June 4, 2009

MAE 105 Quiz #7
(Closed Book and Note, No Cell Phone or Computer)

Problem 1

Consider the following nonhomogeneous diffusion equation:

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} + [\sin(\pi x) + 3\sin(2\pi x)]e^{3t} = 0, \qquad 0 < x < 1, \tag{1}$$

with boundary conditions

$$u(0,t) = 0, \qquad u(1,t) = 0, \tag{2}$$

and the initial condition

$$u(x,0) = f(x). \tag{3}$$

(a) (1 Point) By considering the following eigenvalue problem:

$$\frac{d^2\phi}{dx^2} + \lambda^2\phi = 0,$$

and appropriate boundary conditions, find the eigenfunctions that can be used to solve (1) with boundary conditions (2).

(b) (2 Points) Set $u(x,t) \approx \sum_{n=0}^{\infty} A_n(t)\phi_n(x)$, and $[\sin(\pi x) + 3\sin(2\pi x)] = \sum_{n=0}^{\infty} B_n\phi_n(x)$, where $\phi_n(x)$'s are the eigenfunctions that you obtained in (a), and reduce the PDE (1) into ODE's for the coefficients $A_n(t)$.

[Note: Some of the B_n's may be zero. Even so, there must be an equation associated to each $A_n(t)$. Make sure to include all of these which are necessary to complete the solution for given initial contion; see (d) below].

(c) (2 Points) Solve the ODE's to find $A_n(t)$'s, which must include one integration constant for each n.

(d) (1 Points) Consider the Fourier series representation of the initial condition $u(x,0) = f(x)$ and explain how you would find the integration constants, $A_n(0)$'s, when the function $f(x)$ is given explicitly.

Note: You need the initial condition (3) to evaluate $A_n(0)$.

For an extra point, evaluate all $A_n(0)$'s if $f(x) = x$.