Boundary conditions of third kind

Heat flow in uniform rod

$$\frac{\partial u(x,t)}{\partial t} = k\frac{\partial^2 u(x,t)}{\partial x^2}, \quad k>0, \quad t>0, \quad -L<x<L$$

Vibrating string

$$\frac{\partial^2 u(x,t)}{\partial^2 t} = c^2\frac{\partial^2 u(x,t)}{\partial x^2}, \quad t>0, \quad -L<x<L$$

B.C. $$\begin{cases} u(0,t)=0 \\ \dfrac{\partial u}{\partial x}(L,t) = -hu(L,t) \end{cases}$$

Solve by Separation of Variables

$$u(x,t) = \phi(x)G(t)$$

Substitute into PDE to Obtain:

$$\frac{1}{kG(t)}\frac{dG(t)}{dt} = \frac{1}{\phi(x)}\frac{d^2\phi(x)}{dx^2} = -\lambda$$

Function of time, t, only

Function of distance, x, only

$$\frac{dG(t)}{dt} = -\lambda k G(t) \qquad \frac{d^2\phi(x)}{dx^2} = -\lambda\phi(x)$$

λ : Eigenvalue

$\phi(x)$: Eigenfunction

Physical case λ>0

$$\frac{d^2\phi(x)}{dx^2} + \lambda\phi(x) = 0, \quad plus\ BC's \qquad \lambda > 0$$

$$\phi(0) = 0$$

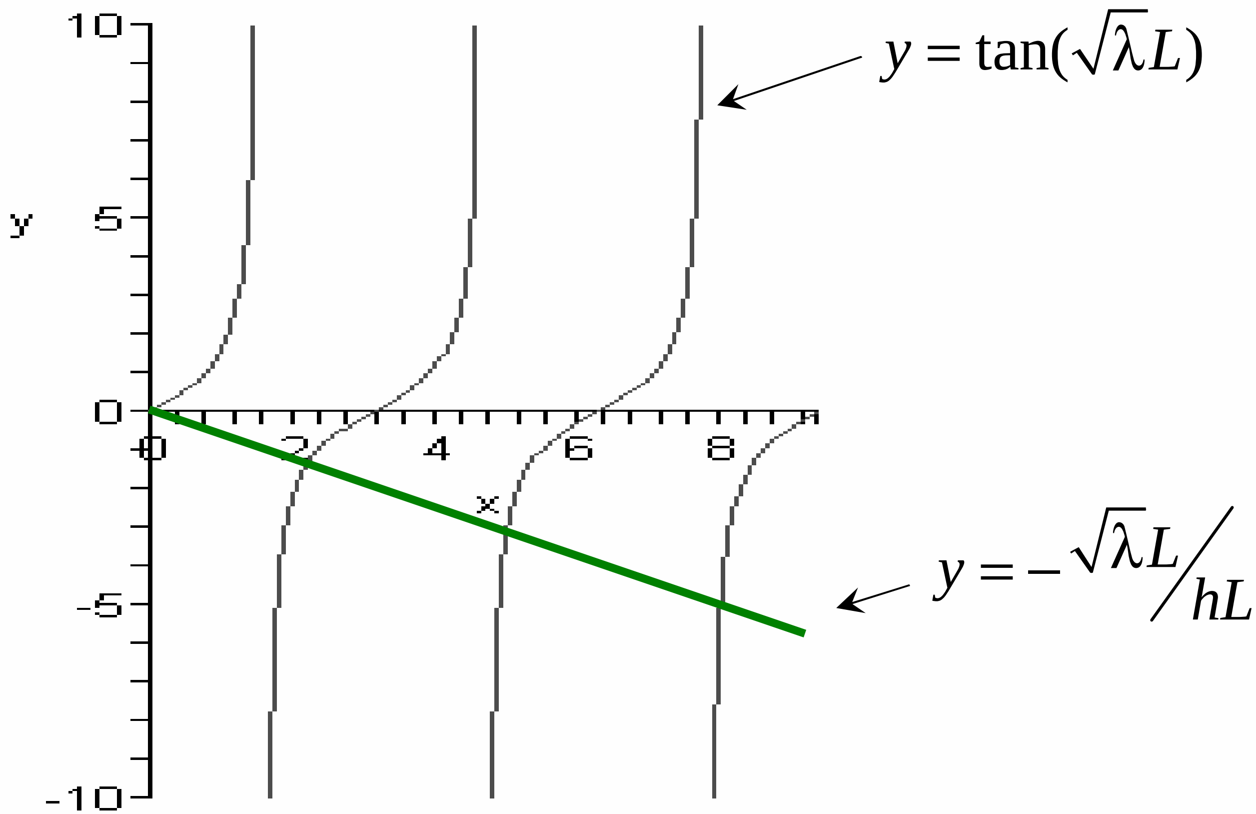
$$\frac{d\phi}{dx}(L) + h\phi(L) = 0$$

$$\phi(x) = c_1 \cos(\sqrt{\lambda} L) + c_2 \sin(\sqrt{\lambda} L)$$

$$\phi(0) = 0 \rightarrow \quad \phi(x) = c_2 \sin(\sqrt{\lambda} L)$$

$$\tan(\sqrt{\lambda}L) = -\frac{\sqrt{\lambda}}{h}$$

Graphical Technic



Time-dependent Part

Heat flow in uniform rod

$$\frac{dG(t)}{dt} = -\lambda k G(t)$$

Vibrating string

$$\frac{d^2G(t)}{dt^2} = -\lambda c^2 G(t)$$