MAE 101B
Fluid Mechanics, Spring 2009
Regarding the Final

Closed book. Closed notes. Bring scientific calculator. You will have to solve 5 questions: 2 on compressible flow (Chap. 9) and 3 on internal/external flows (Chap. 6,7). Formulae sheet that will be provided to you is given. You should know other relevant formulae.

Summary of Material

The relevant sections of the book are also given below. Please consult class notes and text book for the theory and examples. Go over the HW problems, including ungraded ones, and recitation problems for practice.

- **Chap. 6. Viscous Flow in ducts**

  Reynolds number; laminar, transitional and turbulent regimes; entrance length; fully-developed flow (sections 6.1 and 6.2).

  Conservation of energy; simplfication for steady, low-speed flow; frictional head loss, h_f. Definition of (Darcy-Weisbach) friction factor, f (section 6.3, also review 3.6)

  Solution of the incompressible N.S. equations to obtain laminar flow profile in ducts, channels and other simple flows. Use of laminar flow profile to obtain wall shear stress, average velocity, flow rate, power, etc. (section 6.4, also review 4.10)

  Turbulent pipe flow. Reynolds averaging; turbulent stresses. The mean velocity profile in turbulent wall-bounded flows: the friction velocity, u*; viscous (plus units) scaling, multiple regions (inner, logarithmic overlap, outer region). The friction law for smooth pipes. Effect of roughness. Colebrook formula for friction factor. Moody chart. (sections 6.5 & 6.6)

  Pipe flow problems: obtain head loss; obtain flow rate; pipe sizing; pipe length. The need for iteration in some problems when using the Moody chart or the Colebrook formula. (sections 6.7)

  Non circular ducts: procedure for laminar flow; turbulent flow and the use of hydraulic diameter.

  Minor losses: pipe entrance/exit; sudden expansion/contraction; bends, elbows; valves; gradual expansion and contraction. The loss coefficient, K_m (section 6.9). Diffuser performance.

- **Chap. 7. Flow past bodies**

  Concept of a thin viscous boundary layer (BL) at high Re and associated viscous drag force. Laminar BL, transition, and turbulent BL. Various measure of BL thickness: δ, δ^*, and θ. Definitions of shape factor, H, skin friction coefficient, c_f, and drag coefficient, C_D. Karman momentum integral for zero pressure gradient, Eq. (7.5), that relates drag D to the evolution of $\theta(x)$. Use of approximate velocity profiles in Eq. (7.5) to derive laws for the streamwise dependence of boundary layer parameters. (sections 7.1 and 7.2).

  Derivation of the BL equations from the Navier-Stokes equations. Analysis of the laminar BL equations for a ZPG flat plate BL: self-similar evolution as a function of the similarity variable, $\eta = y/h(x) = y/\sqrt{\nu x/U}$; introduction of streamfunction, $\psi(x,y) = Uh(x)f(\eta)$ into BL equations; derivation of the Blasius equation; properties of its solution. (section 7.3)

  The turbulent BL. Logarithmic overlap region in the mean velocity profile, $u^+(y^+)$. Skin friction law, Eq. (7.37), and its power-law approximation. One-seventh power law for the mean velocity and its use in the Karman momentum integral to obtain BL parameters as a function of x.

Formulae for $C_D(Re_L)$: smooth plate result, Eq. (7.45); fully-rough result, Eq. (7.48b); transition drag curves, Eq. (7.49b); analog of Moody chart, Fig. 7.6. (section 7.4).

BL with favorable and adverse pressure gradient. BL separation under adverse pressure gradient. Generalization of Karman momentum integral equation to flows with $dp/dx \neq 0$.

Definition of drag coefficient, C_D, in flow over an immersed body. Components of drag force: friction drag, pressure (form) drag. Effect of streamlining. Separation of laminar BL versus turbulent BL. Simplification of NS equations for creeping flow ($Re << 1$); Stokes law, Eq. (7.64), for drag on a sphere. $C_D(Re)$ for sphere, cylinder and other bodies, Fig. 7.16. Drag of bodies at $Re > 10^4$, Tables 7.2 and 7.3. Automotive drag. Ship drag. Forces on lifting bodies and defns. of lift coefficient, C_L, and drag coefficient, C_D. Airfoils: how lift develops; theoretical result for $C_L(\alpha)$, Eq. (7.67), and it's finite-span extension (7.70); stall; high-lift devices.Use of drag formulae in various mechanics problems.

- **Chap. 9. Compressible flow**

  Mach number. Need for energy equation and equation of state. Basic thermodynamics: ideal gas, specific heats, enthalpy, entropy, isentropic process. Definition of stagnation temperature, stagnation pressure and stagnation density. Adiabatic + steady flow implies total stagnation enthalpy is constant. Adibatiac+steady+frictionless implies entropy is constant. Isentropic flow relationships: Mach number relations for $T_0/T \ldots$ e.g. Eq. (9.34), critical or sonic values.

  Isentropic flow in a a duct with area change. Conservation equations imply difference between subsonic and supersonic behavior. The throat section and (potential) sonic conditions at throat. The behavior of $A/A^*(M)$ and possibility of either subsonic or supersonic flow for the same value of area ratio. Choking i.e; there is a maximum flow rate, Eq. (9.46b).

  The normal shock wave. Jump relationships across a shock. The Rankine-Hugoniot relationship. Mach number relations, eg. $p_2/p_1(M_1)$ in Eq. (9.55). Entropy considerations. Moving normal shock.

  Converging nozzle and converging-diverging nozzle. Different regimes depending on back pressure, Fig. 9.11 and 9.12, and the possibility of shock waves. The behavior of $p(x)$ and $\dot{m}$ for these regimes.

  Constant-area duct flow with friction a.k.a. Fanno flow. The difference between the behavior in subsonic flow and supersonic flow. Critical length. L^*, as function of inlet Mach number. Table B. 3. The behavior if $L > L^*$ and the possibility of frictional choking.

Following equation sheet will be given to you.

Navier Stokes equations in vector form as given below. You should know simplification to get components in Cartesian coordinates.

$$\boldsymbol{\nabla} \cdot \mathbf{v} = 0 \tag{1}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \boldsymbol{\nabla})\,\mathbf{v} = -\frac{1}{\rho_0}\boldsymbol{\nabla} p + \nu \nabla^2 \mathbf{v} \tag{2}$$

Conservation of energy for incompressible flow:

$$\left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_{in} = \left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_{out} + h_f - h_{pump} + h_{turbine} \tag{3}$$

The friction factor is defined by

$$f = \frac{h_f}{V^2/2g}\frac{d}{L} \tag{4}$$

Logarithmic overlap law:

$$\frac{u}{u^*} = \frac{1}{\kappa} \ln \frac{yu^*}{\nu} + B \,, \tag{5}$$

where $\kappa = 0.41$ and $B = 5.2$.

Colebrook formula for turbulent duct flow:

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\epsilon/d}{3.7} + \frac{2.51}{Re_d \sqrt{f}} \right) \tag{6}$$

The Moody chart will be given to you if required.

The 2-D steady BL equations.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_0} \frac{dp}{dx} + \frac{1}{\rho_0} \frac{\partial \tau}{\partial y} \tag{8}$$

For laminar flow, $\tau = \mu \partial u / \partial y$. For turbulent flow, $\tau = \mu \partial u / \partial y - \rho \overline{u'v'}$.

Karman momentum integral for ZPG flat plate BL.

$$\frac{d\theta}{dx} = \frac{c_f}{2} \tag{9}$$

The Blasius equation for a laminar boundary layer

$$f''' + \frac{1}{2} f f'' = 0 \tag{10}$$

and its result for a flat plate:

$$\frac{\delta}{x} = \frac{5.0}{{Re_x}^{1/2}} \quad , \quad c_f = \frac{0.664}{{Re_x}^{1/2}} \quad , \quad C_D = \frac{1.328}{{Re_L}^{1/2}} \tag{11}$$

A boundary layer is turbulent if $Re_x > 5 \times 10^5$. For a turbulent smooth boundary layer:

$$\frac{\delta}{x} = \frac{0.16}{{Re_x}^{1/7}} \quad , \quad c_f(x) = \frac{0.027}{{Re_x}^{1/7}} \quad , \quad C_D = \frac{0.031}{{Re_L}^{1/7}} \tag{12}$$

For a fully rough boundary layer:

$$c_f(x) = \left(2.87 + 1.58 \log \frac{x}{\varepsilon} \right)^{-2.5} \quad , \quad C_D = \left(1.89 + 1.62 \log \frac{L}{\varepsilon} \right)^{-2.5} \tag{13}$$

Fig. 7.6 for $C_D(Re)$ will be given as required.

Tables and figures for computing C_D of bluff bodies will be given as required.

Lift coefficient for airfoil:

$$C_L = \frac{2\pi \sin\left(\alpha + \frac{2h}{\bar{c}}\right)}{1 + 2/AR} \,, \; AR = \frac{b}{\bar{c}} \tag{14}$$

Speed of sound, $c = \sqrt{\gamma R T}$. For air: $R = 287 J/kg - K$, $\gamma = k = 1.4$.

Isentropic state relation:

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1} \right)^{\gamma} = \left(\frac{T_2}{T_1} \right)^{\gamma/(\gamma - 1)} \tag{15}$$

Isentropic flow relationships for air :

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \, , \; \frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)} \, , \; \frac{A}{A^*} = \frac{(1 + 0.2M^2)^3}{1.728M} \tag{16}$$

Shock relationships for air:

$$\frac{p_2}{p_1} = \frac{2.8{M_1}^2 - 0.4}{2.4} \, , \; \frac{\rho_2}{\rho_1} = \frac{2.4{M_1}^2}{0.4{M_1}^2 + 2} = \frac{V_1}{V_2} \, , \; M_2^2 = \frac{0.4{M_1}^2 + 2}{2.8{M_1}^2 - 0.4} \tag{17}$$

Table B.3 for duct flow with friction will be given if required.