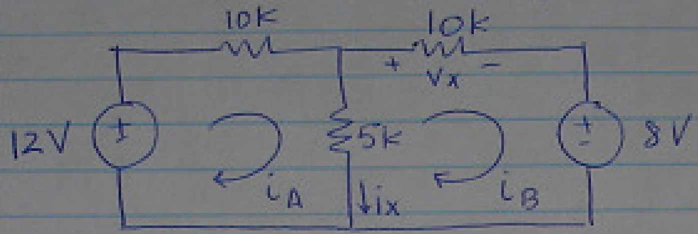
3.9 a) Formulate mesh-current equations for the circuit

b.) Use these equations to find V_x and i_x.



$V_x = 10k\, i_B$

$i_x = i_A - i_B$

KVL in loop A:

$10k\, i_A + 5k(i_A - i_B) - 12 = 0$

$10k\, i_A + 5k\, i_A - 5k\, i_B = 12$

$15k\, i_A - 5k\, i_B = 12$

KVL in loop B:

$10k\, i_B + 8 + 5k(i_B - i_A) = 0$

$10k\, i_B + 5k\, i_B - 5k\, i_A = -8$

$15k\, i_B - 5k\, i_A = -8$

$$\begin{bmatrix} 10k+5k & -5k \\ -5k & 10k+5k \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \end{bmatrix}$$

$i_A = \dfrac{5k\, i_B + 12}{15K}$

$15k\, i_B - \cancel{5K}\left(\dfrac{5k\, i_B + 12}{\cancel{15K}\ 3}\right) = -8$

$15k\, i_B - \dfrac{5k}{3} i_B - 4 = -8$

$45k\, i_B - 5k\, i_B = 3(-4)$

$40k\, i_B = -12$

$i_B = \dfrac{-12}{40k} = \dfrac{-3}{10k} = -0.3\,mA$

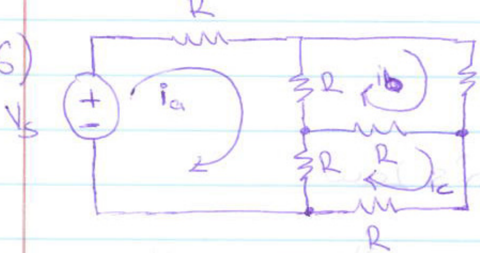
$i_A = \dfrac{5(-0.3)+12}{15k} = \dfrac{10.5}{15k} = 0.7mA$

$V_x = 10k(-0.3mA)$

$\boxed{V_x = 3V}$

$\boxed{i_x = 1mA}$

3.16)



a) Mesh A: $-V_s + Ri_a + R(i_a - i_b) + R(i_a - i_c) = 0$

$\Rightarrow 3Ri_a - Ri_b - Ri_c = V_s$

Mesh B: $R(i_b - i_a) + Ri_b + R(i_b - i_c) = 0$

$\Rightarrow -Ri_a + 3Ri_b - Ri_c = 0$

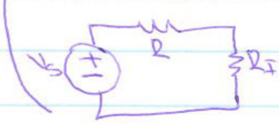
Mesh C: $R(i_c - i_a) + R(i_c - i_b) + Ri_c = 0$

$\Rightarrow 3Ri_c - Ri_a - Ri_b = 0$

$$\Rightarrow \begin{bmatrix} 3R & -R & -R \\ -R & 3R & -R \\ -R & -R & 3R \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ 0 \end{bmatrix}$$

b)

(Note: It can be understood as



It is not problem, you can follow the same way)

The only thing that we need to do is find i_a and apply Ohm's Law ($V_s = i_a \cdot R_{IN}$)

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \frac{1}{R} \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} V_s \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \frac{1}{R} \begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0.25 & 0.5 & 0.25 \\ 0.25 & 0.25 & 0.5 \end{bmatrix} \begin{bmatrix} V_s \\ 0 \\ 0 \end{bmatrix}$$

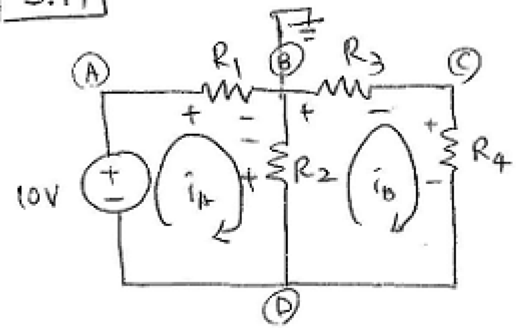
from this equation $i_a = \frac{V_s}{R} 0.5$ $i_b = \frac{V_s}{R} 0.25$ $i_c = \frac{V_s}{R} 0.25$

3.16) b) cont.)

Now apply Ohm's Law

$$V_S = R_{IN} \cdot i_a \Rightarrow R_{IN} = \frac{V_S}{i_a} \Rightarrow R_{IN} = \frac{V_S}{\frac{V_S}{R} \cdot 0.5} = 2R$$

3.17



Find: V_A, V_D, i_A and i_B

$R_1 = R_2 = R_3 = R_4 = 1k\Omega$

$V_C = -2V$ when node B is connected to ground.

Mesh-current eqns:

Mesh (A): $-10 + R_1 i_A + R_2(i_A - i_B) = 0$

Mesh (B): $R_2(i_B - i_A) + R_3 i_B + R_4 i_B = 0$

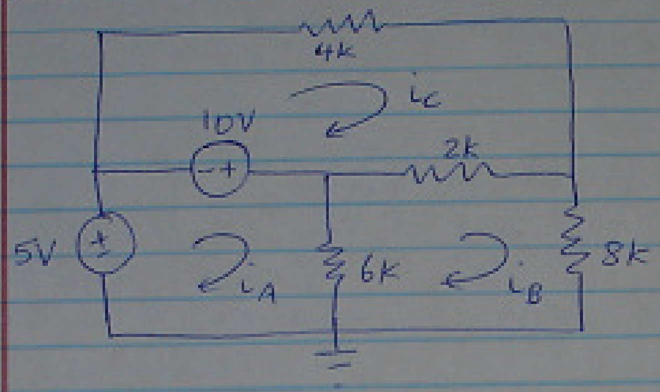
$$\begin{bmatrix} 2000 & -1000 \\ -1000 & 3000 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$\Rightarrow$ $i_A = 6mA$, $i_B = 2mA$

$V_A = R_1 i_A = (1000)(6\times10^{-3} A) = 6\ V$

$V_D = R_2(i_B - i_A) = 1000(2\times10^{-3} - 6\times10^{-3}) = -4V$

3.20 Find the mesh currents i_A, i_B and i_C



KVL in loop A

$-5-10+6k(i_A-i_B)=0$

$6ki_A-6ki_B=15 \implies i_A=\frac{15+6ki_B}{6k}$

loop B: $2k(i_B-i_C)+8ki_B+6k(i_B-i_A)=0$

$16ki_B-6ki_A-2ki_C=0$

loop C: $4ki_C+2k(i_C-i_B)+10=0$

$6ki_C-2ki_B=-10 \implies i_C=\frac{-10+2ki_B}{6k}$

$$\begin{bmatrix} 6k & -6k & 0 \\ -6k & 16k & -2k \\ 0 & -2k & 6k \end{bmatrix}\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix}=\begin{bmatrix} 15 \\ 0 \\ -10 \end{bmatrix}$$

$16ki_B-\cancel{6k}\left(\frac{15+6ki_B}{\cancel{6k}}\right)-\cancel{2k}\left(\frac{-10+2ki_B}{\cancel{6k}\,3}\right)=0$

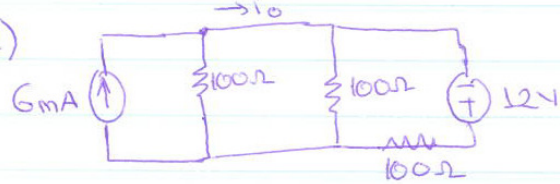
$10ki_B-\frac{2}{3}ki_B-15+\frac{10}{3}=0$

$30ki_B-2ki_B-45+10=0$

$28ki_B=35 \implies \boxed{i_B=1.25mA}$

$\boxed{i_A=3.75mA}$ $\boxed{i_C=-1.25mA}$

3.27)



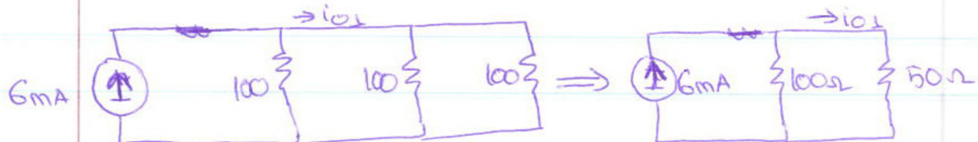
Superposition

Remember:

Voltag Source $\xrightarrow[\text{with}]{\text{Replace}}$ Short circuit

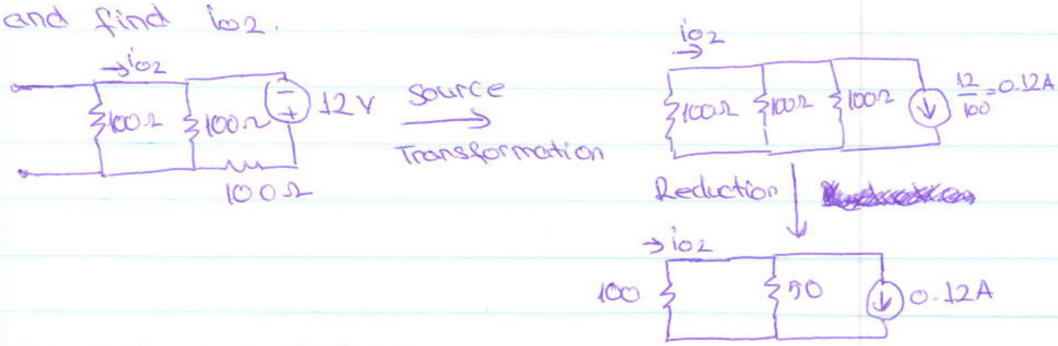
Current Source $\xrightarrow[\text{With}]{\text{Replace}}$ open circuit

<u>Step 1:</u> Replace voltage source with short circuit and find i_{o1}



Two path current division $\quad i_{o1} = \dfrac{100\Omega}{(100+50)\Omega} \cdot 6mA = 0.4mA$

<u>Step 2:</u> Replace current source with open circuit and find i_{o2}.



Two path current division

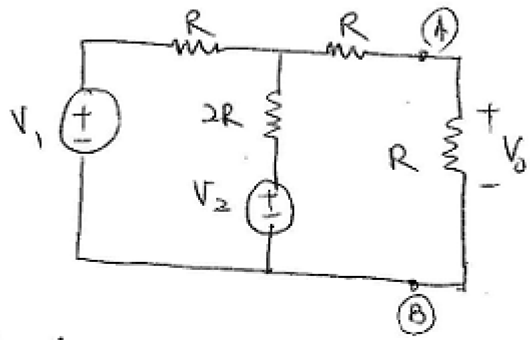
$$i_{o2} = \frac{100\Omega}{(100+50)\Omega} 0.12A = 0.04A$$
$$= 40mA$$

<u>Step 3:</u> $\quad i_o = i_{o1} + i_{o2} = 0.4mA + 40mA$

$$= \underline{\underline{44mA}}$$

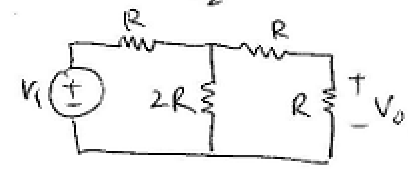
$\boxed{3.30}$

Use the superposition principle to find v_o in terms of v_1, v_2 and R.



Step 1

"turn off" V_2

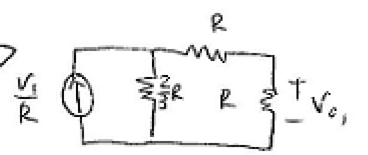


$\Rightarrow$

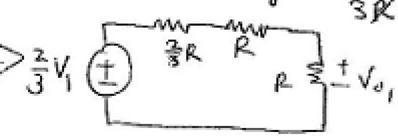
$\Rightarrow$

$$R_{eq} = \frac{(2R)\cancel{R}}{3\cancel{R}} = \frac{2}{3}R$$

$\Rightarrow$



$\Rightarrow$



$$\therefore V_{o_1} = \frac{R}{\left(\frac{2}{3}R+R+R\right)}\left(\frac{2}{3}V_1\right)$$

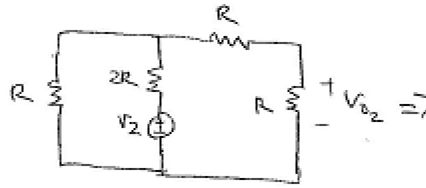
$$= \frac{\cancel{3}}{8}\left(\frac{2}{\cancel{3}}V_1\right)$$

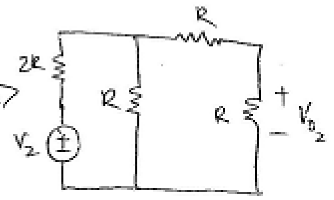
$$= \frac{V_1}{4}$$

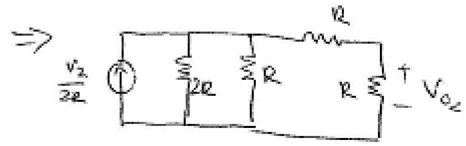
3/

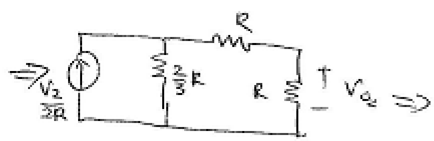
Step 2

"turn off" V_1

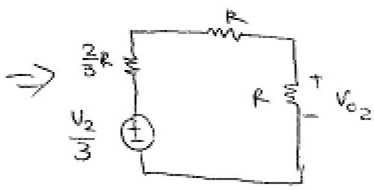








$$R_{eq} = \frac{(2R)\cancel{R}}{3\cancel{R}} = \frac{2}{3}R$$



$$\therefore V_{o_2} = \frac{R}{\left(\frac{2}{3}R + R + R\right)}\left(\frac{V_2}{3}\right)$$

$$= \left(\frac{\cancel{3}}{8}\right)\left(\frac{V_2}{\cancel{3}}\right)$$

$$= \frac{V_2}{8}$$

$$\therefore V_0 = V_{o_1} + V_{o_2}$$

$$= \frac{V_1}{4} + \frac{V_2}{8}$$

$$= \boxed{\frac{1}{8}(2V_1 + V_2)}$$

3.33 $V_s = 10V$, $i_s = 10mA$
$V_o = 2V$, $i_s = \text{off}$
$V_o = 1V$, $V_s = i_s =$ ~~off~~ on

$V_o = k_1 x_1 + k_2 x_2$
$V_o = k_1 V_s + k_2 i_s$

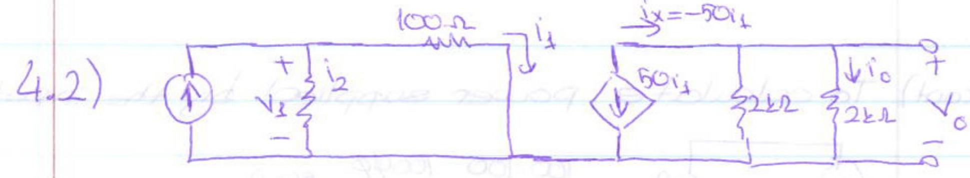
$V_{o1} = k_1 V_s$ when $i_s = \text{off}$
$2V = k_1 10V$
$\boxed{k_1 = 0.2}$

$V_o = k_1 V_s + k_2 i_s$
$1 = (0.2)(10) + k_2 (10mA)$
$1 = 2 + k_2 (10mA)$
$\boxed{k_2 = \frac{-1}{10mA} = -0.1K}$

Changing $V_s = 5V$, $i_s = -10mA$

$V_o = k_1 V_s + k_2 i_s$
$V_o = (0.2)(5) + (-0.1k)(-10mA)$
$= 1 + 1$
$\boxed{V_o = 2V}$

4.2)



Two path current division for source part

$$i_1 = \left(\frac{100}{100+100}\right) i_s = \frac{i_s}{2} \qquad i_2 = i_s - i_1 = \frac{i_s}{2}$$

Two path current division for right hand side

$$i_o = \left(\frac{2}{2+2}\right) i_x = \frac{1}{2}(-50 i_1) = -\frac{25}{2} i_s$$

current gain $\boxed{i_o / i_s = \frac{-25/2\, i_s}{i_s} = \frac{-25}{2}}$

Calculate V_1 and V_o from Ohm's Law

$$V_1 = 100\, i_2 = 100 \frac{i_s}{2} = 50 i_s$$

$$V_o = 2000\left(\frac{-25}{2}\right) i_s = -25000\, i_s$$

Voltage gain $\boxed{V_o / V_1 = \frac{-2500\cancel{0}\, \cancel{i_s}}{5\cancel{0}\, \cancel{i_s}} = -500}$

4.2)(cont) To calculate power supplied by the input source

$$R_{eq} = \frac{100\cdot 100}{100+100} = \frac{100\cancel{0}\cancel{0}}{2\cancel{0}\cancel{0}} = 50\,\Omega$$

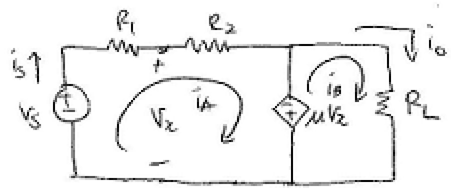
$$\boxed{P_s = (i_s)^2 \cdot 50 = 0.2\,mW}$$

Calculate power delivered to 2kΩ load Resistor.

$$P_o = (i_o)^2\, 2k\Omega = \left(\frac{-25}{\cancel{2}}\cdot \cancel{2}mA\right)^2 2k\Omega$$

$$\boxed{P_o = 1250\,mW = 1.25\,W}$$

4.8



Find an expression for the current gain i_o/i_s in the circuit

Mesh Analysis

Mesh (A): $-V_s + (R_1+R_2)i_A - \mu V_x = 0$ --- ①

Mesh (B): $\mu V_x + i_B R_L = 0$ --- ②

plug $V_x = V_s - i_A R_1$ into ① & ②:

$-V_s + (R_1+R_2)i_A - \mu(V_s - i_A R_1) = 0$ --- ③

$\mu(V_s - i_A R_1) + i_B R_L = 0$ --- ④

From ③,

$$(R_1+R_2+\mu R_1)i_A = V_s(1+\mu)$$

$$i_A = \frac{V_s(1+\mu)}{R_1(1+\mu)+R_2}$$

From ④,

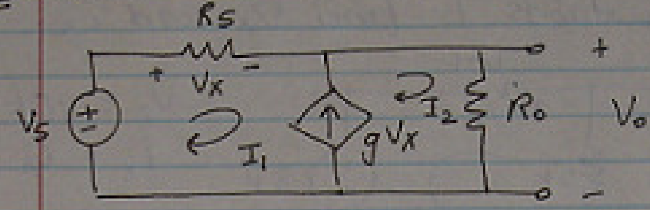
$$i_B = \frac{-\mu(V_s - i_A R_1)}{R_L} = -\frac{\mu}{R_L}\left[V_s - \frac{V_s(1+\mu)}{R_1(1+\mu)+R_2}R_1\right]$$

$$= -\frac{\mu}{R_L}\left[\frac{V_s \cancel{R_1(1+\mu)} + V_s R_2 - \cancel{V_s(1+\mu)R_1}}{R_1(1+\mu)+R_2}\right] = -\frac{\mu}{R_L}\left[\frac{V_s R_2}{R_1(1+\mu)+R_2}\right]$$

$$\therefore \frac{i_o}{i_s} = \frac{i_B}{i_A} = \frac{-\frac{\mu}{R_L}\left[\frac{\cancel{V_s}R_2}{\cancel{R_1(1+\mu)+R_2}}\right]}{\frac{\cancel{V_s}(1+\mu)}{\cancel{R_1(1+\mu)+R_2}}}$$

$$= \boxed{-\frac{\mu}{R_L}\left[\frac{R_2}{(1+\mu)}\right]}$$

4.9) Find an expression for the voltage gain $\frac{V_o}{V_s}$



KVL in loop 1

$$-V_S + V_X + V_o = 0$$
$$V_X = V_S - V_o$$

$$V_X = I_1 R_S$$
$$V_o = I_2 R_o$$

$$I_2 - I_1 = g V_X$$

$$I_1 R_S = V_S - V_o$$
$$(I_2 - g V_X) R_S = V_S - V_o$$
$$\left(\frac{V_o}{R_o} - g V_X\right) R_S = V_S - V_o$$

$$V_o \frac{R_S}{R_o} - g V_X R_S = V_S - V_o$$

$$V_o \frac{R_S}{R_o} - g R_S (V_S - V_o) = V_S - V_o$$

$$V_o \frac{R_S}{R_o} - g R_S V_S + g R_S V_o = V_S - V_o$$

$$V_o \left(\frac{R_S}{R_o} + g R_S + 1\right) = V_S (1 + g R_S)$$

$$\boxed{\frac{V_o}{V_S} = \frac{(1 + g R_S) R_o}{R_S + R_o + g R_S R_o}}$$