

MAE105
Quiz #6
(closed book, closed notes)

Name: _____

Time: 3:35 to 3:55pm

Date: May 31, 2007

Consider the following non-homogeneous PDE:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \sin x, \quad t > 0, \quad 0 < x < 1, \quad (1)$$

with the following non-homogeneous boundary conditions:

$$u(0, t) = 1, \quad u(1, t) = 2. \quad (2)$$

(a) (1.5 Point) First consider the following related ODE:

$$\frac{d^2 w(x)}{dx^2} = -\sin x, \quad w(0) = 1, \quad w(1) = 2, \quad (3)$$

and find an explicit expression for $w(x)$.

(b) (1.5 Point) Now set $u(x, t) = v(x, t) + u_E(x)$, substitute into PDE (1), and, in light of your results above, find an ODE with suitable boundary conditions for $u_E(x)$ such that the resulting PDE for $v(x, t)$ is homogeneous, having homogeneous boundary conditions. Neatly write down the PDE and the boundary conditions for $v(x, t)$.

(c) (1 Point) Use the separation of variable, and the homogeneous boundary conditions, to find the eigenvalues and eigenfunctions necessary to obtain the general series solution for $v(x, t)$. Write down this solution.

(d) (1 Point) If the initial condition for the ORIGINAL problem [i.e., for $u(x, t)$ which satisfies PDE (1) and boundary conditions (2)], is given by

$$u(x, 0) = x,$$

find the initial condition for $v(x, t)$.

(e) (1 Point) Explain how you would use this initial condition for $v(x, t)$ to find all the constants in the corresponding series solution of $v(x, t)$.

For an extra point, find all the constants and write down the complete solution for $u(x, t)$; note that $\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha + \beta) - \cos(\alpha - \beta)]$.