MAE 101B, Spring 2009

Homework 2

Due Thursday, April 16, in class.

Guidelines: Please turn in a *neat* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Required plots should be generated using computer software such as Matlab or Excel.
Please refrain from copying. Refer to the course outline for what constitutes copying.
Use the following fluid properties: water: $\rho = 1000$ kg/m^3; $\mu = 0.001$ kg/ms.

1. Water flows through a round pipe of diameter $d = 10$ cm. The pipe is hydraulically smooth, and the volumetric flow rate is $Q = 0.078$ m^3/s.

a) Determine the velocity V averaged over the cross section and the Reynolds number. Is the flow laminar or turbulent?

b) To obtain a value of the friction factor f proceed in the following way. Calculate a relation between V and u_τ by using the logarithmic velocity profile for the complete flow

$$\frac{u(r)}{u^\star} = \frac{1}{\kappa} \ln \left[\frac{(R-r)u^\star}{\nu} \right] + B \qquad (1)$$

where u_τ is the friction velocity, $R = d/2$ and r is the radial coordinate from the pipe centerline. Relate this result to the friction factor f by using the definition of the friction velocity and a momentum balance along the pipe centerline, and obtain a relationship between f and the Reynolds number $\text{Re}_d = Vd/\nu$. Particularize this result for $B = 5.62$ and, for the given pipe flow, obtain the friction factor f by numerical iteration.

c) Calculate the maximum velocity u_{max} and estimate the thickness of the viscous sublayer.

d) Calculate the wall shear stress and the pressure gradient, and sketch qualitatively the radial distribution of the total (laminar + turbulent) and the turbulent shear stress.

2. Assume a steady incompressible flow at an average velocity V of a fluid of density ρ and viscosity μ in a hydraulically smooth two-dimensional channel of height h.

a) Sketch the Moody diagram, label the laminar and smooth turbulent branches of the friction factor, and explain the difference between both regimes.

b) Let the shear stress on the wall be $\tau_w = 1$ Pa and the fluid be water. Estimate the maximum allowable size of the roughness elements ϵ_{min} of the walls such that the flow remains hydraulically smooth.

c) Sketch how the Moody diagram of a smooth pipe becomes modified due to roughness, and give a physical explanation of why the friction factor becomes independent of the Reynolds number, $\text{Re}_h = \rho V h/\mu$, at sufficiently large Re_h.

3. A UCSD engineer has to design a commercial new steel pipe (see table 6.1 from textbook) to pump water from an infinite reservoir, such that the flow rate of water through the pipe is

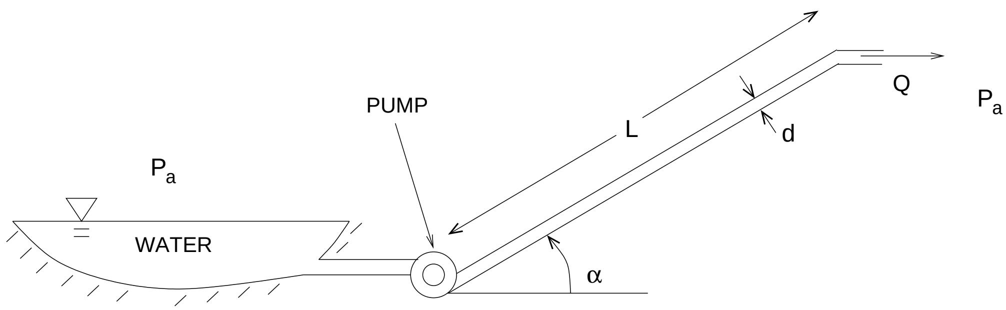

Figure 1: Problem 3.

$Q = 0.1$ m^3/s. For that purpose, an ideal pump of power $\dot{W} = 12$ kW is used. The ambient pressure is P_a, the length of the pipe is $L = 10$ m and the elevation angle is $\alpha = 30°$. Assume turbulent flow (check this assumption after having done all the calculations).

a) Use the energy equation, neglecting kinetic energy at the exit, to show that the diameter of the pipe d can be written as

$$d = \left[\frac{8\rho Q^2 f L}{\pi^2 \left(\frac{\dot{W}}{Q} - \rho g L \sin\alpha \right)} \right]^{1/5} .$$

Also show that the diameter-based Reynolds number can be written as a function of the flow rate Q as $\mathrm{Re}_d = 4\rho Q/\pi d\mu$.

b) Use the two above expressions and the appropriate roughness to calculate the diameter of the pipe by iterating on the Moody chart.

4. A stainless steel pipe of internal diameter $D = 5$ cm carries water with an average velocity $V = 1$ m/s. Compute the head loss and pressure drop per unit of pipe length.

5. An ideal turbine extracts 250 W of power in the configuration shown in the figure. The pipes are made of wrought iron. What is the flow rate Q in m^3/h?. The atmospheric pressure is P_a.

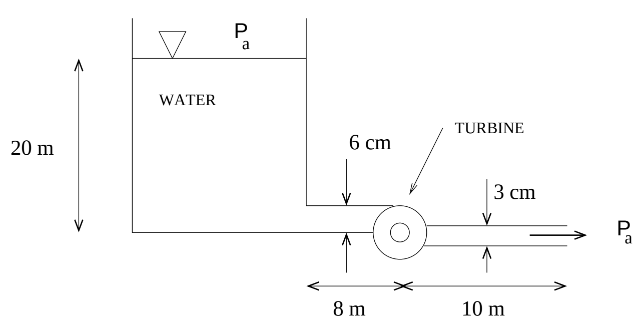

Figure 2: Problem 5.

Ungraded problems From text. 6.40, 6.148.