MAE 101B, Spring 2009
Midterm I, 10:00 AM - 10:50 AM

Guidelines: Closed-book, closed notes, no calculator exam. Give all the formulae used and explain your steps in each problem. Each problem is worth **50** points. Attach question paper to the exam that you turn in.

1. Water flows in the x-direction in a channel between two plat plates as shown in the figure. The bottom plate is stationary. The top plate moves at constant velocity $\hat{U}$, and there is a constant pressure gradient dP/dx. It is also given that

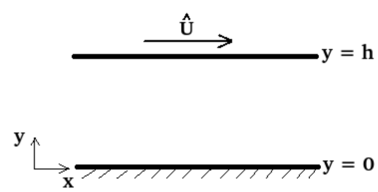
$$\hat{U} = \frac{-h^2}{4\mu}\frac{dP}{dx}. \tag{1}$$

Assume fully-developed, laminar, incompressible flow: $u = u(y)$ and $v = 0$.

a) Start from the x-momentum equation to show that the velocity profile is

$$u\left(y\right) = \hat{U}\left(3\frac{y}{h} - 2\frac{y^2}{h^2}\right). \tag{2}$$

b) Compute shear stress at the walls.

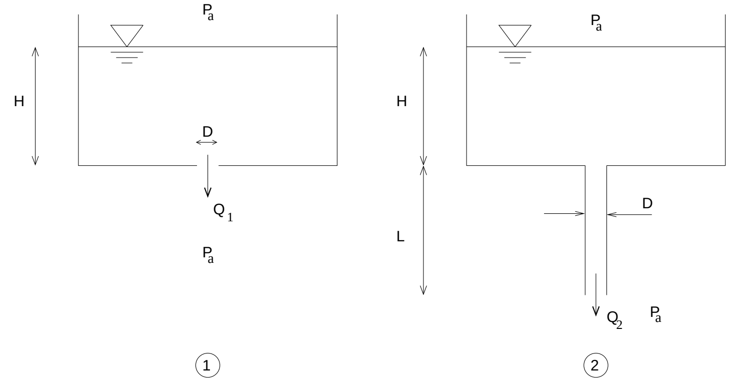


2. A large tank filled with an incompressible fluid is to be drained. Two possible configurations are shown in the figure. In configuration 1, there is a drain hole with small diameter D at the tank bottom. In configuration 2, a rough commercial steel pipe of length L is added to the same drain hole so as to take advantage of a higher available head and increase the flow rate. Neglect minor losses in your analysis. Assume an incompressible fluid of density ρ and viscosity μ. The atmospheric pressure is P_a.

a) Find an expression for the volumetric flow rate Q_1 in configuration 1.

b) Find an expression for the volumetric flow rate Q_2 in configuration 2. Obtain a condition on the value of fH/D, which determines that $Q_2 > Q_1$. Here, f is the pipe friction coefficient.

c) Explain the method to calculate the volumetric flow rate Q_2 (in configuration 2) based on the Moody chart or on the Colebrook formula.



Equation sheet (will be given to you with midterm)

Navier Stokes equations in vector form as given below. You should know simplification to get components in Cartesian coordinates.

$$\boldsymbol{\nabla} \cdot \mathbf{v} = 0 \tag{3}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \boldsymbol{\nabla})\, \mathbf{v} = -\frac{1}{\rho} \boldsymbol{\nabla} p + \nu \nabla^2 \mathbf{v} \tag{4}$$

Conservation of energy for incompressible flow:

$$\left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_{in} = \left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_{out} + h_f - h_{pump} + h_{turbine} \tag{5}$$

The friction factor is defined by

$$f = \frac{h_f}{V^2/2g} \frac{d}{L} \tag{6}$$

Logarithmic overlap law:

$$\frac{u}{u^*} = \frac{1}{\kappa} \ln \frac{y u^*}{\nu} + B\,, \tag{7}$$

where $\kappa = 0.41$ and $B = 5.2$.

Colebrook formula for turbulent duct flow:

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\epsilon/d}{3.7} + \frac{2.51}{Re_d \sqrt{f}}\right) \tag{8}$$