MAE233A Final Exam
(Closed Book & Notes)

Date: June 09, 2009 Time: 11:30 to 2:30pm

Name: ______________________________

Note: In all cases below, assume an isotropic and linearly elastic solid outside the crack tip region.

Problem 1.

Consider a semi-infinite crack in an infinite elastic strip, held in rigid grips and subjected to a total separation, Δ, as shown in Fig. 1. Assume plane strain conditions. Also, assume that the strain energy density vanishes as $x_1 \to -\infty$ and that it attains a constant value, w^0, as $x_1 \to +\infty$.

(a) Write down the expression for the J-integral, and carefully describe the meaning of each term.

(b) Find J using the path shown by the dotted lines.

(c) Find w^0 assuming $\varepsilon_{11} = 0$, $\varepsilon_{22} = \Delta/(2h)$, and find J explicitly.

Problem 2.

Consider a semi-infinite crack subjected to concentrated forces, as shown in Fig. 2. Assume plane strain conditions.

(a) Find the length of the plastic zone.

(b) Find the critical (limit) value of the applied load, P_c, if the critical value of the CTOD is δ_c.

(c) Find the length of the plastic zone when a distributed load of intensity p (measured per unit length per unit thickness) is applied to a part of the crack, as shown in Fig. 3.

Note: For LEFM, we have $K_I = 2\,P/\sqrt{2\,\pi\, l}$.

Problem 3.

Consider an anti-plane shearing of a semi-infinite traction-free crack. The crack is along the negative x_1-axis in the x_1, x_2-plane, with its surfaces parallel to the x_1, x_3-plane. The x_2-axis is normal to the crack surfaces. Assume a power-law nonlinear behavior of the form

$$\gamma/\gamma_0 = (\tau/\tau_0)^n, \tag{1}$$

where $\gamma_0 = \tau_0/\mu$ defines the yield strain in terms of the yield stress and shear modulus.

(a) Which displacement components are non-zero?

(b) Set $u_3 = w(r, \theta)$, where (r, θ) are the polar coordinates. Use the polar coordinates and give, *in polar coordinates, explicitly* the expressions for:

(i) Non-zero strain components in terms of the displacement components;

(ii) Non-zero strain components in terms of the stress components;

(iii) The relevant equation of equilibrium; and

(iv) The complementary energy density, w^c, in terms of the stress components.

(c) Now consider an asymptotic solution of the form

$$w(r, \theta) \sim r^s f(\theta), \qquad (2)$$

and use an energy argument to express s in terms of the exponent n.

(d) Then find the asymptotic expressions for the stress and strain components.

Problem 4.

The specimen shown in Fig. 4, contains two edge cracks and is subjected to tension. Use plane strain conditions, and assume a *perfectly plastic model* for the material.

(a) Write down the boundary conditions for the cracks, as well as for the cental plane connecting the tips of the cracks, i.e., on $x_2 = 0$.

(b) Show that the slip lines shown in Fig. 5 correspond to the boundary conditions in (a).

(c) Find the value of the total load P (measured per unit thickness) if the yield stress in shear is k_0.

Note: Please make sure to show all the details and write neatly.

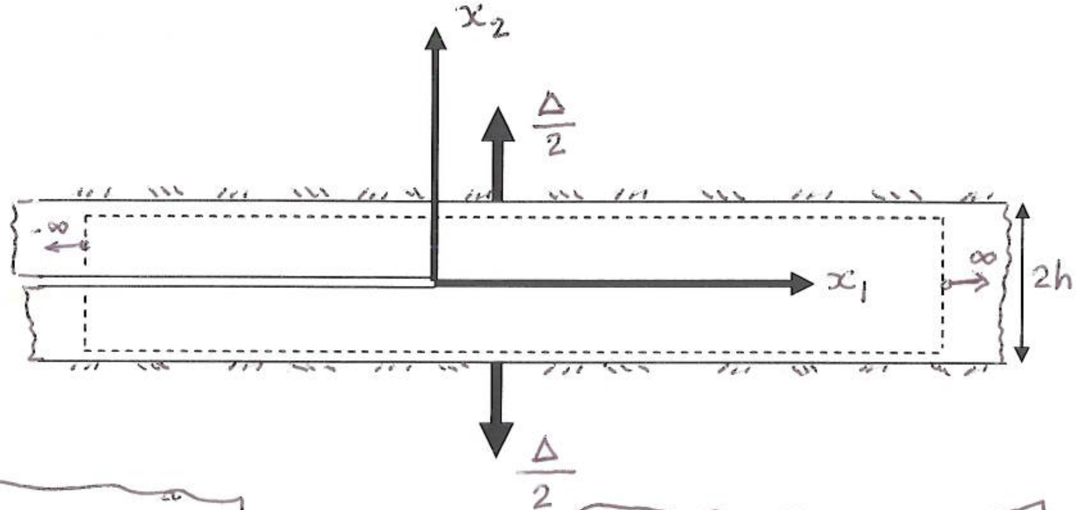

Fig. 1

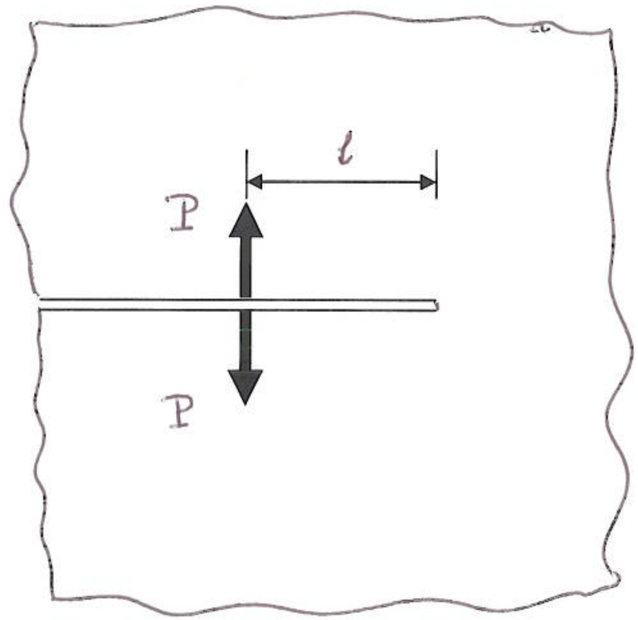

Fig. 2

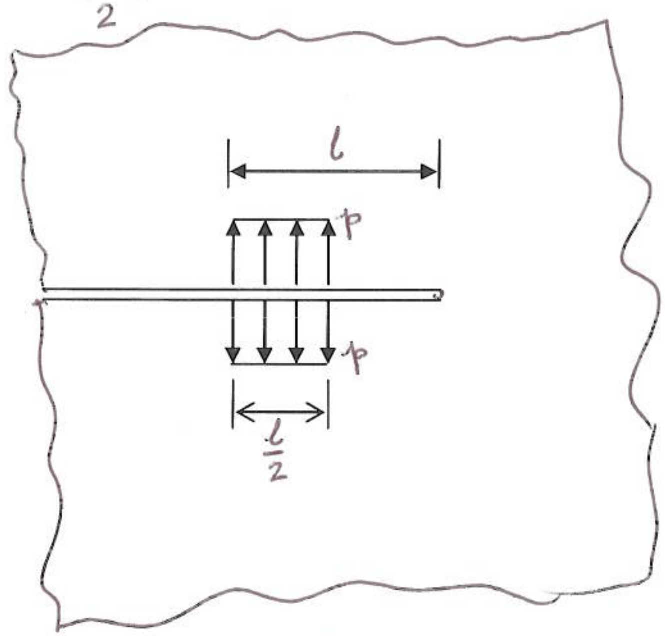

Fig. 3

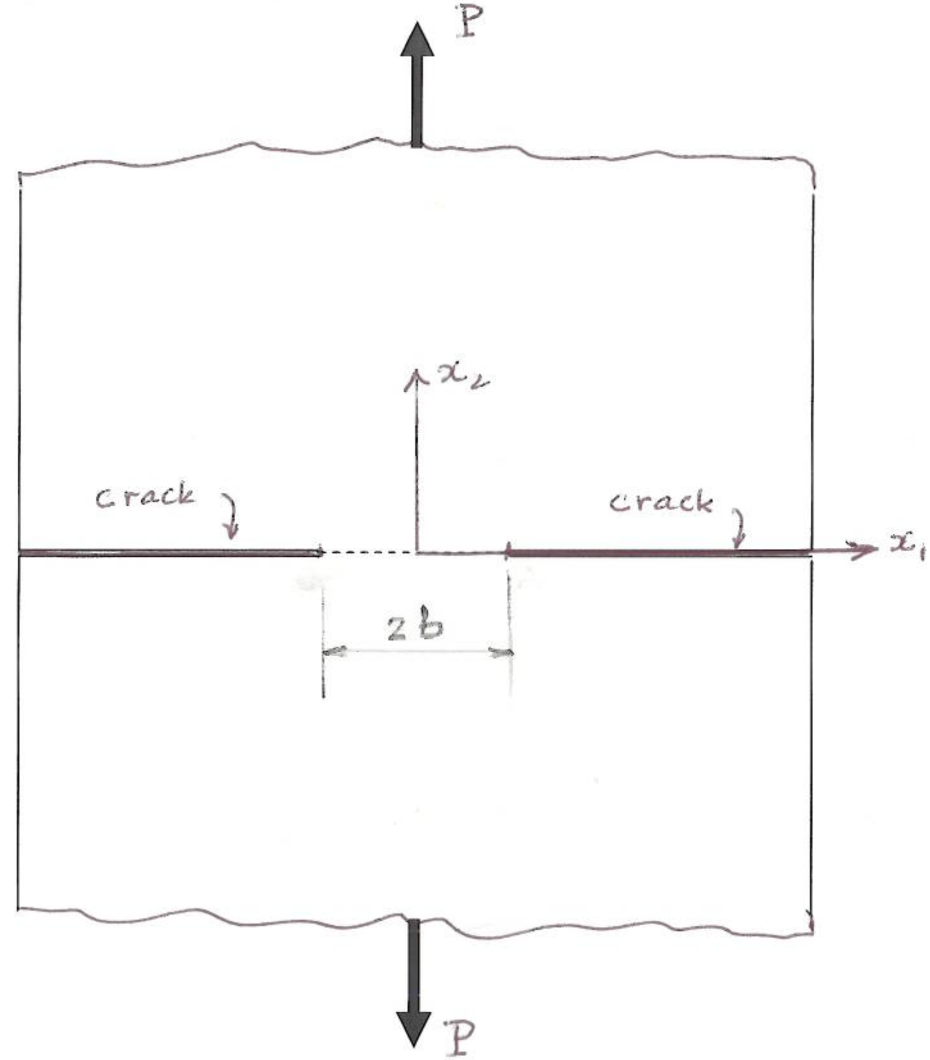

Fig. 4

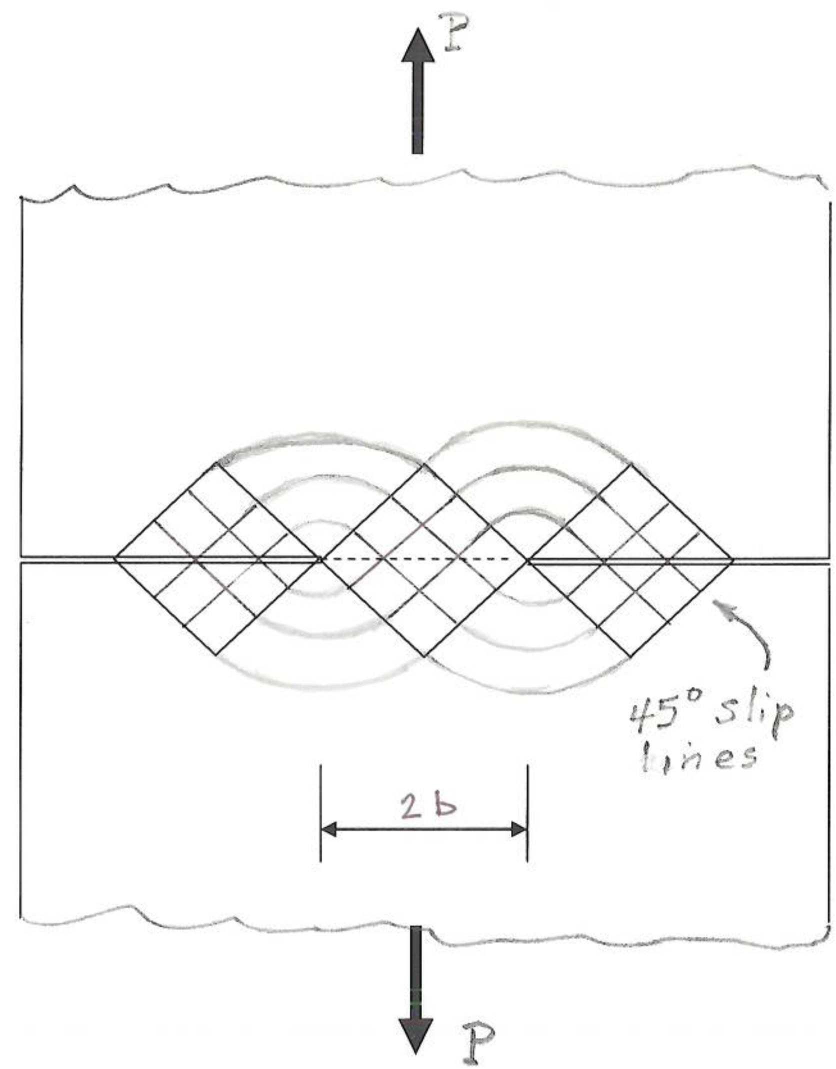

Fig. 5