MAE 140 – Linear Circuits – Winter 2008
Midterm

Instructions

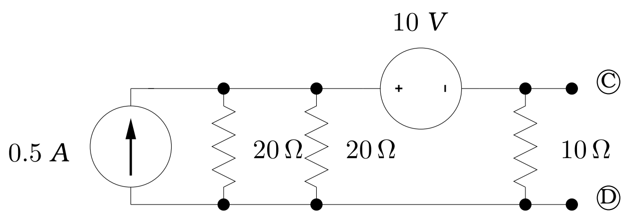
1) This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities.

2) You have 45 minutes.

Question 1 — Equivalent Circuits

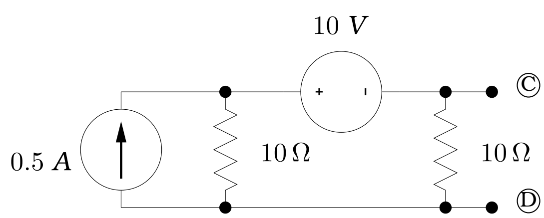
Part (i) [6 marks] Use source transformations and association of resistors to find the Thévenin equivalent to the circuit in Figure 1 as seen from terminals C and D.

First convert the 10 V *voltage source in series with the* $20\,\Omega$ *resistor into a current source of* $I = 10/20 = 0.5$ A *in parallel with a* $20\,\Omega$ *resistor. This results in the following circuit:*



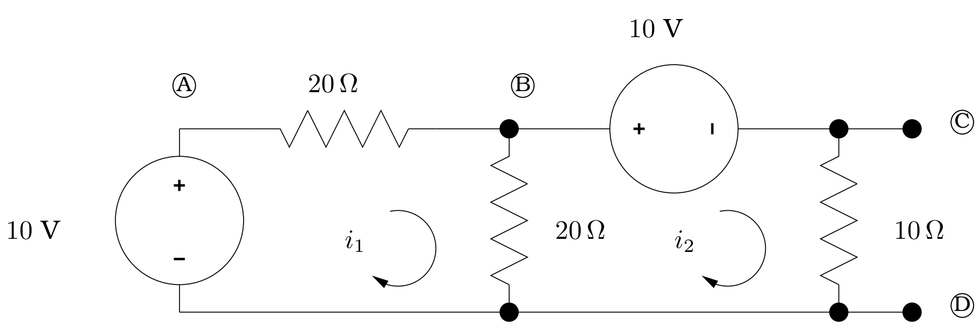
[+ 1 mark]

Associate the two parallel $20\,\Omega$ *resistors into a* $R = 20 \times 20/(20+20) = 10\,\Omega$ *resistor. This results in the following circuit:*

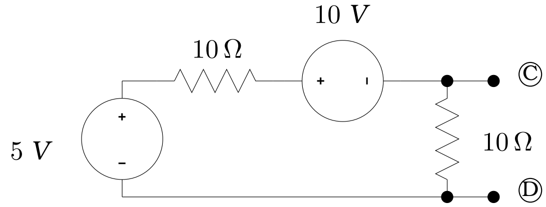


[+ 1 mark]

Figure 1: Circuit for Questions 1, 2 and 3

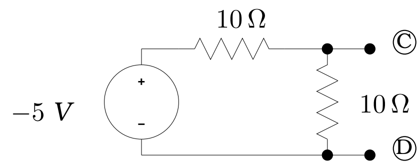


Convert the source of current and resistor back to a source of voltage $V = 10 \times 0.5 = 5$ *V voltage source in series with a* $10\,\Omega$ *resistor. This results in the following circuit:*



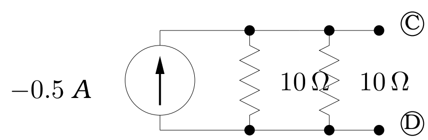
[+ 1 mark]

Associate the voltage sources in series into a $V = 5 - 10 = -5$ *V voltage source. This results in the following circuit:*

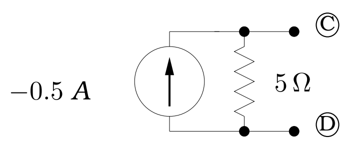


[+ 1 mark]

Convert the source of voltage and resistor back to a source of current $I = -5/10 = -0.5$ *A in parallel with a* $10\,\Omega$ *resistor. This results in the following circuit:*

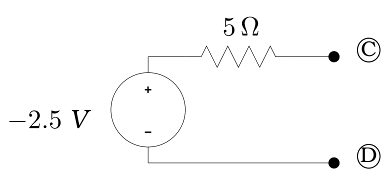


Associate the two $10\,\Omega$ *resistors in parallel. This results in the following circuit:*



[+ 1 mark]

Finally transform back into a source of voltage $V = -0.5 \times 5 = 2.5$ *V in series with a* $5\,\Omega$ *resistor. This results in the following circuit:*



This is the Thévenin equivalent.

[+ 1 mark]

Part (ii) [2 marks] Find the power absorbed by a $5\,\Omega$ resistor that is connected to terminals C and D.

First compute the voltage on the resistor using voltage division:

$$v = \frac{5}{5+5} \times -2.5 = -1.25\,\mathrm{V}.$$

[+ 1 mark]

Then compute power

$$p = \frac{v^2}{R} = \frac{1.25^2}{5} \approx 0.31\,\mathrm{W}.$$

[+ 1 mark]

Question 2 — Nodal and Mesh Analysis

Part (i) [6 marks] Formulate mesh-current equations for the circuit in Figure 1. Use the mesh currents indicated in the figure and clearly indicate the final equations and the unknowns they must be solved for. You do not have to solve any equations!

Circuit contains only voltage sources, so no transformations are needed. Writting the equations by inspection:

$$\begin{bmatrix} 20+20 & -20 \\ -20 & 20+10 \end{bmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} 10 \\ -10 \end{pmatrix},$$

[+ 5 marks]

These equations need to be solved for the mesh currents (variables) i_1 *and* i_2*.*

[+ 1 mark]

Part (ii) [6 marks] Assuming that the node labeled D is the ground node (reference), formulate node-voltage equations for the circuit in Figure 1. Use the node labels provided in the figure and clearly indicate how you handle the presence of voltage sources, the final equations and the unknowns they must be solved for. Do not solve any equations!

Circuit has two voltage sources. One has one terminal grounded so that v_A *is automatically determined, that is*

$$v_A = 10\,\mathrm{V},$$

[+ 1 mark]

The other voltage source determines a relation between v_B *and* v_C *namely*

$$v_B - v_C = 10\,\mathrm{V}.$$

[+ 1 mark]

We do not have to write KCL node A and will end up with a supernode B-C. Writing the node-voltage equations by inspection at nodes B and C:

$$\begin{bmatrix} -\frac{1}{20} & \frac{1}{20} + \frac{1}{20} & 0 \\ 0 & 0 & \frac{1}{10} \end{bmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} i_2 \\ -i_2 \end{pmatrix}.$$

where i_2 *is the current on the voltage source connected between nodes B and C. You can arrive at the supernode equation by adding the above equations, thus eliminating* i_2*:*

$$-\frac{1}{20}v_A + \frac{1}{10}v_B + \frac{1}{10}v_C = 0.$$

[+ 3 marks]

The supernode equation should be solved along with

$$v_A = 10\,\mathrm{V},$$
$$v_B - v_C = 10\,\mathrm{V}.$$

for the node voltage (variables) v_A, v_B, v_C.

[+ 1 mark]

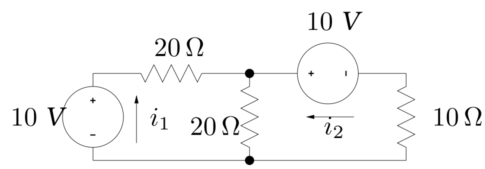
Alternatively, you also get marks if you correctly state four equations in four variables v_A, v_B, v_C and i_2 *need be solved.*

Question 3 — Bonus question

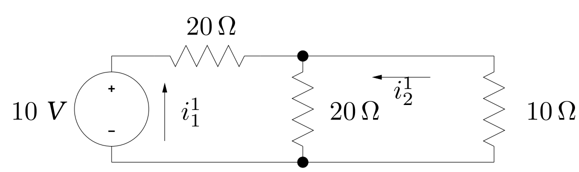
[3 marks] Use superposition to find the total power absorbed by the circuit in Figure 1.

CAUTION: currents and voltages are linear variables but not their product. So one cannot produce add cannot simply add the power in two circuits and hope that it will be the same.

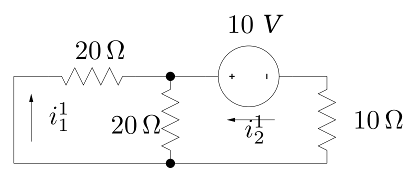
Hence we use superposition to determine currents and voltage. Then we add the voltages and currents and find the power. We will use superposition to solve for i_1 *and* i_2 *in the following circuit:*



First divide the problem in two subproblems. First subproblem is



Second subproblem is



[+1 mark]

Now solve for i_1^1 *and* i_2^1 *in the first subproblem. Current* i_1^1 *is the current on the voltage source which is equal to*

$$i_1^1 = \frac{10}{20 + 20//10} = \frac{10}{20 + 200/30} = \frac{3}{6+2} = 0.375\,\mathrm{A}.$$

Current i_2^1 *can be computed using current division after transforming the source into a* $10/20 = 0.5$ *A source in parallel with two* $20\,\Omega$ *and one* $10\,\Omega$ *resistor:*

$$i_2^1 = -\frac{\frac{1}{10}}{\frac{1}{20} + \frac{1}{20} + \frac{1}{10}}\,0.5 = -\frac{2}{1+1+2}\,0.5 = -0.25\,\mathrm{A}.$$

Now solve for i_1^2 and i_2^2 in the second subproblem. This time current i_2^1 is the current on the voltage source which is equal to

$$i_2^1 = \frac{10}{10 + 20//20} = \frac{10}{10 + 10} = 0.5\,\mathrm{A}.$$

Current i_1^1 can be computed using current division after transforming the source into a $10/10 = 1$ A source in parallel with two $20\,\Omega$ and one $10\,\Omega$ resistor:

$$i_1^2 = -\frac{\frac{1}{20}}{\frac{1}{20} + \frac{1}{20} + \frac{1}{10}} = -\frac{1}{1 + 1 + 2} = -0.25\,\mathrm{A}.$$

[+1 mark]

Now compute i_1 and i_2 using superposition

$$i_1 = i_1^1 + i_1^2 = 0.375 - 0.25 = 0.125\,\mathrm{A}, \qquad i_2 = i_2^1 + i_2^2 = -0.25 + 0.5 = 0.25\,\mathrm{A}.$$

The total power absorbed is equal to the total power delivered

$$p = v_1 i_1 + v_2 i_2 = 1.25 + 2.5 = 3.75\,\mathrm{W}.$$

[+1 mark]