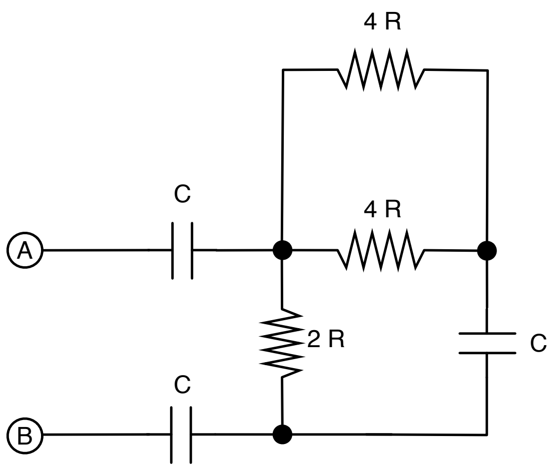
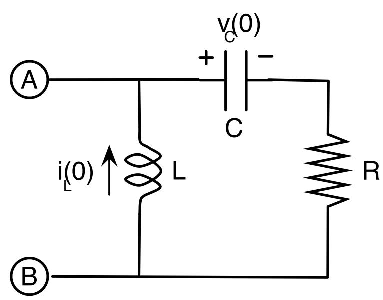
MAE140 – Linear Circuits – Winter 2009
Final, March 17

Instructions

1) This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities.
2) You have 170 minutes.
3) Write your name, student number and instructor.
4) On the questions for which we have given the answers, please provide detailed derivations.



(a) Part (a)



(b) Part (b)

Figure 1: Circuit for Question 1

Questions

1. **Equivalent Circuits**
   (a) [5 points] Assuming zero initial conditions, find the impedance equivalent to the circuit in Figure 1(a) as seen from terminals A and B. The answer should be given as a ratio of two polynomials.
   (b) [5 points] Assuming that the initial conditions of the inductor and capacitor are as indicated in the diagram, redraw the circuit shown in Figure 1(b) in the s-domain. Then use source transformations to find the s-domain Norton equivalent of this circuit as seen from terminals A and B. The transfer functions should be given as a ratio of two polynomials.
   (*Hint: Use an equivalent model for the inductor in which the initial condition appears as a current source, and an equivalent model for the capacitor in which the initial condition appears as a voltage source.*)

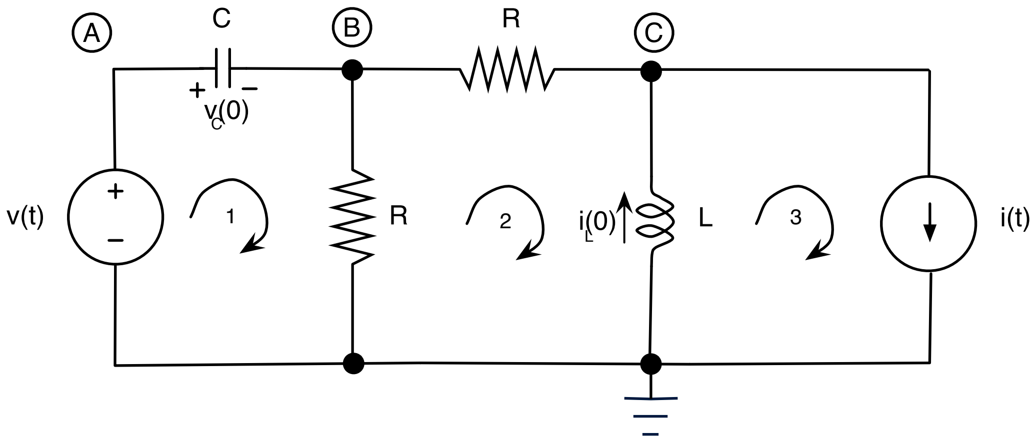


Figure 2: Nodal and Mesh Analysis Circuit

2. **Nodal and Mesh Analysis**

   In this question you do not need to solve any equations, but clearly indicate the equations and the unknown voltages or currents that you should solve for in order to determine the entire circuit.

   (a) [5 points] Formulate node-voltage equations in the s-domain for the circuit in Figure 2. Use the reference node and other labels as shown in the figure. Do not assume zero initial conditions! Transform initial conditions on the capacitor and on the inductor into current sources.

   (b) [5 points] Formulate mesh-current equations in the s-domain for the circuit in Figure 2. Use the currents shown in the figure. Do not assume zero initial conditions! Transform initial conditions on the capacitor and on the inductor into voltage sources.

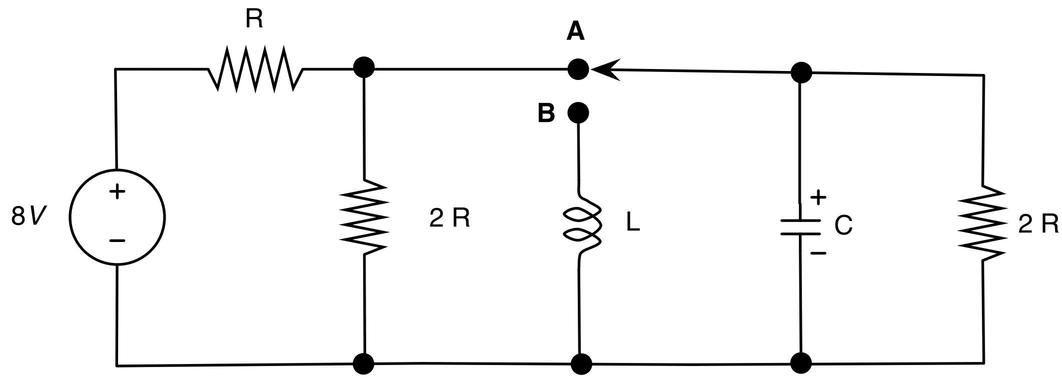


Figure 3: RCL circuit for Laplace Analysis

3. **Laplace Domain Circuit Analysis**

(a) [3 points] Consider the circuit depicted in Figure 3. The voltage source is constant. The switch is kept in position **A** for a very long time. At $t = 0$ it is moved to position **B**. Show that the initial capacitor voltage and inductor currents are given by

$$v_C(0^-) = 4\,\mathrm{V}, \quad i_L(0^-) = 0\,\mathrm{A}.$$

[Show your work]

(b) [2 points] Use these initial conditions to transform the circuit into the s-domain for $t \geq 0$. Use equivalent models for the capacitor and the inductor in which the initial conditions appear as current sources.

[Show your work]

(c) [5 points] Use s-domain circuit analysis and inverse Laplace transforms to show that the capacitor voltage is

$$v_C(t) = (8e^{-2t} - 4e^{-t})u(t)\,\mathrm{V}.$$

when $C = \frac{1}{6}$ F, $L = 3$ H, and $R = 1\ \Omega$.

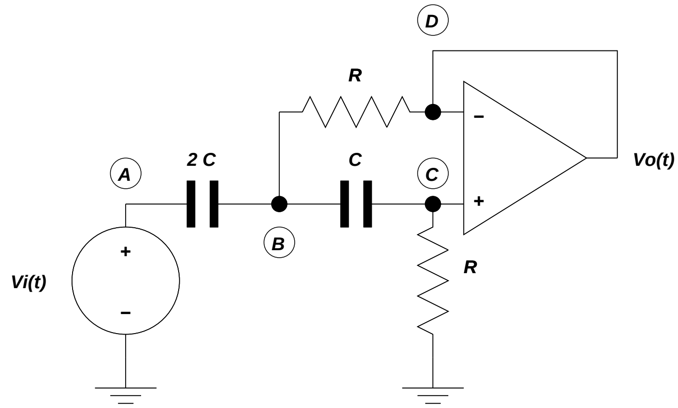


Figure 4: Active Filter Circuit

4. **Active Filter Analysis**

   In this question assume zero initial conditions.

   (a) [1 point] Transform the circuit in Figure 4 to the s-domain.

   (b) [3 points] Formulate node-voltage equations in the s-domain that would allow one to compute the transfer function from $V_i(s)$ to $V_0(s)$. You do not need to solve any equations, but clearly indicate the equations that need to be solved for and the unknowns. Tell how you can find $V_0(s)/V_i(s)$ from the unknowns. Use the labels for the nodes provided in Figure 4.

   (c) [3 points (bonus)] Solve the node-voltage equations you formulated in part (b) to show that the transfer function from $V_i(s)$ to $V_0(s)$ is

$$T(s) = \frac{V_0(s)}{V_i(s)} = \frac{s^2}{s^2 + (3\lambda/2)s + \lambda^2/2}$$

   where $\lambda = 1/(RC)$.

   (d) [3 points] What is the gain and phase at $\omega = 0$ and $\omega \to \infty$? What is the gain at $\omega = \lambda$?

   (e) [3 points] Sketch the gain versus frequency diagram. What type of filter is this circuit? Is the cut-off frequency of the circuit equal, less or greater than λ rad/s?

5. **Active Filter Design**

Consider the transfer function

$$T(s) = \frac{V_0(s)}{V_i(s)} = \frac{s^2}{s^2 + (3\lambda/2)s + \lambda^2/2}$$

of the filter studied in the previous question where the parameter λ is now to be specified by the user. In this question, always assume zero initial conditions.

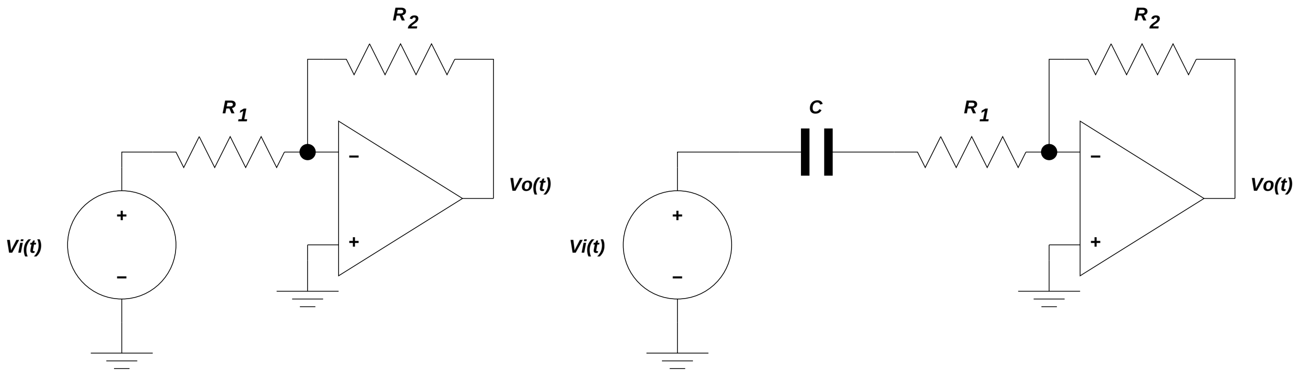
(a) [3 points] Show that the transfer function $T(s)$ can be realized as a product of two first-order high-pass filters of the form

$$T_1(s) = \frac{\pm s}{s + \omega_1}, \qquad T_2(s) = \frac{\pm s}{s + \omega_2}$$

that is, $T(s) = T_1(s) \times T_2(s)$. What is the cut-off frequency and gain of $T_1(s)$ and $T_2(s)$ in terms of λ?

(b) [4 points] Design a circuit that implements $T(s)$ as the product of the two filters $T_1(s)$ and $T_2(s)$ using no more than 2 OpAmps.

(c) [3 points] Find values of the components in your design so that $\lambda = 1000$ Hz.



(a) Circuit I (b) Circuit II

Figure 5: OpAmp Circuits for AC Amplifier

6. **AC Amplifier**

A group of former MAE 140 students needs to build an OpAmp circuit to amplify telephone signals. Because of reasons that were relevant some hundred years ago, your telephone jack has a DC 48 V voltage superimposed to the voice signal. To make things simple in this exam, think now of a "test" signal coming through your telephone line with a fixed frequency of $\omega = 1500$ rad/s. On your telephone jack you would measure a voltage

$$V_i(t) = 48 + 12\cos(\omega\, t) \text{ V}.$$

In order to solve this problem the students have selected two circuits, shown in Figure 5. The power supplies to the OpAmps (not shown in the figure) are $V_{CC} = +12$ V and $-V_{CC} = -12$ V. For impedance matching set $R_1 = 600\ \Omega$.

(a) [2 points] The students built Circuit I with $R_2 = R_1 = 600\ \Omega$ but found out to their surprise that they could not hear the $\omega = 1500$ rad/s tone after they plugged $V_0(t)$ into a speaker. Why that happened?
(*Hint: Recall the role played by V_{CC} and $-V_{CC}$ in an OpAmp!*)

(b) [3 points] By replacing R_2 with a potentiometer, they eventually found a value of resistance R_2 below which they could hear the tone *without any distortion*. What is this critical value of R_2 and what is the corresponding amplitude of the tone fed to the speaker?

(c) [3 points] One student suggested that all they needed to do to fix the problem was to add a capacitor to Circuit I, obtaining Circuit II. In order to explain why this new circuit solves the problem, compute the steady state response $V_0(t)$ as a function of R_1, R_2 and C. Why does this circuit fix the problem?
(*Hint: Because the circuit is linear you can compute the steady-state response to the input $V_i(t)$ as the sum of the steady-state response to the cosine tone plus the steady-state response to the constant 48 V.*)

(d) [2 points] Find values of R_2 and C so that the amplitude of the steady-state cosine function in $V_0(t)$ be 12 V for large ω ($\to \infty$) and at least $12/\sqrt{2}$ V for $\omega = 1500$ rad/s.