

April 14, 2009

MAE 105 Homework #3

Due: Tuesday, 4/21/09

PROBLEM 1:

Consider the heat equation,

$$\frac{\partial u}{\partial t} - 2 \frac{\partial^2 u}{\partial x^2} = 0 \quad t > 0, \quad 0 < x < 2\pi.$$

a. (1 Point): Use the separation of variables, $u(x, t) = G(t) \phi(x)$, to find the necessary ODEs, one for $G(t)$ and another for $\phi(x)$. Choose your ODEs such that the solution for $\phi(x)$ is periodic.

b. (2 Points): Find $G(t)$ and $\phi(x)$.

c. (1 Point): Consider the boundary conditions,

$$u(0, t) = u(2\pi, t), \quad \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(2\pi, t),$$

Write the corresponding boundary conditions for $\phi(x)$.

d. (2 Points) Find all the eigenvalues and the corresponding eigenfunctions, using the boundary conditions.

e. (2 Points) Find the complete solution for $u(x, t)$ when the initial condition is given by

$$u(x, 0) = 5 + 4 \sin 6x - 2 \cos 7x.$$

[Hint: If you understand orthogonality of eigenfunctions, then the answer should be obvious!]

PROBLEM 2 (0.5 Point):

By direct calculation, show that,

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{d\phi}{dr} \right) = \frac{d^2\phi}{dr^2} + \frac{1}{r} \frac{d\phi}{dr}.$$

PROBLEM 3 (6.5 Points):

Solve the Laplace's equation inside a quarter circle,

$$\nabla^2 u = 0, \quad 0 < r < a, \quad 0 < \theta < \pi/4,$$

with boundary conditions

$$u(r, 0) = 0, \quad u(r, \pi/4) = 0, \quad u(a, \theta) = (\theta - \pi/4)\theta.$$

Note 1: To receive full credit, *all steps must be neatly shown*. Writing down the final results will receive no credit.

Note 2: Homeworks must be turned in at the start of due-date class. Late homeworks will be graded but *will receive zero credit*.