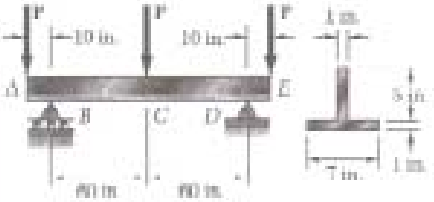
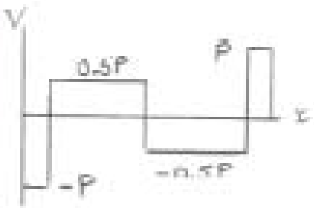
Problem 5.87

5.87 Determine the allowable value of P for the loading shown, knowing that the allowable normal stress is +8 ksi in tension and −18 ksi in compression.



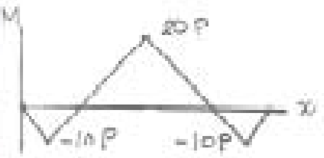
Reactions. $B = D = 1.5P \uparrow$

Shear diagram.
A to B^- $V = -P$
B^+ to C. $V = -P + 1.5P = 0.5P$
C^+ to D^-. $V = 0.5P - P = -0.5P$
D^+ to E. $V = -0.5P + 1.5P = P$



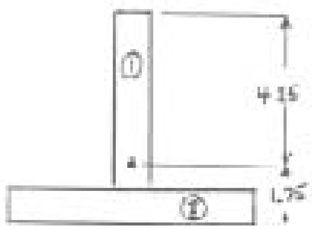
Areas.
A to B. $(10)(-P) = -10P$
B to C. $(60)(0.5P) = 30P$
C to D. $(60)(-0.5P) = -30P$
D to E. $(10)(P) = 10P$

Bending moments. $M_A = 0$
$M_B = 0 - 10P = -10P$
$M_C = -10P + 30P = 20P$
$M_D = 20P - 30P = -10P$
$M_E = -10P + 10P = 0$



Largest positive bending moment : 20P
Largest negative bending moment : −10P

Centroid and moment of inertia.



Part	A, in²	$\bar{y}_0$, in.	$A\bar{y}_0$, in³	d, in.	Ad^2 in⁴	$\bar{I}$, in⁴
1	5	3.5	17.5	1.75	15.3125	10.417
2	7	0.5	3.5	1.25	10.9375	0.583
Σ	12		21		26.25	11.000

$\bar{Y} = \frac{21}{12} = 1.75$ in.

$I = \Sigma Ad^2 + \Sigma \bar{I} = 37.25$ in⁴

Top: $y = 4.25$ in.
Bottom: $y = -1.75$ in.

$\sigma = -\frac{My}{I}$

Top, Tension $8 = -\frac{(-10P)(4.25)}{37.25}$ $P = 7.01$ kips

Top, Comp. $-18 = -\frac{(20P)(4.25)}{37.25}$ $P = 7.89$ kips

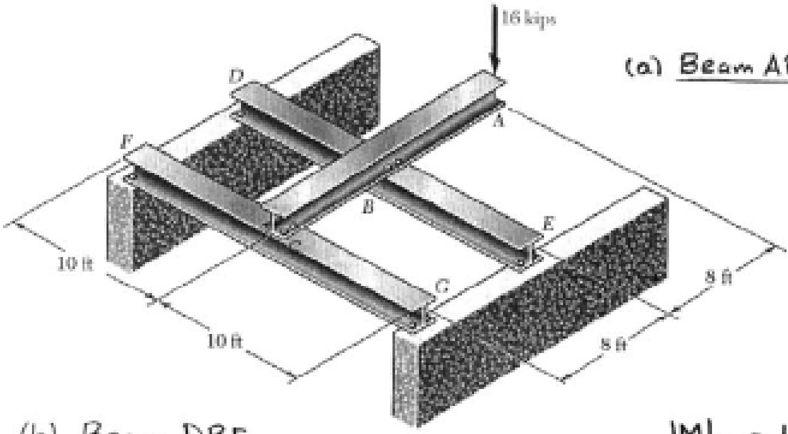
Bot. Tension $8 = -\frac{(20P)(-1.75)}{37.25}$ $P = 8.51$ kips

Bot. Comp $-18 = -\frac{(-10P)(-1.75)}{37.25}$ $P = 38.3$ kips

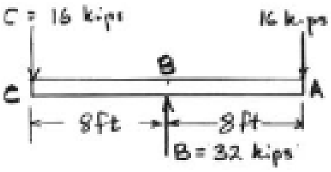
Smallest value of P is the allowable value. $P = 7.01$ kips

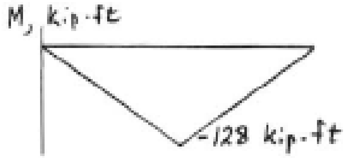
Problem 5.89

5.89 Beam *ABC* is bolted to beams *DBE* and *FCG*. Knowing that the allowable normal stress is 24 ksi, select the most economical wide-flange shape that can be used (*a*) for beam *ABC*, (*b*) for beam *DBE* (*c*) for beam *FCG*.



(a) Beam ABC





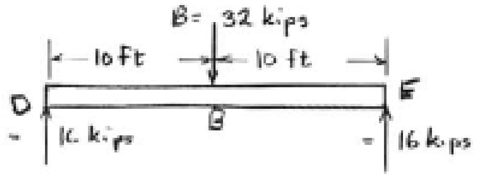
$M_B = -(8)(16) = -128$ kip·ft

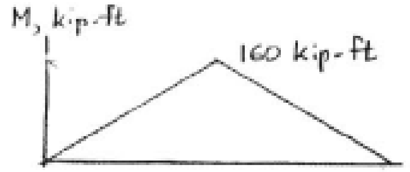
$|M|_{max} = 128$ kip·ft $= 1536$ kip·in.

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{1536}{24} = 64 \text{ in}^3$$

Shape	S, in³	
W 21 × 44	81.6	
W 18 × 50	88.9	
W 16 × 40	64.7	← (a) W16 × 40 ◄
W 14 × 53	77.8	
W 12 × 50	64.7	
W 10 × 68	75.7	

(b) Beam DBE



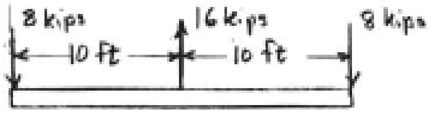


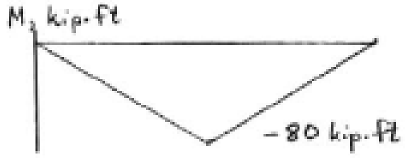
$|M|_{max} = 160$ kip·ft $= 1920$ kip·in

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{1920}{24} = 80 \text{ in}^3$$

Shape	S, in³	
W 21 × 44	81.6	← (b) W 21 × 44 ◄
W 18 × 50	88.3	
W 16 × 57	97.2	
W 14 × 68	103	
W 12 × 72	97.4	
W 10 × 112	126	

(c) Beam FCG





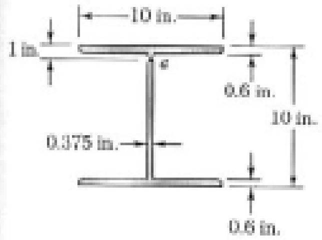
$|M|_{max} = 80$ kip·ft $= 960$ kip·in

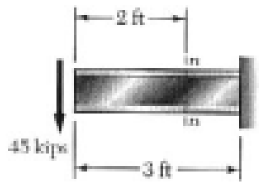
$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{960}{24} = 40 \text{ in}^3$$

Shape	S, in³	
W 18 × 35	57.6	
W 16 × 31	47.2	
W 14 × 30	42.0	← (c) W 14 × 30 ◄
W 12 × 35	45.6	
W 10 × 39	42.1	
W 8 × 48	43.3	

Problem 6.9

6.9 through 6.12 For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.



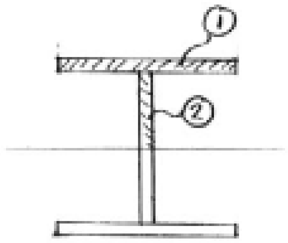


$V = 45$ kips

Part	A (in²)	d (in.)	Ad^2 (in⁴)	$\bar{I}$ (in⁴)
Flange	6.00	4.7	132.54	0.18
Web	3.30	0	0	21.296
Flange	6.00	4.7	132.54	0.18
Σ			265.08	21.656

$$I = \Sigma Ad^2 + \Sigma \bar{I}$$
$$= 265.08 + 21.656$$
$$= 286.736 \text{ in}^4$$

(a)

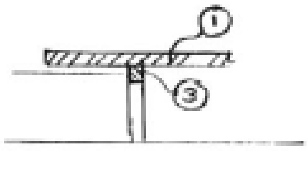


$$Q = A_1\bar{y}_1 + A_2\bar{y}_2$$
$$= (6.00)(4.7) + (0.375)(4.4)(2.2) = 31.83 \text{ in}^3$$

$t = 0.375$ in.

$$\tau_{max} = \frac{VQ}{It} = \frac{(45)(31.83)}{(286.736)(0.375)} = 13.32 \text{ ksi}$$

(b)

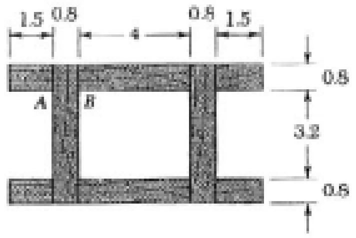


$$Q_a = A_1\bar{y}_1 + A_3\bar{y}_3$$
$$= (6.00)(4.7) + (0.375)(0.4)\left(\frac{4.4+4.0}{2}\right)$$
$$= 28.83 \text{ in}^3$$

$t = 0.375$ in.

$$\tau_a = \frac{VQ_a}{It} = \frac{(45)(28.83)}{(286.736)(0.375)} = 12.07 \text{ ksi}$$

Problem 6.32

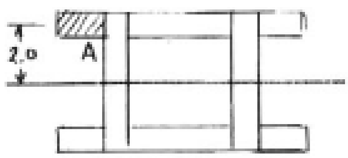


Dimensions in inches

6.32 The built-up beam was made by gluing together several wooden planks. Knowing that the beam is subjected to a 1200-lb shear, determine the average shearing stress in the glued joint (*a*) at *A*, (*b*) at *B*.

$$I = 2\left[\tfrac{1}{12}(0.8)(4.8)^3 + \tfrac{1}{12}(7)(0.8)^3 + (7)(0.8)(2.0)^2\right]$$

$$= 60.143 \text{ in}^4$$

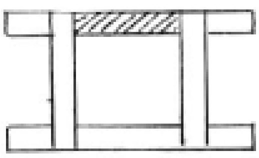


(a) $A_a = (1.5)(0.8) = 1.2 \text{ in}^2 \qquad \bar{y}_a = 2.0 \text{ in.}$

$Q_a = A_a \bar{y}_a = 2.4 \text{ in}^3$

$t_a = 0.8 \text{ in.}$

$$\tau_a = \frac{VQ_a}{I t_a} = \frac{(1200)(2.4)}{(60.143)(0.8)} = 59.9 \text{ psi}$$



(b) $A_b = (4)(0.8) = 3.2 \text{ in}^2 \qquad \bar{y}_b = 2.0 \text{ in.}$

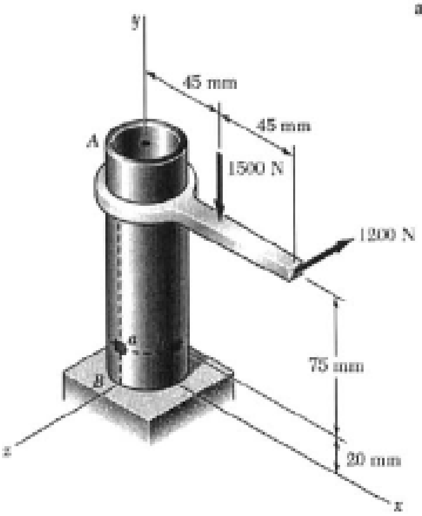
$Q_b = A_b \bar{y}_b = (3.2)(2.0) = 6.4 \text{ in}^3$

$t_b = (2)(0.8) = 1.6 \text{ in.}$

$$\tau_b = \frac{VQ_b}{I t_b} = \frac{(1200)(6.4)}{(60.143)(1.6)} = 79.8 \text{ psi}$$

Problem 8.39

8.39 Two forces are applied to the pipe AB as shown. Knowing that the pipe has inner and outer diameters equal to 35 and 42 mm, respectively, determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*.

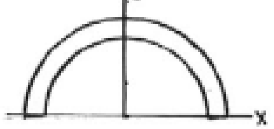


$c_o = \frac{d_o}{2} = 21 \text{ mm}, \quad c_i = \frac{d_i}{2} = 17.5 \text{ mm}$

$A = \pi(c_o^2 - c_i^2) = 423.33 \text{ mm}^2$

$J = \frac{\pi}{2}(c_o^4 - c_i^4) = 158.166 \times 10^3 \text{ mm}^4$

$I = \frac{1}{2}J = 79.083 \times 10^3 \text{ mm}^4$



For semicircle with semicircular cutout

$Q = \frac{2}{3}(c_o^3 - c_i^3)$

$Q = 2.6011 \times 10^3 \text{ mm}^3$

At the section containing points a and b

$P = -1500 \text{ N} \qquad V_z = -1200 \text{ N} \qquad V_x = 0$

$M_z = -(45 \times 10^{-3})(1500) = -67.5 \text{ N·m}$

$M_x = -(75 \times 10^{-3})(1200) = -90 \text{ N·m}$

$T = (90 \times 10^{-3})(1200) = 108 \text{ N·m}$

(a) $\sigma = \frac{P}{A} - \frac{M_x c}{I} = \frac{-1500}{423.33 \times 10^{-6}} - \frac{(-90)(21 \times 10^{-3})}{79.083 \times 10^{-9}} = 20.4 \text{ MPa}$ ◄

$\tau = \frac{Tc}{J} + \frac{V_x Q}{It} = \frac{(108)(21 \times 10^{-3})}{158.166 \times 10^{-9}} + 0 = 14.34 \text{ MPa}$ ◄

(b) $\sigma = \frac{P}{A} + \frac{M_z c}{I} = \frac{-1500}{423.33 \times 10^{-6}} + \frac{(-67.5)(21 \times 10^{-3})}{79.083 \times 10^{-9}} = -21.5 \text{ MPa}$ ◄

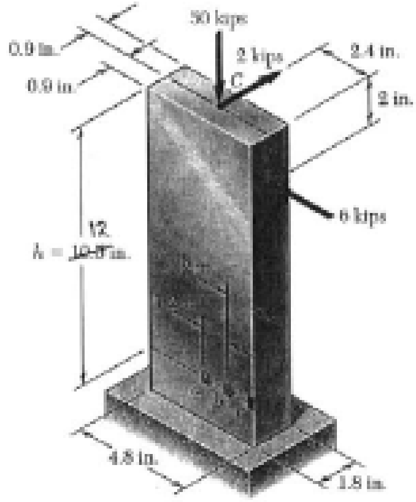
$\tau = \frac{Tc}{J} + \frac{|V_z| Q}{It} = \frac{(108)(21 \times 10^{-3})}{158.166 \times 10^{-9}} + \frac{(1200)(2.6011 \times 10^{-6})}{(79.083 \times 10^{-9})(7 \times 10^{-3})}$

$= 19.98 \text{ MPa}$ ◄

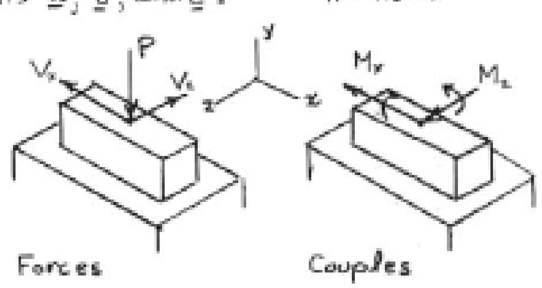
Problem 8.48

8.48 Solve Prob. 8.47, assuming that $h = 12$ in.

8.47 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.



Calculate forces and couples at section containing points $\underline{a}$, $\underline{b}$, and $\underline{c}$. $h = 12$ in.



$P = 50$ kips $\qquad V_x = 6$ kips $\qquad V_z = 2$ kips

$M_z = (12 - 2)(6) = 60$ kip·in

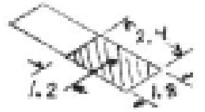
$M_x = (12)(2) = 24$ kip·in

Section properties. $A = (1.8)(4.8) = 8.64 \text{ in}^2$

$I_x = \frac{1}{12}(4.8)(1.8)^3 = 2.3328 \text{ in}^4 \qquad I_z = \frac{1}{12}(1.8)(4.8)^3 = 16.5888 \text{ in}^4$

Stresses $\qquad \sigma = -\frac{P}{A} + \frac{M_z x}{I_z} + \frac{M_x z}{I_x} \qquad \tau = \frac{V_x Q}{I_z t}$

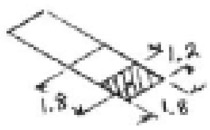
(a) Point $\underline{a}$: $\quad x = 0, \quad z = 0.9$ in., $\quad Q = (1.8)(2.4)(1.2) = 5.184 \text{ in}^3$



$$\sigma = -\frac{50}{8.64} + 0 + \frac{(24)(0.9)}{2.3328} = 3.47 \text{ ksi}$$

$$\tau = \frac{(6)(5.184)}{(16.5888)(1.8)} = 1.042 \text{ ksi}$$

(b) Point $\underline{b}$: $\quad x = 1.2$ in. $\quad z = 0.9$ in, $\quad Q = (1.8)(1.2)(1.8) = 3.888 \text{ in}^3$



$$\sigma = -\frac{50}{8.64} + \frac{(60)(1.2)}{16.5888} + \frac{(24)(0.9)}{2.3328} = 7.81 \text{ ksi}$$

$$\tau = \frac{(6)(3.888)}{(16.5888)(1.8)} = 0.781 \text{ ksi}$$

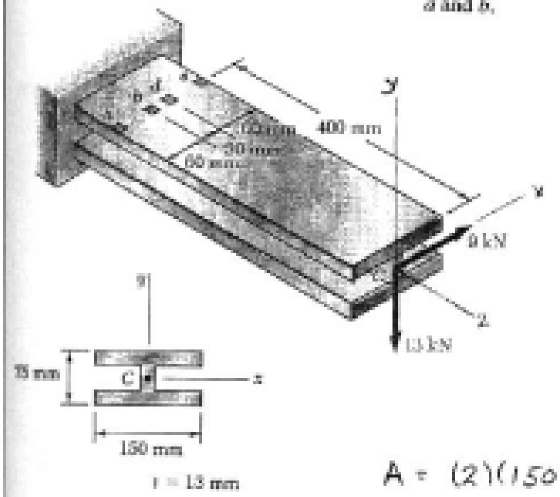
(c) Point $\underline{c}$: $\quad x = 2.4$ in., $\quad z = 0.9$ in., $\quad Q = 0$

$$\sigma = -\frac{50}{8.64} + \frac{(60)(2.4)}{16.5888} + \frac{(24)(0.9)}{2.3328} = 12.15 \text{ ksi}$$

$$\tau = 0$$

Problem 8.53

8.53 Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *a* and *b*.



Equivalent force-couple system at section containing points a and b.

$F_x = 9 \text{ kN}, \quad F_y = -13 \text{ kN}, \quad F_z = 0$

$M_x = (0.400)(13 \times 10^3) = 5200 \text{ N}\cdot\text{m}$

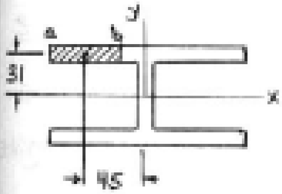
$M_y = 0.400)(9 \times 10^3) = 3600 \text{ N}\cdot\text{m}$

$M_z = 0$

$A = (2)(150)(13) + (13)(75-26) = 4537 \text{ mm}^2 = 4537 \times 10^{-6} \text{ m}^2$

$$I_x = 2\left[\tfrac{1}{12}(150)(13)^3 + (150)(13)(37.5-6.5)^2\right] + \tfrac{1}{12}(13)(75-26)^3 = 3.9303 \times 10^6 \text{ mm}^4 = 3.9303 \times 10^{-6} \text{ m}^4$$

$$I_y = 2 \cdot \tfrac{1}{12}(13)(150)^3 + \tfrac{1}{12}(75-26)(13)^3 = 7.3215 \times 10^6 \text{ mm}^4 = 7.3215 \times 10^{-6} \text{ m}^4$$



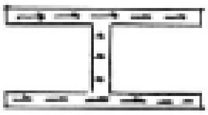
For point $\underline{a}$, $\quad Q_x = 0 \qquad Q_y = 0$

For point $\underline{b}$, $\quad A^* = (60)(13) = 780 \text{ mm}^2$

$\bar{x} = -45 \text{ mm} \qquad \bar{y} = 31 \text{ mm}$

$Q_x = A^*\bar{y} = 24.18 \times 10^3 \text{ mm}^3 = 24.18 \times 10^{-6} \text{ m}^3$

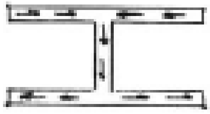
$Q_y = A^*\bar{x} = -35.1 \times 10^{-3} \text{ mm}^3 = -35.1 \times 10^{-6} \text{ m}^3$



At point $\underline{a}$, $\sigma_a = \dfrac{M_x y}{I_x} - \dfrac{M_y x}{I_y}$

$$= \frac{(5200)(37.5 \times 10^{-3})}{3.9303 \times 10^{-6}} - \frac{(3600)(-75 \times 10^{-3})}{7.3215 \times 10^{-6}} = 86.5 \text{ MPa}$$

$\tau_a = 0$



At point $\underline{b}$, $\sigma_b = \dfrac{M_x y}{I_x} - \dfrac{M_y x}{I_y}$

$$= \frac{(5200)(37.5 \times 10^{-3})}{3.9303 \times 10^{-6}} - \frac{(3600)(-15 \times 10^{-3})}{7.3215 \times 10^{-6}} = 57.0 \text{ MPa}$$

$$\tau_b = \frac{|V_x||Q_y|}{I_y t} + \frac{|V_y||Q_x|}{I_x t} = \frac{(9 \times 10^3)(35.1 \times 10^{-6})}{(7.3215 \times 10^{-6})(13 \times 10^{-3})} + \frac{(13 \times 10^3)(24.18 \times 10^{-6})}{(3.9303 \times 10^{-6})(13 \times 10^{-3})}$$

$$= 3.32 \text{ MPa} + 6.15 \text{ MPa} = 9.47 \text{ MPa}$$