

MAE105
Quiz #2

(closed book, closed notes, no computer, no calculator)

Name: _____

Time: 3:35 to 3:55pm

Date: April 16, 2009

(a) (1 Point) Use the separation of variables, $u(x, t) = \phi(x)G(t)$, to find two ODE's, one defining the time variation and the other the spatial variation of the general solution of the following PDE:

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0 \quad t > 0, \quad 0 < x < \pi,$$

in such a way that the solution for $G(t)$ would DECAY in time.

(b) (1 Point) Solve the two ODE's to find the general expressions for $G(t)$ and $\phi(x)$.

Consider the following boundary conditions:

$$\frac{\partial u}{\partial x}(0, t) = 0, \quad u(\pi, t) = 0.$$

(c) (0.5 Point) Apply these boundary conditions to find the corresponding boundary conditions for $\phi(x)$.

(d) (1 Point) Find the eigenvalues and eigenfunctions corresponding to $\phi(x)$.

(e) (0.5 Point) Write down the general infinite series solution for $u(x, t)$.

(f) (1 Point) Obtain the final solution that satisfies the following initial condition:

$$u(x, 0) = 2 \cos\left(\frac{3x}{2}\right) + 3 \cos\left(\frac{7x}{2}\right).$$

Note: To receive full credit, *all steps must be neatly shown, following the requested procedure.* Writing down the final results will receive no credit.