

MAE105
Second Midterm Exam
(open book, closed notes, no computers, no cell phones)

Time: 3:35 to 4:50pm

Date: May 12, 2009

Problem 1:

(a) (1 Point) Use the separation of variables, $u(x, t) = \phi(x) G(t)$, to find two ODE's, one defining the time variation and the other the spatial variation of the general solution of the following PDE:

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0 \quad t > 0, \quad 0 < x < \pi,$$

in such a way that the solution for $G(t)$ would DECAy in time.

(b) (1 Point) Solve the two ODE's to find the general expressions for $G(t)$ and $\phi(x)$.

Consider the following boundary conditions:

$$\frac{\partial u}{\partial x}(0, t) = 0, \quad \frac{\partial u}{\partial x}(\pi, t) = 0.$$

(c) (1 Point) Apply these boundary conditions to find the corresponding boundary conditions for $\phi(x)$.

(d) (1 Point) Find the eigenvalues and eigenfunctions corresponding to $\phi(x)$.

(e) (1 Point) Write down the general infinite series solution for $u(x, t)$.

(f) (2 Point) Obtain the final solution that satisfies the following initial condition:

$$u(x, 0) = 1 + x.$$

Calculate and give explicit expressions for all the coefficients, and then write down the final series solution with explicit coefficients.

Problem 2

(a) (0.5 Point) Write down the two dimensional Laplace's PDE in polar coordinates, i.e., in the r, θ -plane.

(b) (1 Point) Use separation of variables, $u(r, \theta) = \phi(\theta) f(r)$, to find an ODE for $\phi(\theta)$ and an ODE for $f(r)$, such that $\phi(\theta)$ is periodic in θ .

(c) (1 Point) Write down the general solution of the ODE of $\phi(\theta)$ that must include two integration constants.

(d) (1 Point) Noting that the ODE for $f(r)$ is *equidimensional*, find its general solution that must include two integration constants.

Consider now a section of a circle defined by

$$0 < \theta < \pi/3, \quad 0 < r < R, \tag{5}$$

where a and b are constants. Consider also the following boundary conditions:

$$u(r, 0) = 0, \quad u(r, \pi/3) = 0, \tag{6}$$

$$|u(0, \theta)| < \infty, \quad u(R, \theta) = 2 \sin 6\theta. \tag{7}$$

(e) (1.5 Point) Apply the boundary conditions (6) and use these to find the associated eigenvalues and eigenfunctions.

(f) (1.5 Points) Write down the general infinite series solution of $u(r, \theta)$. Then apply the first boundary condition in (7) to this general series solution to simplify the series. Make sure to write down the final infinite series.

(g) (1.5 Points) Now use the second condition in (7) to find all the non-zero coefficients (using orthogonality) and write down the final solution for $u(r, \theta)$.