MAE105
Quiz # 5
(closed book, closed notes, no computers, no cell phones)

Time: 3:35 to 3:55pm

Date: May 21, 2009

Problem 1

(a) (1 Points) Find a general expression $x = x(t)$, for the characteristic of the following PDE:

$$\frac{\partial u}{\partial t} + 2\frac{\partial u}{\partial x} = 0.$$

[Note that your expression must include a constant of integration, say, $x(0) = x_0$.]

(b) (1 Point) In the x, t-plane, $t > 0$, sketch the characteristic curve that starts from $x_0 = 1/2$ at $t = 0$ and find the general solution of this PDE.

(c) (0.5 Point) Specialize the solution in (b) such that at $t = 0$, we have $u(x_0, 0) = u_0 = \sin(\pi x_0)$.

(d) (0.5 Point) Find the value of $u(x, t)$ at $x = 3/2$ and $t = 1/2$.

[Show all details.]

Problem 2

Consider the wave equation

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0 \tag{1}$$

in an infinite domain, $-\infty < x < \infty$, and for $t > 0$, with the initial conditions

$$u(x, 0) = f(x) = \cos(\pi x),$$

and

$$\frac{\partial u}{\partial t}(x, 0) = g(x) = 0.$$

(a) (0.5 Point) Draw the (x, t)-plane, and below the x-axis, draw the initial conditions for $-2 < x < 2$, as discussed in the class.

(b) (0.5 Point) Write down the expressions for the two characteristics which are straight lines in this case.

(c) (1 Points) In the x, t-plane, draw the characteristics that pass through the point $x = 2/3$, $t = 4/3$, and find the value of $u(2/3, 4/3)$.

[Show all details.]

Note that the general solution of the wave equation (1) in an infinite space is

$$u(x, t) = \tfrac{1}{2}[f(x-t) + f(x+t)] + \tfrac{1}{2}\int_{x-t}^{x+t} g(\bar{x})d\bar{x}.$$