MAE 214A, Winter 2009

Homework 3

Due Thursday, March 5, in class

Guidelines: Please turn in a *neat* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution.

1. Let Y have a lognormal PDF, with μ, σ^2 denoting the expected value and variance of $\ln(Y)$, respectively. Show that the variance and mean of Y are related by

$$\frac{< (Y - < Y >)^2 >}{< Y >^2} = e^{\sigma^2} - 1 \tag{1}$$

2. A lognormal distribution is a reasonable model for the PDF of the instantaneous dissipation, $\epsilon_0 = 2\nu s'_{ij} s'_{ij}$. The expected value, $< \epsilon_0 >$, is equivalent to the turbulent dissipation rate, ϵ. Experiments shows that the kurtosis (normalized fourth moment) of $\partial u'/\partial x$ increases with Reynolds number, being given by $\mathcal{K} = Re_\lambda^{3/8}$.

a. Let $Re_\lambda = 1000$, a moderate value that can be obtained in the laboratory. What is the value of $\epsilon_{0,rms}/\epsilon$? **Hint:** Use the result, Eq. 1, of problem 1.

b. Let $Re_\lambda = 1000$. What are the probabilities that ϵ_0 is larger than $\alpha\epsilon$, where the coefficient $\alpha = 1, 2, 5, 10$, respectively? Comment on your results.

3. What is the inertial subrange of the energy spectrum? Why is a high Reynolds number necessary for having an inertial subrange? Derive a powerlaw for the energy spectrum, $E(\omega)$, in the inertial subrange. Here ω is the frequency in radians/sec.

4.

a. Starting from the definition of the turbulent dissipation rate, show that the *isotropic* estimate of ϵ is given by

$$\epsilon = 15\nu \frac{u^2}{\lambda^2} \, .$$

Here λ is the Taylor microscale associated with the lateral correlation function, $g(r)$.

b. What is the invisicid estimate of ϵ? Give a physical argument for the inviscid estimate.