MAE 140 – Linear Circuits – Winter 2009
Midterm

Instructions

1) This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities.
2) You have 70 minutes.
3) Write your name, student number and instructor.

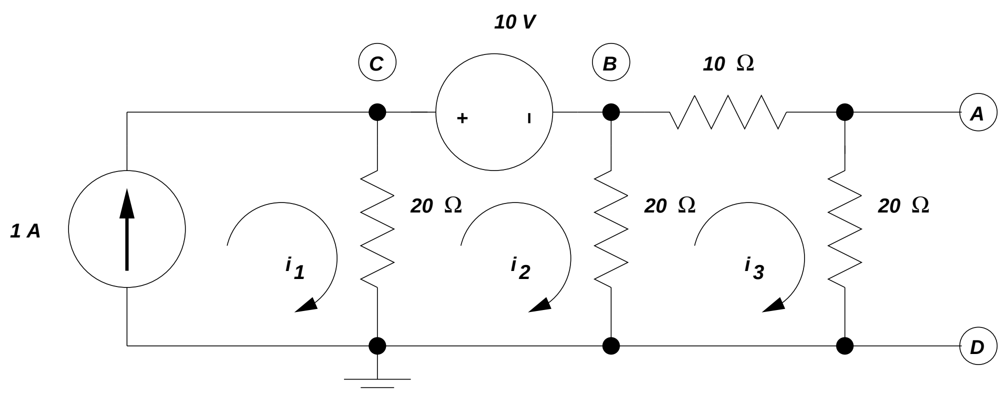


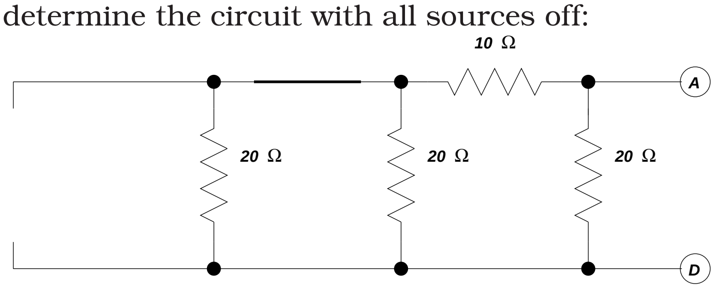
Figure 1: Circuit for questions 1-4.

Questions

1. **Equivalent circuits**

   (a) (2 points) Turn off all the sources in the circuit of Figure 1 and find the equivalent resistance as seen from terminals A and D.

   Solution: First determine the circuit with all sources off: (1 point)

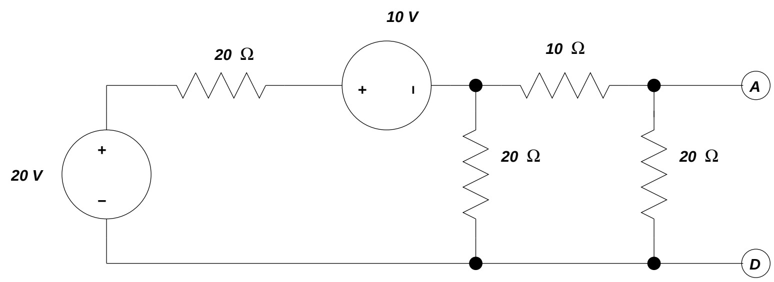
   

   The equivalent resistance is then

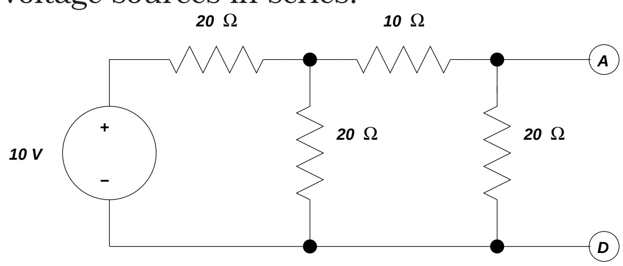
   $$[(20//20) + 10]//20 = [10 + 10]//20 = 20//20 = 10\ \Omega$$ (1 point)

   (b) (3 points) Find the Thévenin equivalent as seen from terminals A and D.

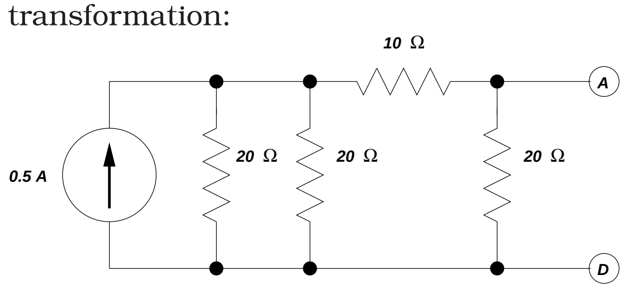
   Solution: The simplest possible way may be to use a source transformation on the current source to obtain the following circuit: (1 point)



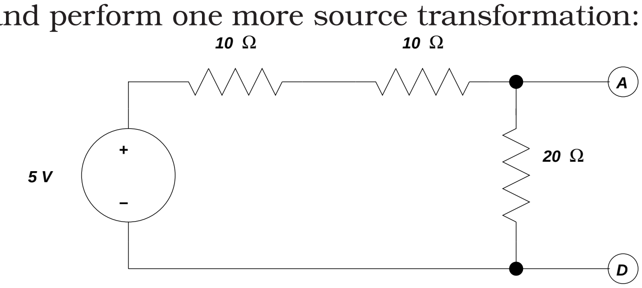
Then associate the voltage sources in series:



and apply a source transformation: (1 point)



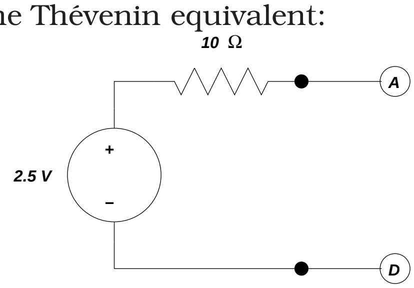
Associate $20//20$ and perform one more source transformation:



Calculate the open-circuit voltage V_T directly using voltage division

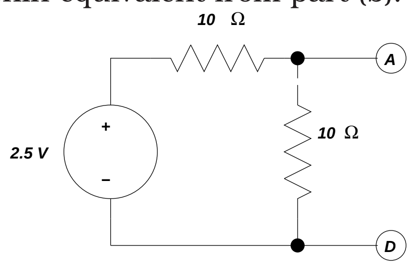
$$V_T = \frac{20}{10 + 10 + 20} 5 \text{ V} = 2.5 \text{ V}$$

and use part (a) to draw the Thévenin equivalent: (1 point)



(c) (1 point) Find the power absorbed by a $10\,\Omega$ resistor if connected to terminals A and D.

Solution: Using the Thévenin equivalent from part (b):



Compute the voltage on the resistor using voltage division:

$$v = \frac{10}{10+10} 2.5 = 1.25V$$

then the power absorbed is

$$p = \frac{v^2}{10} = \frac{1.25^2}{10} \approx 156 \text{ mW}$$ (1 point)

2. **Mesh current analysis**

(a) (6 points) Formulate mesh-current equations for the circuit in Figure 1. Use the mesh currents shown in the figure and clearly indicate how you handle the presence of a current source, the final equations and the unknowns they must be solved for. **Do not modify the circuit or the labels in any way**. **Do not use source transformation**. **Do not solve any equations**!

Solution: Because the current source appears in a single mesh we have

$$i_1 = 1 \text{ A}, \quad \text{(eq. 1)}$$

which determines i_1. (1 point)

The remaining two currents can be determined by the mesh-current equations

$$\begin{bmatrix} -20 & 20+20 & -20 \\ 0 & -20 & 20+10+20 \end{bmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} -10 \\ 0 \end{pmatrix} \quad \text{(eq. 2)}$$

which were obtained by inspection of the circuit. (3 points)

One must solve (eq. 1) and (eq. 2) for currents i_1, i_2 and i_3. That is $i_1 = 1$ and solve

$$\begin{bmatrix} 40 & -20 \\ -20 & 50 \end{bmatrix} \begin{pmatrix} i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$$

for i_2 and i_3. (2 points)

3. **Nodal voltage analysis**

(a) (6 points) Assuming that the node labeled D is the ground node (reference), formulate node-voltage equations for the circuit in Figure 1. Use the node labels provided in the figure and clearly indicate how you handle the presence of a voltage source, the final equations, and the unknowns they must be solved for. **Do not modify the circuit or the labels in any way**. **Do not use source transformation**. **Do not solve any**

equations! *Hint: Use a super-node.*

Solution: We start by writing the node-voltage equations by inspection

$$\begin{bmatrix} \frac{1}{10}+\frac{1}{20} & -\frac{1}{10} & 0 \\ -\frac{1}{10} & \frac{1}{10}+\frac{1}{20} & 0 \\ 0 & 0 & \frac{1}{20} \end{bmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} 0 \\ -i_v \\ 1+i_v \end{pmatrix} \qquad \text{(eq. 3)}$$

where i_v is the unknown current on the voltage source. (2 points)

Furthermore, we have the relationship

$$v_C - v_B = 10 \text{ V} \qquad \text{(eq. 4)}$$

because of the voltage source. (1 point)

In order to determine the node-voltages we can form the *super-node* equation formed by adding the second and third rows of (eq. 4)

$$\begin{bmatrix} -\frac{1}{10} & \frac{1}{10}+\frac{1}{20} & \frac{1}{20} \end{bmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = 1 \qquad \text{(eq. 5)}$$

which should be solved for v_A, v_B and v_C along with the first equation of (eq. 3) and (eq. 4). (2 points)

The final equations are

$$\begin{bmatrix} \frac{1}{10}+\frac{1}{20} & -\frac{1}{10} & 0 \\ -\frac{1}{10} & \frac{1}{10}+\frac{1}{20} & \frac{1}{20} \\ 0 & -1 & 1 \end{bmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 10 \end{pmatrix}$$

to be solved for v_A, v_B and v_C. (1 point)

4. **Bonus Question**

   (a) (1 point) If you were allowed to modify the circuit in Figure 1, describe what would you do in order to avoid having to use a super-node in Question 3? **Do not write or solve any equations**!

Solution: One could choose nodes B (or C) as references. (1 point)

In this way v_C (or v_B) would be determined and one would simply not write KCL at node C. This is method # 2.

Alternatively, a more complicate solution would be to leave D as the reference node and apply a source transformation on the current source and then associate the voltage sources as done in part (b) of Question 1.