

MAE105
Quiz #3
(closed book, closed notes)

Name: _____

Time: 3:35 to 3:55pm

Date: April 30, 2009

Consider the following eigenvalue problem:

$$\frac{d^2\phi}{dx^2} + \lambda^2\phi = 0, \quad 0 < x < \pi, \quad (1)$$

$$\phi(0) - \phi'(0) = 0, \quad \phi'(\pi) = 0 \quad (2)$$

(a) (1 Point) Find the general solution of ODE (1) which involves two arbitrary constants.

(b) (1 Point) Apply boundary conditions (2) to obtain two equations for the two constants in the general solution.

(c) (1 Point) Using the results in (b), find an equation whose roots are the eigenvalues of the problem.

(d) (1 Point) Show graphically how one may obtain the eigenvalues of the equation you obtained in (c); just sketch the graphs to show you understand the basic approach.

(e) (1 Point) What is the limiting value of λ_n as n becomes very large?

For an extra 1/2 point, start with the ODE (1), multiply through by $\phi(x)$, integrate the result from 0 to π , use integration by parts and the boundary conditions (2) to obtain the Rayleigh quotient of this eigenvalue problem.

For another extra 1/2 point, use $\phi_{appr}(x) = \cos(x) - 1$ to approximate the first eigenfunction, substitute into the Rayleigh quotient, evaluate this quotient (which would be an estimate for the smallest eigenvalue).

Note: To receive full credit, *all steps must be neatly shown*. Writing down the final results will receive no credit.

