# MAE 223, Spring 2009

## Homework 1

Due Friday, April 24, 5PM in my office

**1.** The convection-diffusion equation for temperature evolution,

$$T_t + uT_x = \alpha T_{xx}$$

where $u$ and $\alpha$ are positive constants is to be solved numerically. A third-order, Runge-Kutta scheme is to be used for the time integration with a second-order, central approximation for the spatial derivatives. Derive a condition (in nondimensional terms) for the time step so that the numerical method is stable.

**2.** The Laplace equation, $\nabla^2 p = 0$ is to be solved for $p$ in a rectangular domain. The boundary conditions are $p = 1$ on the left side, $p = 0$ on the right side, and $\partial p/\partial y = 0$ on top and bottom sides. Take the initial guess for the pressure to be

$$p(x, y) = \sum_{k=1}^{k=8} \sum_{l=1}^{l=8} \sin\left(\frac{2\pi kx}{L_x} + \frac{2\pi ly}{L_y}\right)$$

where $L_x$ and $L_y$ are the sides of the domain. Use a $16 \times 16$ grid to discretize the domain and a second-order accurate scheme. The exact solution is trivial. The objective is to study the iterations and computational time required for convergence from the chosen initial guess to the exact solution.

**a)** Let $\Delta x = 0.1$, $\Delta y = 0.1$. Write a code for point Gauss-Seidel and line Gauss- Seidel iteration. Compare the two methods.

**b)** Repeat (a) with $\Delta x = 0.1$, $\Delta y = 0.01$.

**c)** Write a multigrid solver with 3 grid levels. Repeat (a) and (b) with the multigrid solver.