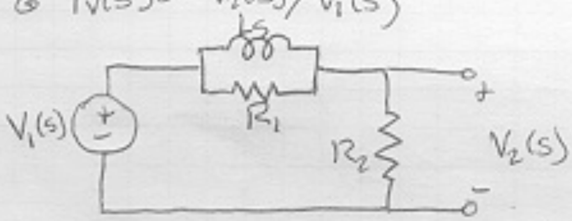
Homework VIII | Ch. 11: 2, 4, 7, 28, 53, 58
Ch. 12: 4, 10, 15, 28

P.11.2 find driving pt. impedance & $T_V(s) = V_2(s)/V_1(s)$



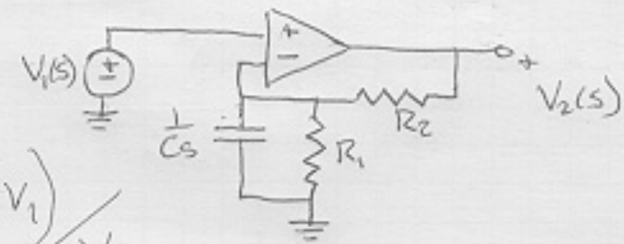
$$Z_{EQ} = R_2 + \frac{R_1 L s}{R_1 + Ls}$$

$$\boxed{Z_{EQ} = \frac{(R_1+R_2)Ls + R_1R_2}{R_1 + Ls}}$$

$V_2(s) = T_V(s)\,V_1(s)$ → voltage divider $T_V(s) = \dfrac{R_2}{Z_{EQ}} =$

$$\boxed{T_V(s) = \frac{R_2(Ls + R_1)}{(R_1+R_2)Ls + R_1R_2}}$$

P.11.4 find driving pt. impedance & $T_V(s)$



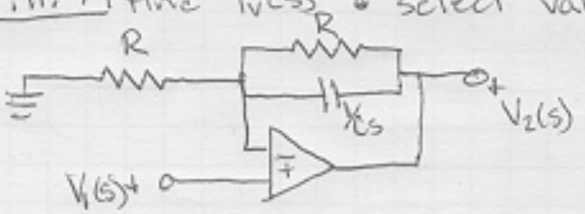
OP-AMP Golden rule $I_N = I_P = 0$

$\therefore \boxed{Z_{EQ} = \infty}$

$$T_V(s) = \frac{V_2(s)}{V_1(s)} = \frac{\left(\dfrac{R_2 + \left(\frac{1}{1/R_1 + Cs}\right)}{\left(\frac{1}{1/R_1 + Cs}\right)} V_1\right)}{V_1}$$

$$\boxed{T_V(s) = \frac{R_1R_2Cs + R_1 + R_2}{R_1}}$$

P.11.7 find $T_V(s)$ & select values of R & C so there is a pole @ $s = -500\ \frac{rad}{s}$



inverting amplifier

$$T_V(s) = \frac{\frac{R}{RCs+1} + R}{R}$$

$$\boxed{T_V(s) = \frac{RCs + 2}{RCs + 1}}$$

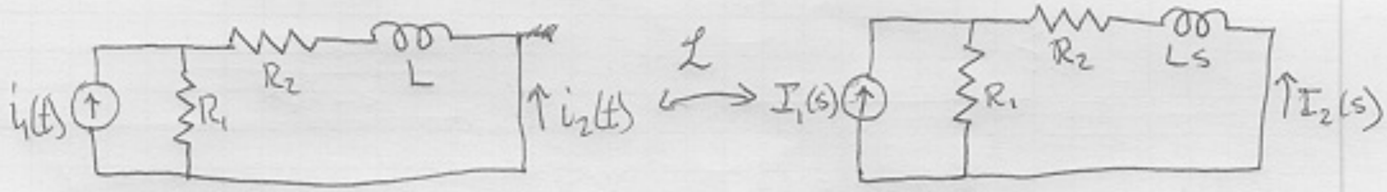
any values of R & C such that $RC = \frac{1}{500}$

ex.

$R = 1\,k\Omega$

$C = 2\,\mu F$

P.11.28 If circuit is in steady state with $i_1(t) = 10\cos 500t$ mA & $R_1 = 100\Omega$, $R_2 = 400\Omega$, & $L = 100$mH find $i_{2ss}(t)$. Repeat with $i_1(t) = 10 \cos 5000t$ mA



with input $\quad i(t) = A\cos(\omega t)$

steady state output will have the form $\quad i_{ss}(t) = A\,|T(j\omega)|\cos(\omega t + \theta)$

where $\theta = \angle T(j\omega)$

so what is $T(s)$ → use current divider

$$T(s) = \frac{\left(\frac{1}{R_2 + Ls}\right)}{\frac{1}{R_1} + \frac{1}{R_2 + Ls}} = \frac{R_1}{Ls + R_1 + R_2} = \frac{R_1/L}{s + \frac{R_1 + R_2}{L}}$$

with given values $\quad T(s) = \frac{1000}{s + 5000}$

$$|T(j500)| = \frac{1000}{\sqrt{500^2 + 5000^2}} = \frac{2}{\sqrt{101}}$$

$$\theta(j500) = 0 - \tan^{-1}\left(\frac{500}{5000}\right)$$

$i_1(t) = 10\cos(500t)$ mA

$i_{2ss}(t) = -10\,|T(j500)|\cos(500t + \theta)$ mA

$\quad = 10\,|T(j500)|\cos(500t + \pi + \theta)$ mA

$\boxed{i_{2ss}(t) \approx 1.99\cos(500t + 3.041)\text{ mA}}$

$i_1(t) = 10\cos(5000t)$ mA

$$|T(j5000)| = \frac{1000}{\sqrt{5000^2 + 5000^2}} = \frac{1}{5\sqrt{2}}$$

$$\theta = 0 - \tan^{-1}\left(\frac{5000}{5000}\right)$$

$i_{2ss}(t) = 10\,\frac{1}{5\sqrt{2}}\cos(5000t + \pi + \theta)$ mA

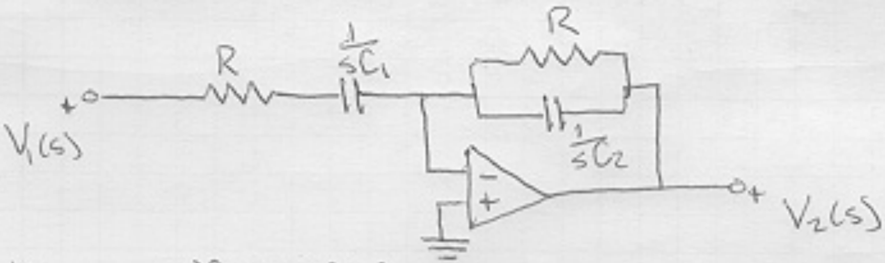
$\boxed{i_{2ss}(t) \approx 1.41\cos(5000t + 2.356)\text{ mA}}$

$\boxed{P.11.53}$ <u>Design</u> a circuit to fit $T_v(s) = \pm \frac{1000s}{(s+500)(s+1000)}$

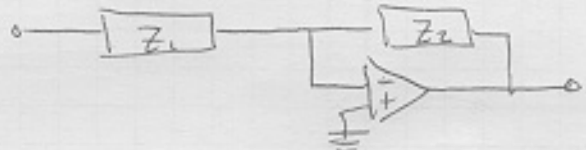
using only resistors, capacitors & not more than on OP-AMP
-scale it so that the final design uses only 20kΩ resistors

$$T_v(s) = \frac{k_1}{s+500} \quad \frac{k_2 s}{s+1000}$$

<u>ex.</u>



inverting amplifier ~~of form~~



$$\text{Gain} = -\frac{Z_2}{Z_1}$$

in example $Z_2 = \frac{1}{\frac{1}{R}+sC_2} = \frac{R}{RC_2s+1}$

$$Z_1 = R + \frac{1}{sC_1} = \frac{RC_1s+1}{C_1 s}$$

$$T_v(s) = -\frac{Z_2}{Z_1} = -\frac{R}{RC_2s+1} \quad \frac{C_1 s}{RC_1s+1}$$

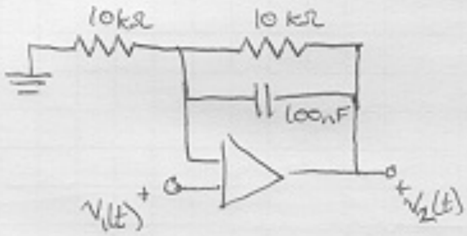
$$= -\frac{1}{RC_2} \quad \frac{1}{s+\frac{1}{RC_2}} \quad \frac{s}{s+ 1/RC_1}$$

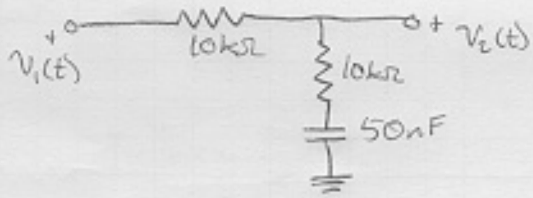
if $R = 20k\Omega$
$C_1 = 100nF$
$C_2 = 50nF$

$$T_v(s) = -\frac{1000s}{(s+500)(s+1000)}$$

$\boxed{\text{P.11.58}}$ Show that both circuits satisfy the transfer function

$$T_v(s) = K\left(\frac{s+2000}{s+1000}\right)$$





non-inverting amplifier

$$\text{Gain} = T_v(s) = \frac{\dfrac{R/Cs}{R + 1/Cs} + R}{R}$$

$$= \frac{RCs + 2}{RCs + 1}$$

$$T_v(s) = \frac{s + 2/RC}{s + 1/RC}$$

$$T_v(s) = \frac{s+2000}{s+1000}$$

voltage divider

$$V_2(s) = \frac{R + \frac{1}{Cs}}{R + R + 1/Cs} V_1(s)$$

$$T_v(s) = \frac{R + 1/Cs}{2R + 1/Cs}$$

$$= \frac{RCs+1}{2RCs+1}$$

$$= \frac{1}{2} \frac{s + 1/RC}{s + \frac{1}{2RC}}$$

$$T_v(s) = \frac{1}{2} \frac{s+2000}{s+1000}$$

b) Which circuit would you choose to drive a 1kΩ load?

OP-AMP circuit

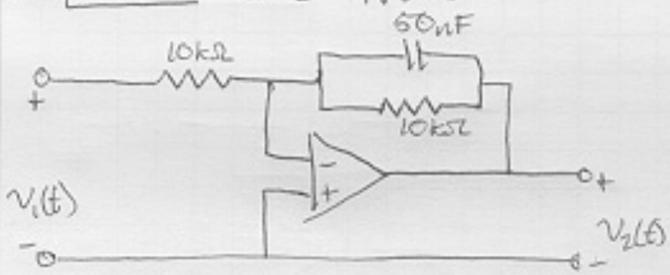
c) Which would you pick if driven by a 50Ω source?

OP-AMP circuit

d) Do you agree with the statement that the transfer function would be $|T_v(s)|^2$ if the two circuits were connected in cascade, regardless of order? Explain.

Yes. Neither order would suffer from loading.

$\boxed{P.12.4}$ find $T_v(s)$



inverting amplifier

$$T_v(s) = -\frac{\left(\frac{R}{RCs+1}\right)}{R}$$

$$= -\frac{1}{RCs+1}$$

$$T_v(s) = -\frac{1}{RC}\frac{1}{s+1/RC}$$

$$\boxed{T_v(s) = -\frac{2000}{s+2000}}$$

What type of filter is this?

Low-Pass

a)

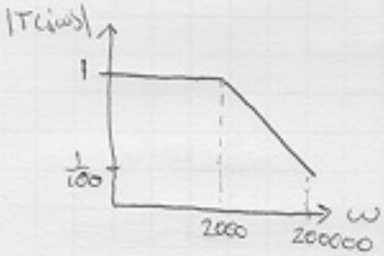
$$\boxed{\text{dc gain} = |T(0)| = 1}$$

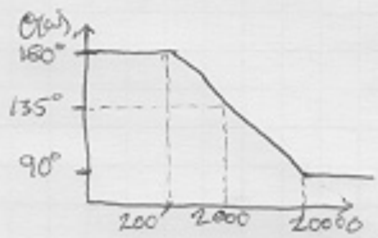
$$\boxed{\text{Infinite frequency gain} = |T(\infty)| = 0}$$

$$\boxed{\text{cutoff frequency} = \text{absolute value of pole} = 2000\ \tfrac{rad}{s}}$$

b) Draw a straight line approx. for gain & phase of $T_v(j\omega)$

$$\theta(\omega) = \pi - \tan^{-1}\left(\frac{\omega}{2000}\right)$$





c) What element values would you change to double the passband gain with out changing the cutoff frequency?

halve the left most resistor

i.e. make it 5kΩ

$\boxed{\text{P.12.10}}$ a) Identify the gain response, cutoff frequency & passband gain for $T(s) = \dfrac{10/s}{100/s + 1/20}$

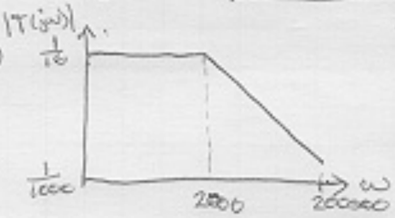
b) Sketch straight line gain response & estimate the gain for $0.5\omega_c$, ω_c, $2\omega_c$

a) $|T(0)| \to 1/10$
$|T(\infty)| \to 0$ $\boxed{\text{Low - Pass}}$

$\boxed{\omega_c = 2000 \ \frac{rad}{s}}$

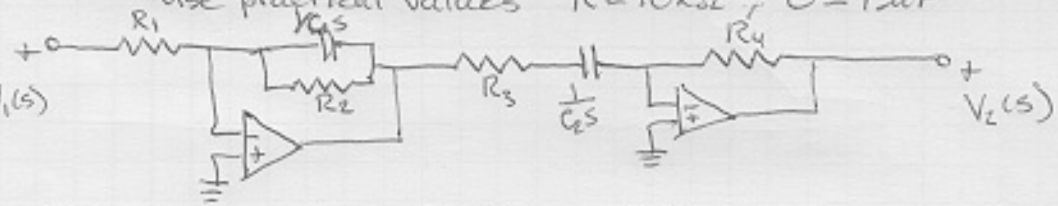
passband gain $\boxed{|T(0)| = \frac{1}{10}}$

b)



$|T(j0.5\omega_c)| \approx 1/10$
$|T(j\omega_c)| \simeq \frac{1}{10\sqrt{2}}$
$|T(j2\omega_c)| \simeq 1/50$

$\boxed{\text{P.12.15}}$ for the circuit, identify the elements that control the two cutoff frequencies & select element values so that the passband gain is 10 & the cutoff frequencies are 100 $\frac{rad}{s}$ & 2500 $\frac{rad}{s}$. Use practical values $R \geq 10k\Omega$, $C \leq 1\mu F$



cascade connection of high-pass & low-pass

$$T(s) = T_1(s) \cdot T_2(s)$$

$$T_1(s) = -\frac{R_2}{R_1}\frac{1}{s+\frac{1}{R_2C_1}} \qquad T_2(s) = -\frac{R_4 \cancel{C_2}}{R_3 \cancel{C_2}}\frac{s}{s+\frac{1}{R_3C_2}}$$

$$T_v(s) = \frac{R_2R_4}{R_1R_3}\frac{1}{s+1/R_2C_1}\frac{s}{s+1/R_3C_2}$$

$R_2, R_3, C_1, \& C_2$ determine the cutoff frequencies

ex. select

$R_2 = 10k\Omega$, $C_1 = 40nF$ $\quad \omega_{c2} = 2500 \frac{rad}{s}$
$R_3 = 10k\Omega$, $C_2 = 1\mu F$ $\quad \omega_{c1} = 100 \frac{rad}{s}$

take $\omega_{c1} \ll \omega \ll \omega_{c2}$

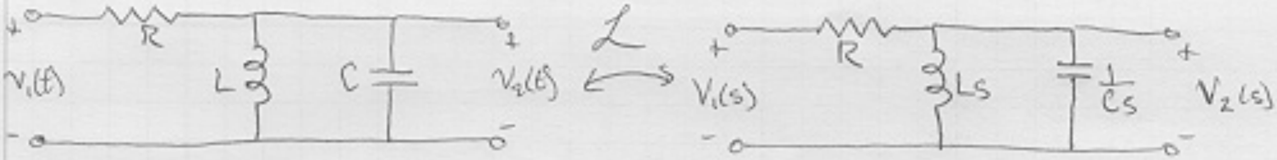
$$T(j\omega) = \frac{R_4}{R_1}\frac{1}{j\omega}\frac{j\omega}{\omega_{c2}} = \frac{R_4}{R_1}\frac{1}{2500} \overset{\text{set}}{=} 10$$

$$R_4 = 2500 R_1$$

choose

$R_4 = 25 M\Omega$
$R_1 = 10 k\Omega$

$\boxed{\text{P.12.28}}$ for the bandpass circuit find $T_V(s)$, B, Q, ω_{c1} & ω_{c2} when $R = 50\,\Omega$, $L = 50\,\mu H$ & $C = 20\,nF$



voltage divider

$$V_2(s) = \frac{\left(\frac{1}{\frac{1}{Ls}+Cs}\right)}{R + \left(\frac{1}{\frac{1}{Ls}+Cs}\right)} V_1(s) \Rightarrow T_V(s) = \frac{\left(\frac{1}{\frac{1}{Ls}+Cs}\right)}{R + \left(\frac{1}{\frac{1}{Ls}+Cs}\right)}$$

$$= \frac{Ls}{RLCs^2 + Ls + R}$$

$$\boxed{T_V(s) = \frac{1}{RC}\,\frac{s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}}$$

$$T_V(s) = \frac{1}{R}\,\frac{1}{Cs + \frac{1}{Ls} + \frac{1}{R}}$$

the cutoff frequencies occur when

$$\left(Cs + \frac{1}{Ls}\right)\Big|_{s=j\omega} = \pm\frac{j}{R}$$

$$\Rightarrow -\omega^2 \pm \frac{1}{RC}\omega + \frac{1}{LC} = 0$$

by quadratic formula

$$\omega_{c1} = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \simeq 6.18 \times 10^5 \ \frac{rad}{s}$$

$$\omega_{c2} = +\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \simeq 1.62 \times 10^6 \ \frac{rad}{s}$$

$$B = \omega_{c2} - \omega_{c1} = \frac{1}{RC} = 10^6 \ \frac{rad}{s}$$

$$Q = \frac{\omega_0}{B} = R\sqrt{\frac{C}{L}} = 1$$