MAE 101B
Fluid Mechanics, Spring 2009
Regarding Midterm 2

Closed book. Closed notes. No calculator. Formulae sheet that will be provided to you is given. You should know other relevant formulae.

Summary of Material

The relevant sections of the book are also given below. Please consult class notes and text book for the theory. Go over the HW problems, including ungraded ones, and recitation problems for practice. Regarding the Blasius laminar boundary layer solution, you need to know its derivation and understand its solution. You should be familiar with the Karman momentum integral so as to be able to use it. You do not need to study Thwaite's analysis for the midterm but will need to do so for the final. You do not need to be able to derive the Karman momentum integral or Thwaite's method in the exams.

- **Chap. 7. Flow past bodies**

  Concept of a thin viscous boundary layer (BL) at high Re and associated viscous drag force. Laminar BL, transition, and turbulent BL. Various measure of BL thickness: δ, δ^*, and θ. Definitions of shape factor, H, skin friction coefficient, c_f, and drag coefficient, C_D. Karman momentum integral for zero pressure gradient, Eq. (7.5), that relates drag D to the evolution of $\theta(x)$. Use of approximate velocity profiles in Eq. (7.5) to derive laws for the streamwise dependence of boundary layer parameters. (sections 7.1 and 7.2).

  Derivation of the BL equations from the Navier-Stokes equations. Analysis of the laminar BL equations for a ZPG flat plate BL: self-similar evolution as a function of the similarity variable, $\eta = y/h(x) = y/\sqrt{\nu x/U}$; introduction of streamfunction, $\psi(x, y) = Uh(x)f(\eta)$ into BL equations; derivation of the Blasius equation; properties of its solution. (section 7.3)

  The turbulent BL. Logarithmic overlap region in the mean velocity profile, $u^+(y^+)$. Skin friction law, Eq. (7.37), and its power-law approximation. One-seventh power law for the mean velocity and its use in the Karman momentum integral to obtain BL parameters as a function of x. Formulae for $C_D(Re_L)$: smooth plate result, Eq. (7.45); fully-rough result, Eq. (7.48b); transition drag curves, Eq. (7.49b); analog of Moody chart, Fig. 7.6. (section 7.4).

  BL with favorable and adverse pressure gradient. BL separation under adverse pressure gradient. Generalization of Karman momentum integral equation to flows with $dp/dx \neq 0$. Thwaite's approximate method for laminar BL: introduction of additional parameter, λ, that influences the velocity profile; use of Eq. (7.54) to predict BL parameters as a function of x and Eq. (7.55) to predict separation point. (section 7.5)

  Definition of drag coefficient, C_D, in flow over an immersed body. Components of drag force: friction drag, pressure (form) drag. Effect of streamlining. Separation of laminar BL versus turbulent BL. Simplification of NS equations for creeping flow ($Re << 1$); Stokes law, Eq. (7.64), for drag on a sphere. $C_D(Re)$ for sphere, cylinder and other bodies, Fig. 7.16. Drag of bodies at $Re > 10^4$, Tables 7.2 and 7.3. Automotive drag. Ship drag. Use of drag formulae in various mechanics problems.

Equation sheet (will be given to you with midterm)

Navier Stokes equations in vector form as given below. You should know simplification to get components in Cartesian coordinates.

$$\boldsymbol{\nabla} \cdot \mathbf{v} = 0 \tag{1}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \boldsymbol{\nabla})\, \mathbf{v} = -\frac{1}{\rho_0} \boldsymbol{\nabla} p + \nu \nabla^2 \mathbf{v} \tag{2}$$

The 2-D steady BL equations.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_0} \frac{dp}{dx} + \frac{1}{\rho_0} \frac{\partial \tau}{\partial y} \tag{4}$$

For laminar flow, $\tau = \mu \partial u/\partial y$. For turbulent flow, $\tau = \mu \partial u/\partial y + \rho \overline{u'v'}$.

Karman momentum integral for ZPG flat plate BL.

$$\frac{d\theta}{dx} = \frac{C_f}{2} \tag{5}$$

The Blasius equation for a laminar boundary layer

$$f''' + \frac{1}{2} f f'' = 0 \tag{6}$$

and its result for a flat plate:

$$\frac{\delta}{x} = \frac{5.0}{{Re_x}^{1/2}} \quad , \quad c_f = \frac{0.664}{{Re_x}^{1/2}} \quad , \quad C_D = \frac{1.328}{{Re_L}^{1/2}} \tag{7}$$

For a turbulent smooth boundary layer:

$$\frac{\delta}{x} = \frac{0.16}{{Re_x}^{1/7}} \quad , \quad c_f(x) = \frac{0.027}{{Re_x}^{1/7}} \quad , \quad C_D = \frac{0.031}{{Re_L}^{1/7}} \tag{8}$$

For a fully rough boundary layer:

$$c_f(x) = \left(2.87 + 1.58 \log \frac{x}{\varepsilon}\right)^{-2.5} \quad , \quad C_D = \left(1.89 + 1.62 \log \frac{L}{\varepsilon}\right)^{-2.5} \tag{9}$$

Fig. 7.6 for $C_D(Re)$.

Tables and figures for computing C_D of bluff bodies will be given as required.