

MAE 105 Quiz #4
Total Time: 20 minutes
(Closed Book & Notes; No Calculators or Computers or Cell Phones)

Name: _____

Time: 3:35 to 3:55pm

Date: MAY 07, 2009

(a) (1 Point) Use the separation of variables, $u(x, t) = \phi(x)G(t)$, to find two ODE's, one defining the time variation and the other the spatial variation of the general solution of the following PDE:

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0 \quad t > 0, \quad 0 < x < \pi,$$

in such a way that the solution for $\phi(x)$ is periodic in x (note that this choice will ensure that $G(t)$ be periodic in time).

(b) (1 Point) Solve the two ODE's to find the general expressions for $G(t)$ and $\phi(x)$.

Consider the following boundary conditions:

$$u(0, t) = 0, \quad \frac{\partial u}{\partial x}(\pi, t) = 0.$$

(c) (1 Point) Apply these boundary conditions to find the corresponding boundary conditions for $\phi(x)$. Find the eigenvalues and eigenfunctions corresponding to $\phi(x)$.

(d) (0.5 Point) Write down the general infinite series solution for $u(x, t)$.

(e) (1.5 Points) Consider the following initial conditions:

$$u(x, 0) = x, \quad \frac{\partial u}{\partial t}(x, 0) = 0.$$

Find all the coefficients of the infinite series solution of $u(x, t)$, and write down the final solution series.

Note: To receive full credit, please show *all steps and write neatly*.

