MAE 101B, Spring 2009

Homework 4

Due Thursday, May 7, in class.

Guidelines: Please turn in a *neat* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Required plots should be generated using computer software such as Matlab or Excel.
Please refrain from copying. Refer to the course outline for what constitutes copying.
Use the following fluid and solid properties:
water: $\rho = 1000$ kg/m^3; $\mu = 0.001$ kg/ms.
steel: $\rho = 9000$ kg/m^3;

1. a) Show that the integration over y of the x-momentum equation in the boundary-layer approximation, supplied with the continuity equation, leads to the von Kármán integral equation

$$U\frac{d}{dx}\left(\frac{\theta^2}{\nu}\right) + 2(2+H)\frac{\theta^2}{\nu}\frac{dU}{dx} - 2T = 0. \quad (1)$$

In this formulation, ν is the kinematic viscosity, $U(x)$ is the outer stream velocity, $\theta(x)$ is the momentum thickness, $H = \delta^*(x)/\theta(x)$ and $T(x) = \tau_w\theta/\mu U$ are the shape factors with $\delta^*(x)$ the displacement thickness, and x and y are the tangential and normal coordinates to the wall respectively as depicted in Fig. 1.

b) State in words, and by making use of a couple of flow sketches, the influences of the sign of the pressure gradient dP/dx on the boundary layer separation, and relate your conclusions to the influences of the sign of the outer stream velocity variations dU/dx by using the x-momentum equation in the inviscid outer stream. Explain why the boundary-layer separation point x_s, downstream from which backflow may occur, can be estimated as $\tau_w(x_s) = 0$, and show that at this point the vorticity along the wall changes from clockwise (upstream) to counter-clockwise (downstream).

2. The Falkner-Skan type of flows

$$U(x)/U_o = (x/x_o)^\alpha, \quad (2)$$

represent a large family of canonical flows such as the plane-parallel flow ($\alpha = 0$) - as in the Blasius problem -, the stagnation-point flow ($\alpha = 1$) or the corner flow ($\alpha < 0$) as depicted in Fig. 2. In this formulation, x_o is a reference point where the velocity U_o and momentum thickness θ_o are known.

a) Based on your results in part b) of Problem 1, state in words which of these three above mentioned flow examples will likely undergo boundary layer separation.

b) Equation (1) of Problem 1 can be easily integrated to obtain the momentum thickness $\theta(x)$ if $H(x)$ and $T(x)$ are constants as in the Blasius solution ($\alpha = 0$). Unfortunately this is not the case in more general flows $\alpha \neq 0$, although all the experimental and numerical data show that T and H mainly depend on the parameter $\lambda = (\theta^2/\nu)dU/dx$, which is a measure of the square of the momentum thickness. Thwaites (1949) collected all the experimental data available at that time and obtained correlations for T and H as

$$T = (\lambda + 0.09)^{0.62} \quad \text{and} \quad 2T - 2(H+2)\lambda = 0.45 - 6.0\lambda, \quad (3)$$

so that, according to part b) of Problem 1, boundary-layer separation occurs when $\lambda = -0.09$. This is referred to as the Thwaites method. Combine equations (1) and (3) to show that the momentum thickness can be expressed as

$$\frac{\theta^2}{\nu} = \frac{\theta_o^2}{\nu}\left[\frac{U(x_o)}{U(x)}\right]^6 + \frac{0.45}{U^6(x)}\int_{x_o}^{x} U^5(x)dx. \quad (4)$$

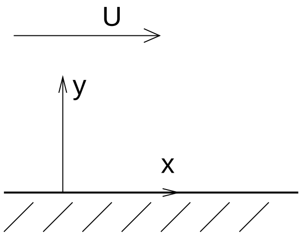
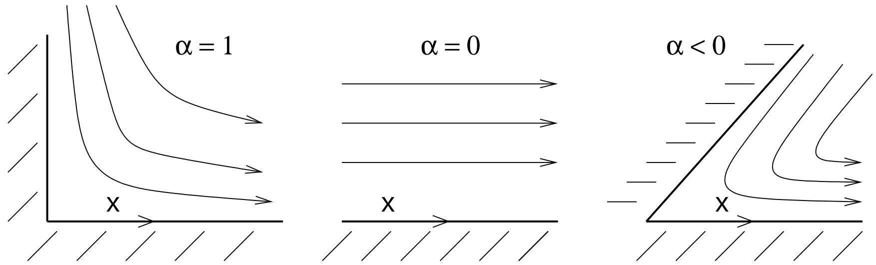


Figure 1



(a) (b) (c)

Figure 2

c) Particularize equation (4) for the Falkner-Skan type flows (2) and show that the resulting momentum thickness is

$$\frac{\theta^2}{\nu} = \frac{x_o}{U_o}\left(\frac{x}{x_o}\right)^{1-\alpha}\left[\frac{0.45}{1+5\alpha} + \left(\frac{U_o\theta_o^2}{\nu x_o} - \frac{0.45}{1+5\alpha}\right)\left(\frac{x}{x_o}\right)^{-(1+5\alpha)}\right]. \qquad (5)$$

Based on this result, show that the critical value of α below which boundary-layer separation occurs is $\alpha \approx -0.1$ (18° inclination corner flow). (Note: the terms involving $x^{-(1+5\alpha)}$ in (5) can be neglected if $\alpha \gtrsim -0.2$, i.e. for negative and slightly positive pressure gradients).

d) Analyze the case $\alpha = 0$ and compare the value of θ obtained by the Thwaites method with the exact Blasius analysis result given by equation (7.30) from the textbook. Based on this comparison, comment on the accuracy of the Thwaites method.

3. Water flows over a flat plate of length $L = 10$ m at a free stream velocity $U = 10$ m/s subject to a null pressure gradient.

a) Is the flow laminar or turbulent?. Calculate the transition point $(x/L)_{tr}$.

b) The von Kármán equation (1) from Problem 1 part a) can also be used in turbulent flows. Use this equation and the turbulent boundary-layer velocity profile proposed by Prandtl, $u/U = [y/\delta(x)]^{1/7}$, to obtain an expression of the friction coefficient $c_f(x)$, knowing that c_f can be approximately related to the boundary layer thickness δ as $c_f = 0.02\mathrm{Re}_\delta^{-1/6}$.

c) Plot the turbulent and laminar friction factors as a function of $\log(\mathrm{Re}_x)$ in the same figure. From your plot and the definition of c_f, we see that if the rest of variables are held constant, the wall shear stress is proportional to the velocity as $\tau_w \sim U^n$; what is n for turbulent and laminar flows?.

4. Calculate the power needed to launch a cone-shaped torpedo, of length $L = 3$ m and opening angle $\theta = 20°$, at an initial speed of 25 m/s from a submarine immersed in water.

5. Assuming high Reynolds number regime, calculate the terminal velocity U_∞ of a steel sphere of radius $R = 1$ cm with a measured drag coefficient $C_D = 0.8$ falling down in water. After having done your calculations, check that the high Reynolds number assumption is correct.

Ungraded problems From text. 7.59, 7.66.