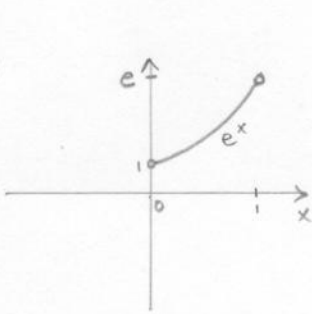
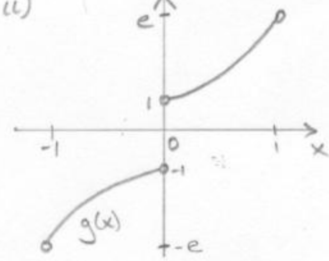
MAE 105 - Problem Solving Session 4/27/09

① a) $f(x) = e^x$, $0 < x < 1$

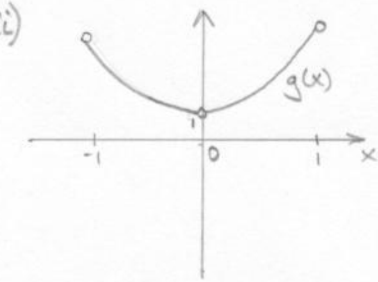
i)



ii)



iii)



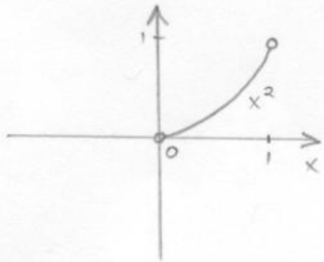
odd: $f(x) = -f(-x)$

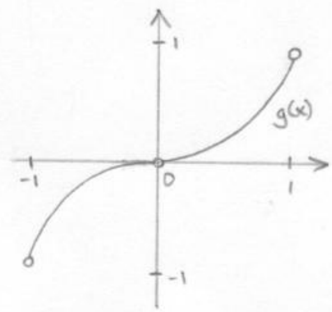
$$g(x) = \begin{cases} -e^{-x} & -1 < x < 0 \\ e^x & 0 < x < 1 \end{cases}$$

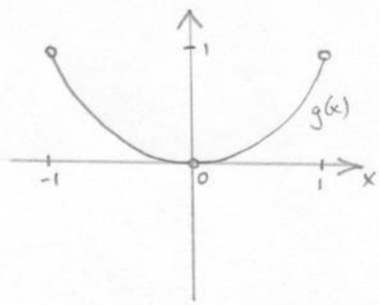
even: $f(x) = f(-x)$

$$g(x) = \begin{cases} e^{-x} & -1 < x < 0 \\ e^x & 0 < x < 1 \end{cases}$$

b) $f(x) = x^2$ $0 < x < 1$







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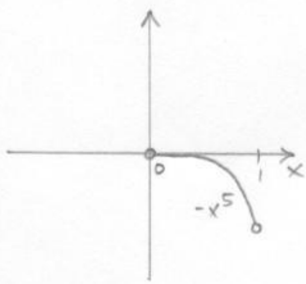
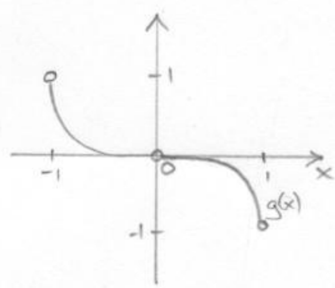
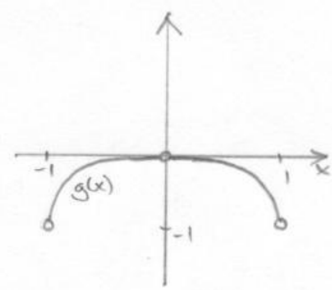
even: $f(x) = f(-x)$

$$g(x) = \begin{cases} x^2 & -1 < x < 0 \\ x^2 & 0 < x < 1 \end{cases}$$

c) $f(x) = -x^5$







odd: $f(x) = -f(-x)$

$$g(x) = \begin{cases} -x^5 & -1 < x < 0 \\ -x^5 & 0 < x < 1 \end{cases}$$

even: $f(x) = f(-x)$

$$g(x) = \begin{cases} x^5 & -1 < x < 0 \\ -x^5 & 0 < x < 1 \end{cases}$$

② Represent $f(x)=1+x$ $(-\pi<x<\pi)$ with a Fourier series.

Fourier series: $f(x)=a_0+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$

Find a_0) $$\int_{-\pi}^{\pi}(1+x)\,dx=\int_{-\pi}^{\pi}a_0\,dx+\sum_{n=1}^{\infty}\int_{-\pi}^{\pi}(a_n\cos nx+b_n\sin nx)\,dx$$

$$\left[x+\frac{x^2}{2}\right]_{-\pi}^{\pi}=\left[a_0x\right]_{-\pi}^{\pi}+\sum_{n=1}^{\infty}\left[\frac{a_n}{n}\sin nx\right]_{-\pi}^{\pi}+\sum_{n=1}^{\infty}\left[\frac{-b_n}{n}\cos nx\right]_{-\pi}^{\pi}$$

$$2\pi=2\pi a_0+(0-0)+\sum_{n=1}^{\infty}\left(-\frac{b_n}{n}(-1)^n+\frac{b_n}{n}(-1)^n\right)$$

$$a_0=1$$

Find a_n) $$\int_{-\pi}^{\pi}(1+x)\cos nx\,dx=\cancel{\int_{-\pi}^{\pi}\cos nx\,dx}+\int_{-\pi}^{\pi}a_n\cos^2 nx\,dx+\cancel{\int_{-\pi}^{\pi}b_n\sin nx\cos nx\,dx}\quad(\text{for } m=n)$$

check using trig identity and integrating; you will get 0

$\cos^2\theta=\frac{1+\cos 2\theta}{2}$

$$\cancel{\int_{-\pi}^{\pi}\cos nx\,dx}+\int_{-\pi}^{\pi}x\cos nx\,dx=0+a_n\pi+0$$

$$0+\cancel{\left[\frac{x}{n}\sin nx\right]_{-\pi}^{\pi}}-\int_{-\pi}^{\pi}\frac{1}{n}\sin nx\,dx=a_n\pi$$

$$\left[\frac{1}{n^2}\cos nx\right]_{-\pi}^{\pi}=a_n\pi$$

$$\frac{1}{n^2}(-1)^n-\frac{1}{n^2}(-1)^n=a_n\pi$$

$$a_n=0$$

Find b_n) $$\int_{-\pi}^{\pi}(1+x)\sin nx\,dx=\cancel{\int_{-\pi}^{\pi}\sin nx\,dx}+\int_{-\pi}^{\pi}b_n\sin^2 nx\,dx\quad(\text{for } m=n)$$

$$\cancel{\int_{-\pi}^{\pi}\sin nx\,dx}+\left[-\frac{x}{n}\cos nx\right]_{-\pi}^{\pi}+\int_{-\pi}^{\pi}\frac{1}{n}\cos nx\,dx=b_n\pi$$

$$\left(-\frac{\pi}{n}(-1)^n-\frac{\pi}{n}(-1)^n\right)+\cancel{\left[-\frac{1}{n^2}\sin nx\right]_{-\pi}^{\pi}}=b_n\pi$$

$$b_n=-\frac{2(-1)^n}{n}$$

$$\Rightarrow\ \boxed{f(x)=1-\sum_{n=1}^{\infty}\frac{2(-1)^n}{n}\sin nx}$$

graph: