MAE 101B, Spring 2009

Homework 6

Due Tuesday, June 2, in class.

Guidelines: Please turn in a *neat* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Required plots should be generated using computer software such as Matlab or Excel.
Please refrain from copying. Refer to the course outline for what constitutes copying.
Use the following fluid properties (assumed to be constant here):
air: $\gamma = 1.4$, $R = 287$ J/kgK, $c_p = 1$ kJ/kgK.
CO_2: $\gamma = 1.3$, $R = 189$ J/kgK, $c_p = 0.83$ kJ/kgK.

1. A freely standing, stationary shock wave entrains CO_2 at a Mach number $\mathrm{Ma}_1 = 2.0$, static pressure $P_1 = 1$ atm and static temperature $T_1 = 300$ K.

a) Is the flow isentropic through the shock wave? Are the stagnation pressure and stagnation temperature constant through the shock wave?

b) Calculate the Mach number downstream the shock, Ma_2, and the stagnation pressure and stagnation temperature P_{02} and T_{02} there.

2. A lightning produces an initial shock that propagates into still air at a pressure. The pressure just inside the shock wave is 200 atmospheres. Ignore curvature effects in the front.

a) What is the speed of the shock front?

b) What is the absolute fluid velocity just inside the shock?

3. Consider a small rocket thruster for spacecraft propulsion composed by a large combustion chamber welded to a convergent-divergent exhaust nozzle of exit and throat areas $A_e = 10^{-2}\mathrm{m}^2$ and $A_t = A_e/4$ respectively. Exhaust gases (assume air properties) flow throughout the nozzle to the ambient atmosphere, where the pressure is P_b. If the combustion chamber is at an average temperature $T_o = 1000$K and pressure $P_o = 10$ atm.

a) Calculate the back pressure P_s needed for having sonic flow in the throat and subsonic flow everywhere else. Obtain the mass flow rate $\dot{m}$ of air in these conditions.

b) Calculate the back pressure P_d for design operation conditions. Obtain the mass flow rate $\dot{m}$ of air in these conditions.

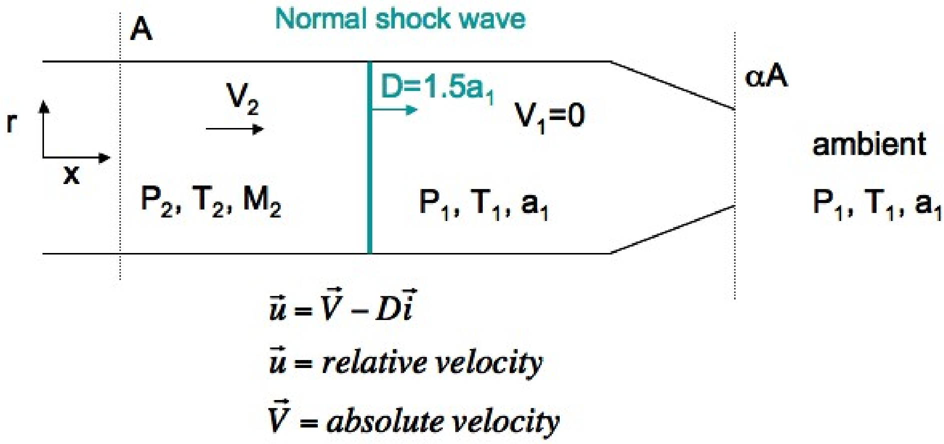
c) For this particular problem, it is known that, at the back pressure $P_b = 2.94$ atm, there is a normal shock wave standing just at the nozzle exit (you can try and calculate it as an ungraded exercise); this pressure is denoted as P_{sw} in the present notation ($P_{sw} = 2.94$ atm). Describe the flow pattern inside the nozzle in the regime $P_{sw} < P_b < P_s$, and sketch the flow at the exit of the nozzle for $P_d < P_b < P_{sw}$. What would be the exit flow pattern observed in a sea-level ignition test ($P_b = 1$ atm)?.

4. Air flows adiabatically in a 2 cm diameter duct. The average friction factor is 0.015. At the entrance, $V_1 = 900$ m/s, $T_1 = 225$ K and $P_1 = 2$ atm.

a) What is the length of the tube at which the flow becomes choked? What is the mass flow rate?

b) What is the length of the tube at which the exit Mach number is $\mathrm{Ma}_2 = 1.8$? What is the mass flow rate?

5. A pipe of cross section A ends on a convergent-divergent nozzle with throat area αA, as shown in the figure. The pipe is initially filled with air at rest at known pressure p_1 and known temperature T_1 (speed of sound $a_1^2 = \gamma R T_1$). The nozzle is discharging to the ambient atmosphere, which also is at pressure p_1 and temperature T_1. A normal shock wave propagates along the pipe at speed $D = 1.5 a_1$ towards the exit nozzle.



a) Calculate p_2/p_1, T_2/T_1 and M_2, knowing that M_1 and M_2 must be calculated with the relative velocity of the fluid with respect to the shock wave.

b) Calculate $\widehat{M_2}$, and $\hat{p}_{02}/p_1$. In this formulation, the symbol ˆ represents a variable calculated with respect to the absolute velocity of the fluid.

c) Once the shock wave has already travelled throughout the entire length of the pipe and nozzle and has disappeared in the surrounding atmosphere, the air inside the pipe is left moving at the velocity V_2. In these conditions, calculate α so that the nozzle throat is choked ($A^\star = \alpha A$).

Ungraded problems From text. 9.69, 9.92.