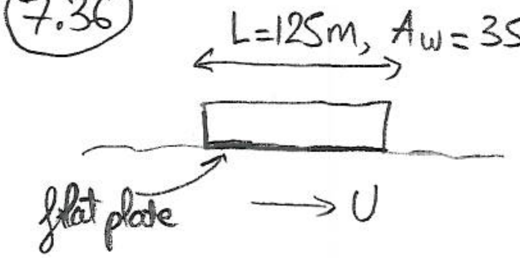
Problem Session 4

(7.36)



Given: → $L = 125\,m$

→ $A_W = 3500\,m^2$

→ propeller: $\dot{W} = 1.1\,MW$

→ seawater at 20°C

→ all drag due to friction.

Find: $U = ?$

→ for seawater, $\rho = 1025\ kg/m^3$, $\mu = 1.07 \times 10^{-3}\,\frac{kg}{m\text{-}s}$ (table A3, Appendix).

→ $Re_L = \frac{\rho U L}{\mu} = \frac{(1025\ kg/m^3)\,U\,(125m)}{(1.07 \times 10^{-3}\ kg/m\text{-}s)} = 1.2 \times 10^8\,U$ ← more likely turbulent

→ for tubulent flow: eqn 7.45 gives:

→ $C_D = \frac{0.031}{Re_L^{1/7}} = \frac{0.031}{(1.2 \times 10^8 U)^{1/7}} = 2.17 \times 10^{-3}\,U^{-1/7}$

→ Drag force:

→ $F_D = \frac{1}{2} C_D\, \rho\, A_W U^2 = \frac{1}{2}(2.17 \times 10^{-3}\,U^{-1/7})(1025\,\frac{kg}{m^3})(3500\,m^2)\,U^2$

$\Rightarrow F_D = 3892\,U^{13/7}$

→ propeller: $\dot{W} = F_D U = (3892\,U^{13/7})\,U = 3892\,U^{20/7}$

$\Rightarrow U = \left(\frac{\dot{W}}{3892}\right)^{7/20} = \left(\frac{1.1 \times 10^6\,W}{3892}\right)^{7/20} = \boxed{7.2\,m/s}$ → flow IS Indeed turbulent

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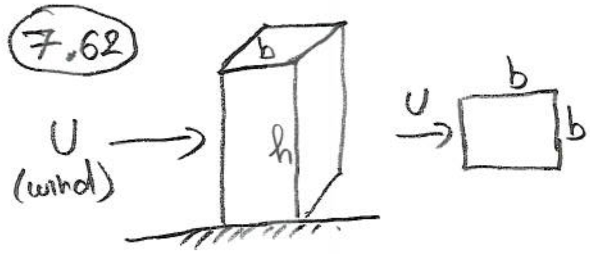
(7.62)



Given: → $h = 52\,m$

→ $F_D = 90 \times 10^3\,N$

→ $U = 90\,mph = 40.2\,m/s$

Find: $b = ?$

→ for air at sea level, $\rho = 1.2\ kg/m^3$, $\mu = 1.8 \times 10^{-5}\,\frac{kg}{m\text{-}s}$ (table A2, Appendix)

$\rightarrow Re_b = \dfrac{\rho U b}{\mu} = \dfrac{(1.2\ kg/m^3)(40.2\ m/s)b}{(1.8\times10^{-5}\ kg/m\text{-}s)} = 2.68\times10^6\, b$

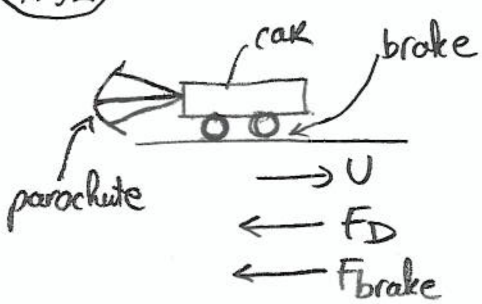
$\rightarrow$ since Re is large ($>10^4$), table A.3 chapter 7 gives $C_D = 2.1$

$\rightarrow F_D = \frac{1}{2} C_D \rho \underbrace{A_w}_{\text{projected area}} U^2 = \frac{1}{2} C_d \rho b h U^2$

$$\Rightarrow b = \frac{2F_D}{\rho C_D h U^2} = \frac{2(90\times10^3 N)}{(1.2\,kg/m^3)(2.1)(52m)(40.2m/s)^2} = \boxed{0.85m}$$

$\rightarrow Re_b \gg 10^4$

7.92



Given: $\rightarrow m = 1500\ kg$

$\rightarrow C_D A|_{car} = 0.4 m^2$

$\rightarrow U_0 = 50\ m/s$

$\rightarrow$ complete stop in $t_f = 8s$

Find: $\rightarrow d_p = ?$

$\rightarrow$ air at sea level, $\rho = 1.2\ kg/m^3$, $\mu = 1.8\times10^{-5}\ kg/m\text{-}s$.

$\rightarrow$ drag is due to car and parachute.

$\rightarrow$ table A3 of Chapter 7 gives $C_{D,p} = 1.2$

$$\rightarrow F_D = \frac{1}{2}\rho C_{D,p} A_w U^2 + \frac{1}{2}\rho C_D A|_{car} U^2$$

$$\Rightarrow F_D = \frac{1}{2}(1.2)\left[(1.2)\left(\frac{\pi}{4} d_p^2\right) + (0.4)\right] U^2$$

$$\Rightarrow F_D = (0.565 d_p^2 + 0.24) U^2$$

$$\rightarrow \Sigma F = m\frac{dU}{dt} = -F_{br} - F_D$$

$$\Rightarrow \frac{dU}{dt} = \frac{-(5000N)}{(1500\,kg)} - \underbrace{\frac{(0.565 d_p^2 + 0.24)}{1500}}_{a} U^2$$

$$\Rightarrow \frac{dU}{dt} = -3.33 - aU^2$$

$\rightarrow$ Integrate, $\displaystyle\int_{U_0}^{0} \frac{dU}{3{,}33 + aU^2} = -\int_0^{t_f} dt$

$\rightarrow$ integration table gives: $\displaystyle\int \frac{dx}{Ax^2+B} = \frac{1}{\sqrt{AB}} \tan^{-1}\left(\sqrt{\frac{A}{B}}\,x\right)$

$$\Rightarrow \quad -t_f = \frac{1}{\sqrt{3{,}33a}} \tan^{-1}\left(U\sqrt{\frac{a}{3{,}33}}\right)\Bigg|_{U_0}^{0}$$

$$\Rightarrow \quad t_f = \frac{1}{\sqrt{3{,}33a}} \tan^{-1}\left(U_0\sqrt{\frac{a}{3{,}33}}\right)$$

$$\Rightarrow \quad 8 = \frac{1}{\sqrt{3{,}33a}} \tan^{-1}\left(50\sqrt{\frac{a}{3{,}33}}\right)$$

(plot, iteration, matlab).

$\rightarrow$ guess $d_p = 3m \Rightarrow a = 3{,}55 \times 10^{-3} \Rightarrow RHS = 9{,}39 > 8$.

$\rightarrow$ guess $d_p = 4m \Rightarrow a = 6.19 \times 10^{-3} \Rightarrow RHS = 7{,}92 < 8$.

$\rightarrow$ $\boxed{d_p = 3.9m}$, $\Rightarrow a = 5.89 \times 10^{-3} \Rightarrow RHS = 8$.