MAE 105
Quiz #6
(closed book and notes, no cell phones, no computers)

Date: May 28, 2009

Time: 3:35 to 3:55pm

Consider the wave equation

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, \qquad t > 0, \qquad 0 < x < \pi,$$

with the boundary conditions

$$u(0, t) = u(\pi, t) = 0,$$

and initial conditions

$$u(x, 0) = \begin{cases} \sin(2x) & \text{for } 0 < x < \pi / 2 \\ 0 & \textit{otherwise}, \end{cases} \qquad \frac{\partial u}{\partial t}(x, 0) = \begin{cases} 1 & \text{for } \pi/4 < x < 3\pi / 4 \\ 0 & \textit{otherwise}. \end{cases}$$

(a) (1 Point) Draw the (x, t)-plane, and below the x-axis, sketch the initial conditions in two graphs, as discussed in the class and in your book.

(b) (2 Point) Extend the initial conditions such that the boundary conditions are satisfied for all $t > 0$, if we use the general solution for the infinite domain. Draw the characteristics that pass through the following points, write down the general solution for the infinite domain, and show that, with the extended initial data, the boundary conditions are in fact satisfied; you must calculate $u(x, t)$ at the following points and check your results against the boundary conditions:

$(x = 0,\ t = \pi/2)$ $\quad (x = \pi,\ t = \pi/2)$

$(x = 0,\ t = 4\pi/3)$ $\quad (x = \pi,\ t = 3\pi/2)$

(c) (2 Points) Draw the relevant characteristics and find the values of:

$u(\pi/4, \pi/4)$ $\quad u(4\pi/5, \pi/4)$

$u(\pi/4, \pi/2)$ $\quad u(4\pi/5, \pi/2)$

Note: The general solution for the infinite domain is given by:

$$u(x, t) = \tfrac{1}{2}\left(f(x - ct) + f(x + ct)\right) + \frac{1}{2c}\int_{x-ct}^{x+ct} g(\bar{x})\, d\bar{x},$$

where c is the wave speed and $f(x)$ and $g(x)$ are the prescribed initial displacement and velocity respectively.