \cdot 6.4$$
$$+ (-0.625P)(-0.78125) \cdot 5$$

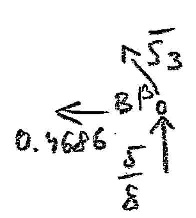


$$\bar{S}_2 \sin\alpha + \frac{3}{8} = 0$$

$$\bar{S}_2 = -0.6$$

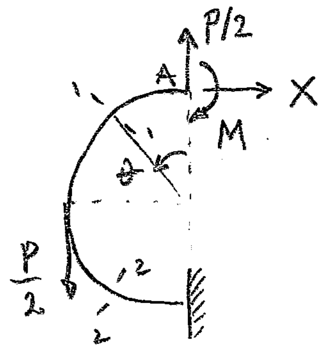
$$\bar{S}_1 + \bar{S}_2 \cos\alpha = 0 \rightarrow \bar{S}_1 = 0.4686$$

$$v_c = \frac{0.776\,P}{EI}$$



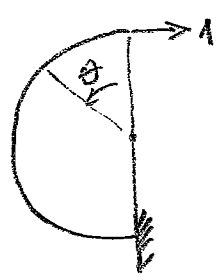
$$\bar{S}_3 \sin\beta + \frac{5}{8} = 0 \rightarrow \bar{S}_3 = -0.78125$$

10.26)

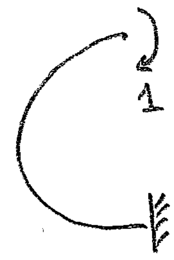


$$M_1 = \frac{P}{2} R \sin\theta - XR(1-\cos\theta) - M \quad , \; 0 \le \theta \le \frac{\pi}{2}$$

$$M_2 = \frac{P}{2} R - XR(1-\cos\theta) - M \quad , \; \frac{\pi}{2} \le \theta \le \pi$$



$$\overline{M}_1 = -R(1-\cos\theta)$$
$$\overline{M}_2 = -R(1-\cos\theta)$$



$$\overline{M}_1 = -1$$
$$\overline{M}_2 = -1$$

$$u_A = 0 : \quad \int_0^{\pi/2} \frac{M_1 \overline{M}_1}{EI} R\,d\theta + \int_{\pi/2}^{\pi} \frac{M_2 \overline{M}_2}{EI} R\,d\theta = 0$$

$$\varphi_A = 0 : \quad \int_0^{\pi/2} \frac{M_1 \cdot \overline{M}_1}{EI} R\,d\theta + \int_{\pi/2}^{\pi} \frac{M_2 \overline{M}_2}{EI} R\,d\theta = 0$$

$$\int_0^{\pi/2} \left[\frac{P}{2} R \sin\theta - XR(1-\cos\theta) - M\right](-R)(1-\cos\theta) R\,d\theta + \int_{\frac{\pi}{2}}^{\pi} \left[\frac{P}{2} R - XR(1-\cos\theta) - M\right](-R)(1-\cos\theta) R\,d\theta =$$

$$\int_0^{\pi/2} \left[\frac{P}{2} R \sin\theta - XR(1-\cos\theta) - M\right](-1) R\,d\theta + \int_{\pi/2}^{\pi} \left[\frac{P}{2} R - XR(1-\cos\theta) - M\right](-1) R\,d\theta = 0$$

Integrating and solving two algebraic eqs. for $(X;M)$ one obtains

$$X = \frac{P}{2\pi} \quad , \quad M = \frac{PR}{4} \; .$$