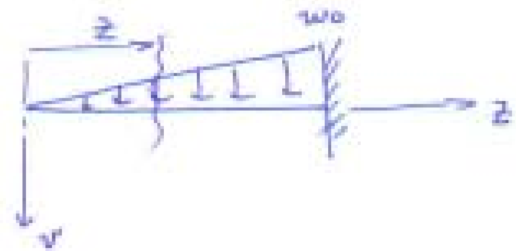


$w(z) = w_0 \frac{z}{L}$

$$EI v'' = -M(z) \,, \quad M(z) = -\frac{1}{2} z \cdot w(z) \cdot \frac{z}{3} = -\frac{1}{6} w_0 \frac{z^3}{L}$$

$$EI v'' = \frac{1}{6} \frac{w_0}{L} z^3$$

$$EI v' = \frac{1}{24} \frac{w_0}{L} z^4 + C_1$$

$$EI v = \frac{1}{120} \frac{w_0}{L} z^5 + C_1 z + C_2$$

$$v'(L) = 0 \rightarrow \quad 0 = \frac{1}{24} w_0 L^3 + C_1 \implies C_1 = -\frac{1}{24} w_0 L^3$$

$$v(L) = 0 \rightarrow \quad 0 = \frac{1}{120} w_0 L^4 - \frac{1}{24} w_0 L^4 + C_2 \implies C_2 = \frac{1}{30} w_0 L^4$$

$$\therefore \quad EI v = \frac{1}{120} \frac{w_0}{L} z^5 - \frac{1}{24} w_0 L^3 z + \frac{1}{30} w_0 L^4$$

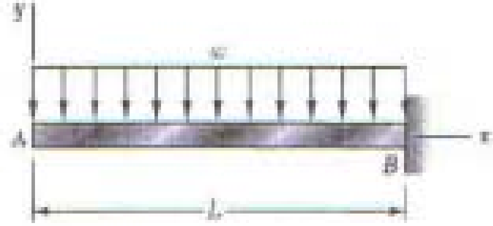
$$v = \frac{w_0}{120 EI L} \left(z^5 - 5 L^4 z + 4 L^5 \right)$$

$$v' = \frac{w_0}{24 EI L} \left(z^4 - L^4 \right) .$$

$$v(0) = \frac{w_0 L^4}{30 EI} \quad ; \quad v'(0) = -\frac{w_0 L^3}{24 EI} .$$

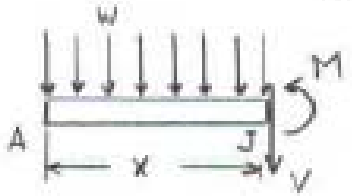
Problem 9.3

9.1 through 9.4 For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.



$[x = L,\ y = 0]$

$[x = L,\ \frac{dy}{dx} = 0]$



$$\circlearrowleft \Sigma M_J = 0: \quad (wx)\frac{x}{2} + M = 0$$

$$M = -\frac{1}{2}wx^2$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + C_1$$

$$[x = L,\ \frac{dy}{dx} = 0] \qquad 0 = -\frac{1}{6}wL^3 + C_1$$

$$C_1 = \frac{1}{6}wL^3$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{6}wL^3$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{6}wL^3x + C_2$$

$$[x = L,\ y = 0] \qquad 0 = -\frac{1}{24}wL^4 + \frac{1}{6}wL^4 + C_2 = 0$$

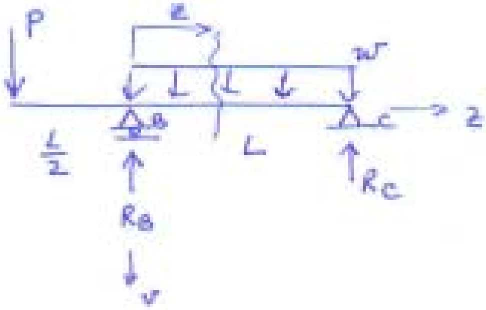
$$C_2 = \left(\frac{1}{24} - \frac{1}{6}\right)wL^4 = -\frac{3}{24}wL^4$$

(a) Elastic curve. $\quad y = -\frac{w}{24EI}(x^4 - 4L^3x + 3L^4)$ ◄

(b) y @ $x = 0$: $\quad y_A = -\frac{3wL^4}{24EI} = -\frac{wL^4}{8EI} \qquad y_A = \frac{wL^4}{8EI}\downarrow$ ◄

(c) $\frac{dy}{dx}$ @ $x = 0$: $\quad \left.\frac{dy}{dx}\right|_A = \frac{wL^3}{6EI} \qquad \theta_A = \frac{wL^3}{6EI}$ ◄

9.7



$P = \frac{wL}{5}$

$R_B + R_C = wL + \frac{wL}{5}$

$R_B + R_C = \frac{6}{5} wL$

$\Sigma M_C = 0: \; P \cdot \frac{3L}{2} - R_B L + \frac{wL^2}{2} = 0$

$R_B = \frac{4}{5} wL \quad , \quad R_C = \frac{2}{5} wL$

Portion BC:

$EI v'' = -M(z) \quad , \quad M(z) = R_C (L - z) - w \frac{(L-z)^2}{2}$

$M(z) = -\frac{1}{2} w z^2 + \frac{3}{5} wLz - \frac{1}{10} wL^2$

$EI v'' = \frac{1}{2} w z^2 - \frac{3}{5} wLz + \frac{1}{10} wL^2$

$EI v' = \frac{1}{6} w z^3 - \frac{3}{10} wLz^2 + \frac{1}{10} wL^2 z + C_1$

$EI v = \frac{1}{24} w z^4 - \frac{1}{10} wLz^3 + \frac{1}{20} wL^2 z^2 + C_1 z + C_2$

$v(0) = 0 \Rightarrow C_2 = 0$

$v(L) = 0 \Rightarrow 0 = \frac{1}{24} wL^4 - \frac{1}{10} wL^4 + \frac{1}{20} wL^4 + C_1 L \Rightarrow C_1 = \frac{wL^3}{120}$

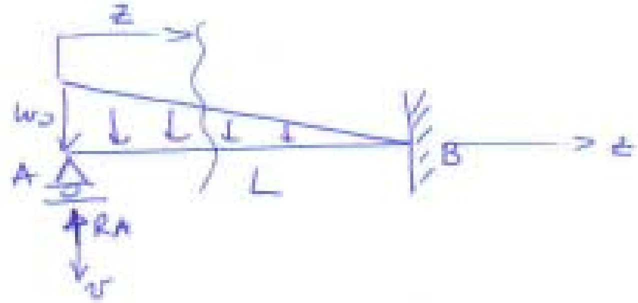
$\therefore \; v = \frac{w}{EI} \left(\frac{1}{24} z^4 - \frac{1}{10} L z^3 + \frac{1}{20} L^2 z^2 + \frac{1}{120} L^3 z \right)$

$v' = \frac{w}{EI} \left(\frac{1}{6} z^3 - \frac{3}{10} L z^2 + \frac{1}{10} L^2 z + \frac{1}{120} L^3 \right)$

b) $v\left(\frac{L}{2}\right) = \frac{w}{EI} \left[\frac{1}{24} \left(\frac{L}{2}\right)^4 - \frac{1}{10} L \left(\frac{L}{2}\right)^3 + \frac{1}{20} L^2 \left(\frac{L}{2}\right)^2 + \frac{1}{120} L^3 \frac{L}{2} \right] = \frac{13 wL^4}{1920 EI}$.

c) $v'(0) = \frac{w}{EI} \cdot \frac{L^3}{120}$.

9.22



$$w(z) = \frac{w_0}{L}(L-z)$$

$$EI\, v^{IV} = w(z)$$

$$EI\, v^{IV} = \frac{w_0}{L}(L-z)$$

$$EI\, v''' = \frac{w_0}{L}\left(Lz - \frac{z^2}{2}\right) + C_1$$

$$EI\, v'' = \frac{w_0}{L}\left(L\frac{z^2}{2} - \frac{z^3}{6}\right) + C_1 z + C_2$$

$$EI\, v' = \frac{w_0}{L}\left(L\frac{z^3}{6} - \frac{z^4}{24}\right) + C_1\frac{z^2}{2} + C_2 z + C_3$$

$$EI\, v = \frac{w_0}{L}\left(L\frac{z^4}{24} - \frac{z^5}{120}\right) + C_1\frac{z^3}{6} + C_2\frac{z^2}{2} + C_3 z + C_4$$

B.C.'s :

$$v(0) = 0 \longrightarrow C_4 = 0$$

$$M(0) = 0 \rightarrow v''(0) = 0 \longrightarrow C_2 = 0$$

$$\left.\begin{aligned} v'(L) = 0 &\rightarrow 0 = \frac{w_0}{L}\left(\frac{L^4}{6} - \frac{L^4}{24}\right) + C_1\frac{L^2}{2} + C_3 \\ v(L) = 0 &\longrightarrow 0 = \frac{w_0}{L}\left(\frac{L^5}{24} - \frac{L^5}{120}\right) + C_1\frac{L^3}{6} + C_3 L \end{aligned}\right\} \Rightarrow \begin{aligned} C_1 &= -\frac{11}{40} w_0 L \\ C_3 &= \frac{w_0 L^3}{80} \end{aligned}$$

Reaction at A:

$$R_A = V(0) = -EI\, v'''(0) = -C_1 = \frac{11}{40} w_0 L.$$

9.22

$\rightarrow$ Second way

$$EI\,v'' = -M(z)\ , \quad M(z) = R_A \cdot z - z\,w(z)\frac{z}{2} - \frac{1}{2}[w_0 - w(z)]\frac{z}{3}z$$

$$w(z) = \frac{w_0}{L}(L-z) = w_0 - w_0\frac{z}{L}$$

$$M(z) = R_A z - \frac{1}{2}w_0 z^2 + \frac{1}{6}w_0\frac{z^3}{L}$$

$$EI\,v'' = -R_A z + \frac{1}{2}w_0 z^2 - \frac{1}{6}w_0\frac{z^3}{L}$$

$$EI\,v' = -R_A\frac{z^2}{2} + \frac{1}{6}w_0 z^3 - \frac{1}{24}w_0\frac{z^4}{L} + C_1$$

$$EI\,v = -R_A\frac{z^3}{6} + \frac{1}{24}w_0 z^4 - \frac{1}{120}w_0\frac{z^5}{L} + C_1 z + C_2$$

$$v(0) = 0 \rightarrow C_2 = 0$$

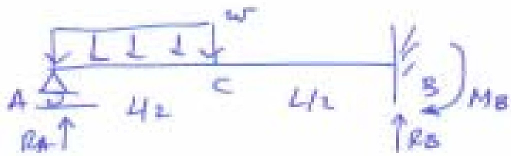
$$\left.\begin{array}{l} v'(L) = 0 \rightarrow 0 = -R_A\frac{L^2}{2} + \frac{1}{6}w_0 L^3 - \frac{1}{24}w_0 L^3 + C_1 \\ v(L) = 0 \rightarrow 0 = -R_A\frac{L^3}{6} + \frac{1}{24}w_0 L^4 - \frac{1}{120}w_0 L^4 + C_1 L + 0 \end{array}\right\} (*)$$

Solve (*) for C_1 and R_A to obtain

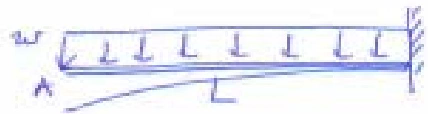
$$C_1 = \frac{1}{80}w_0 L^3$$

$$R_A = \frac{11}{40}w_0 L\ .$$

9.26

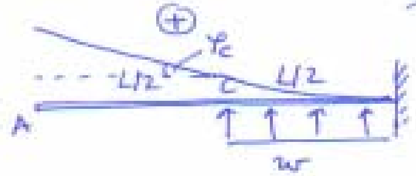


By superposition, above is equal to (use Table 4 App. D page 762)



$v_A = \frac{wL^4}{8EI}$ ↓ down

⊕

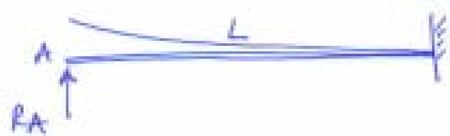


$v_A = v_C + \frac{L}{2}\cdot\varphi_C$ ↑ up

$= \frac{w(L/2)^4}{8EI} + \frac{w(L/2)^3}{6EI}\cdot\frac{L}{2}$

$v_A = \frac{7}{384}\frac{wL^4}{EI}$

⊕



$v_A = \frac{R_A\cdot L^3}{3EI}$ ↑ up

Total $v_A = 0 \Rightarrow \frac{wL^4}{8EI} - \frac{7}{384}\frac{wL^4}{EI} - \frac{R_A L^3}{3EI} = 0$

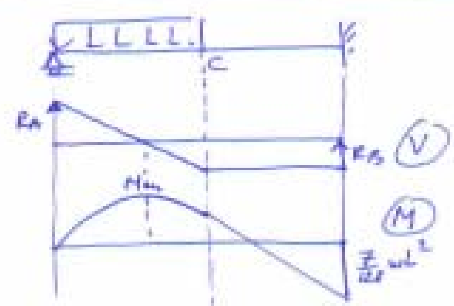
$\Rightarrow \quad R_A = \frac{41}{128}wL.$

$R_A + R_B = w\frac{L}{2} \Rightarrow R_B = \frac{23}{128}wL$

$R_B\cdot L - M_B - w\frac{L}{2}\cdot\frac{L}{4} = 0 \Rightarrow M_B = \frac{7}{128}wL^2$

$M_C = 0.0351\,wL^2$

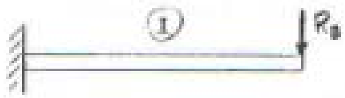
$M_{max} = M\left(\frac{41L}{128}\right) = 0.0513\,wL^2$



Problem 9.83

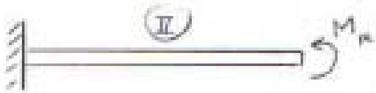
9.83 and 9.84 For the beam shown, determine the reaction at *B*.

Beam is second degree indeterminate. Choose R_B and M_B as redundant reactions.



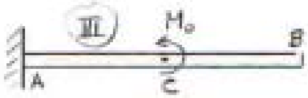
Loading I. Case 1 of Appendix D.

$$(y_B)_I = -\frac{R_B L^3}{3EI}, \quad (\theta_B)_I = -\frac{R_B L^2}{2EI}$$



Loading II. Case 3 of Appendix D.

$$(y_B)_{II} = \frac{M_B L^2}{2EI}, \quad (\theta_B)_{II} = \frac{M_B L}{EI}$$



Loading III. Case 3 applied to portion AC.

$$(y_C)_{III} = \frac{M_0 (L/2)^2}{2EI} = \frac{M_0 L^2}{8EI}$$

$$(\theta_C)_{III} = \frac{M_0 (L/2)}{EI} = \frac{M_0 L}{2EI}$$

Portion CB remains straight.

$$(y_B)_{III} = (y_C)_{III} + \frac{L}{2}(\theta_C)_{III} = \frac{3}{8}\frac{M_0 L^2}{EI}$$

$$(\theta_B)_{III} = (\theta_C)_{III} = \frac{1}{2}\frac{M_0 L^2}{EI}$$

Superposition and constraint:

$$y_B = (y_B)_I + (y_B)_{II} + (y_B)_{III} = 0$$

$$-\frac{L^3}{3EI} R_B + \frac{L^2}{2EI} M_B + \frac{3}{8}\frac{M_0 L^2}{EI} = 0 \qquad (1)$$

$$\theta_B = (\theta_B)_I + (\theta_B)_{II} + (\theta_B)_{III} = 0$$

$$-\frac{L^2}{2EI} R_B + \frac{L}{EI} M_B + \frac{1}{2}\frac{M_0 L}{EI} = 0 \qquad (2)$$

Solving (1) and (2) simultaneously,

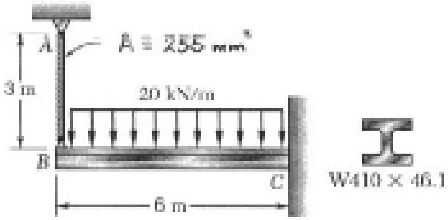
$$R_B = \frac{3}{2}\frac{M_0}{L} \downarrow$$

$$M_B = \frac{1}{4} M_0$$

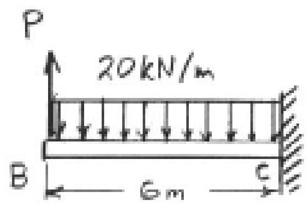
9.92

Problem 9.89

9.89 The cantilever beam BC is attached to the steel cable AB as shown. Knowing that the cable is initially taught, determine the tension in the cable caused by the distributed load shown. Use $E = 200$ GPa.



Let P be the tension developed in member AB and δ_B be the elongation of that member.



$$\delta = \frac{PL}{EA} = \frac{(P)(3)}{(200\times10^{9})(254.47\times10^{-6})} = 58.946\times10^{-9}\,P$$

Beam BC: $I = 156\times10^{6}\ \text{mm}^4 = 156\times10^{-6}\ \text{m}^4$

$EI = (200\times10^{9})(156\times10^{-6}) = 31.2\times10^{6}\ \text{N}\cdot\text{m}^2$

Loading I. 20 kN/m downward.

Refer to Case 2 of Appendix D.

$$(y_B)_1 = -\frac{wL^4}{8EI} = -\frac{(20\times10^{3})(6)^4}{(8)(31.2\times10^{6})} = -103.846\times10^{-3}\ \text{m}$$

Loading II. Upward force P at point B.

Refer to Case 1 of Appendix D.

$$(y_B)_2 = \frac{PL^3}{3EI} = \frac{P(6)^3}{(3)(31.2\times10^{6})} = 2.3077\times10^{-6}\,P$$

By superposition, $y_B = (y_B)_1 + (y_B)_2$

Also, matching the deflection at B, $y_B = -\delta$

$$-103.846\times10^{-3} + 2.3077\times10^{-6}\,P = -58.946\times10^{-9}\,P$$

$$2.3666\times10^{-6}\,P = -103.846\times10^{-3} \qquad P = 43.9\times10^{3}\ \text{N}$$

$P = 43.9$ kN