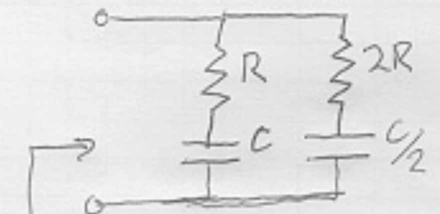
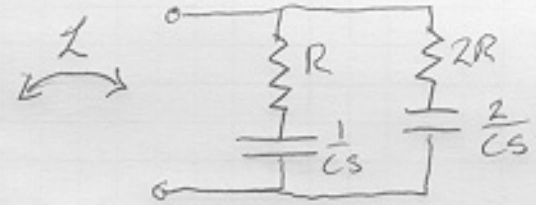
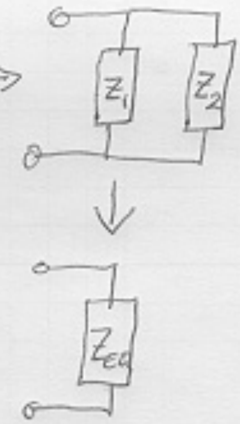
Homework VII Ch.10: 2, 5, 10, 17, 20, 21, 25, 28, 31, 32

P.10.2 find Z_{EQ}, expressed as a rational function & locate poles & zeros







Z_{EQ}

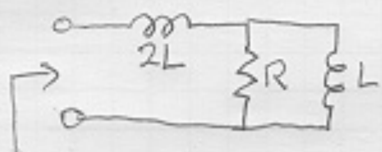
$$Z_1 = R + \frac{1}{Cs} = \frac{RCs+1}{Cs}$$

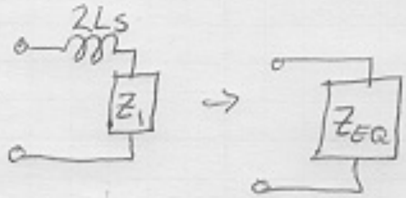
$$Z_2 = 2R + \frac{2}{Cs} = \frac{2(RCs+1)}{Cs}$$

$$Z_{EQ} = \left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)^{-1} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$\boxed{Z_{EQ} = \frac{2}{3}\frac{(RCs+1)}{Cs} \quad \text{poles: } 0 \quad \text{zeros: } -\frac{1}{RC}}$$

P.10.5 same analysis



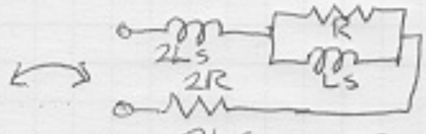


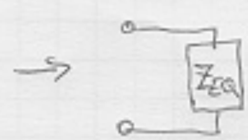
$$Z_1 = \left(\frac{1}{R} + \frac{1}{Ls}\right)^{-1} = \frac{RLs}{R+Ls}$$

$$Z_{EQ} = Z_1 + 2Ls$$
$$= \frac{RLs}{R+Ls} + 2Ls$$

$$\boxed{Z_{EQ} = \frac{2L^2s^2 + 3RLs}{R+Ls} \quad \text{poles: } -R/L \quad \text{zeros: } 0, -\tfrac{3}{2}R/L}$$

P.10.10 find Z_{EQ}, select R & L so that there is a pole @ $s = -1000\ \frac{rad}{s}$. Locate the zeros for your selection of R & L.





$$Z_{EQ} = 2Ls + \frac{RLs}{R+Ls} + 2R$$

$$\boxed{Z_{EQ} = \frac{(2Ls+R)(Ls+2R)}{R+Ls}}$$

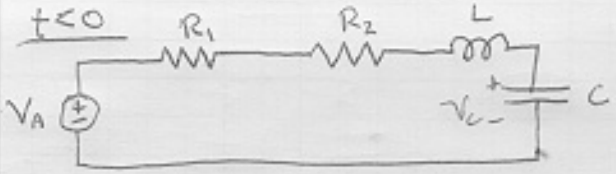
pole: $-\frac{R}{L}$

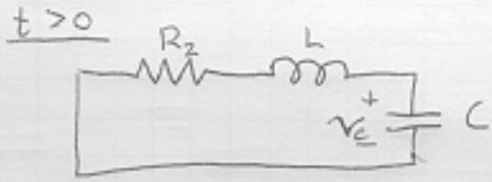
zeros: $-\frac{R}{2L}, -\frac{2R}{L}$

ex. $R = 10k\Omega$
$L = 10H$

$$\boxed{\text{Zeros} = -500\ \tfrac{rad}{s},\ -2000\ \tfrac{rad}{s}}$$

P.10.17 For circuit below

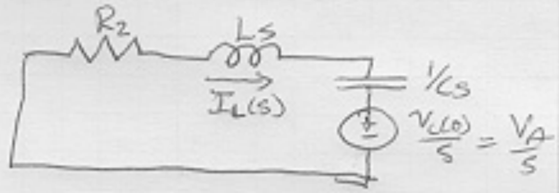




a) Solve for $I_L(s)$, $t>0$

initial conditions $v_C(0)=V_A$

$i_L(0)=0$



$$I_L(s)=\frac{-(V_A/s)}{R_2+Ls+\frac{1}{Cs}}$$

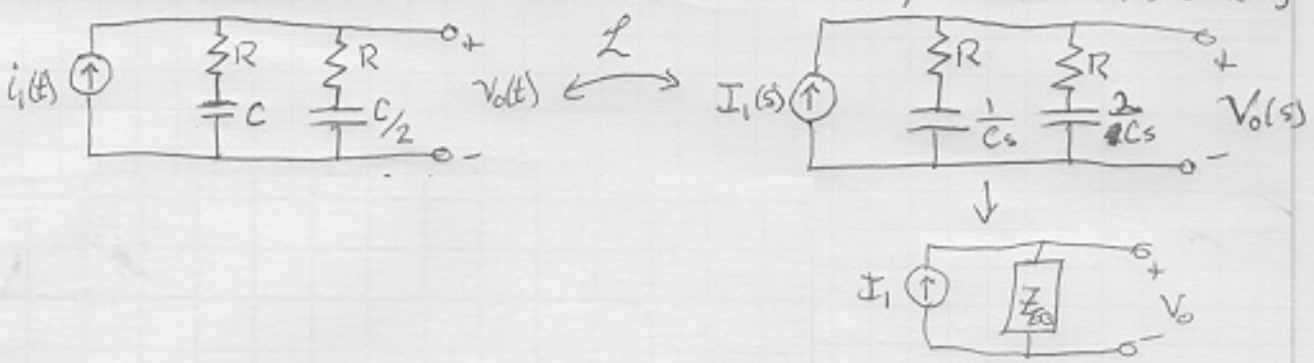
$$\boxed{I_L(s)=\frac{-V_A C}{LCs^2+R_2Cs+1}}$$

b) find $i_L(t)$ for $R_1=R_2=500\,\Omega$, $L=250\,mH$, $C=4\,\mu F$, $V_A=15\,V$

$$I_L(s)=\frac{60}{s^2+2000s+1000000}=\frac{60}{(s+1000)^2}$$

$$\boxed{i_L(t)=60t\,e^{-1000t}u(t)}$$

P.10.20 find the zero state s-domain relationship between $I_1(s)$ & $V_0(s)$

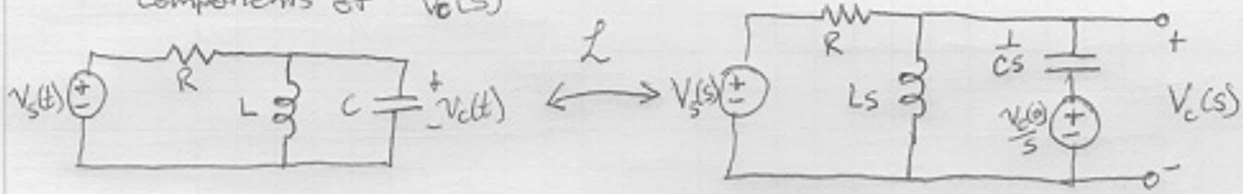


$$Z_{EQ}=\frac{(R+\frac{1}{Cs})(R+\frac{2}{Cs})}{R+\frac{1}{Cs}+R+\frac{2}{Cs}}=\frac{(RCs+1)(RCs+2)}{Cs(2RCs+3)}$$

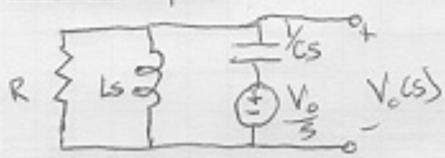
$$V_0(s)=Z_{EQ}I_1(s)$$

$$\boxed{V_0(s)=\frac{(RCs+1)(RCs+2)}{Cs(2RCs+3)}}$$

P.10.21 With iC's $V_c(0) = V_0$, $i_L(0) = 0$. Transform the circuit into the s-domain & use superposition to find the zero state & zero input components of $V_c(s)$



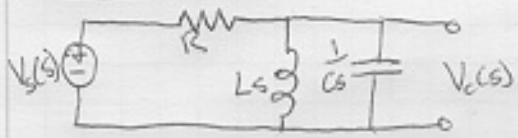
Zero-input



$$V_c(s) = \frac{\left(\frac{RLs}{R+Ls}\right)}{\frac{1}{Cs}+\frac{RLs}{R+Ls}} \frac{V_0}{s}$$

$$\boxed{V_c(s) = \frac{V_0 RLCs}{RLCs^2+Ls+R}}$$

Zero-state

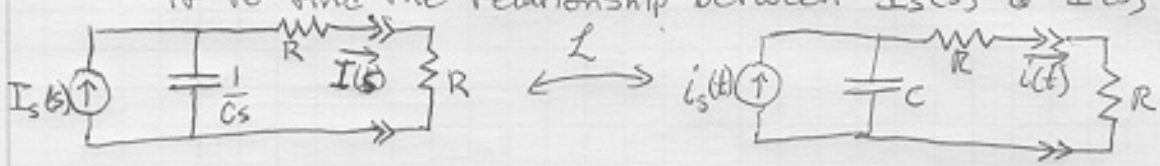


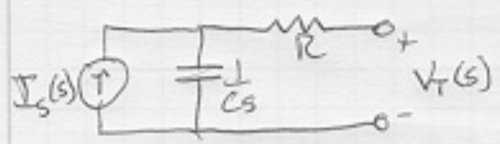
$$V_c(s) = \frac{\left(\frac{L}{C}\frac{1}{Ls+\frac{1}{Cs}}\right)}{R+\left(\frac{L}{C}\frac{1}{Ls+\frac{1}{Cs}}\right)} V_s(s)$$

$$\boxed{V_c(s) = \frac{Ls}{RLCs^2+Ls+R} V_s(s)}$$

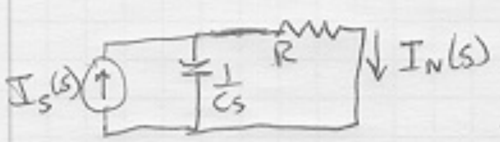
P.10.25

find the zero state Thevenin equivalent ~~the~~ & use it to find the relationship between $I_s(s)$ & $I(s)$



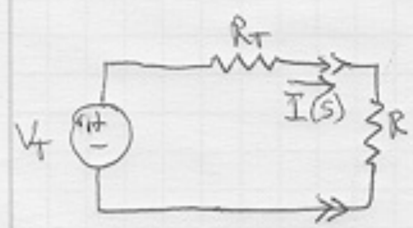


$$V_T = \frac{I_s}{Cs}$$



$$I_N(s) = \frac{(1/R)}{(1/R)+Cs} I_s(s) = \frac{I_s(s)}{RCs+1}$$

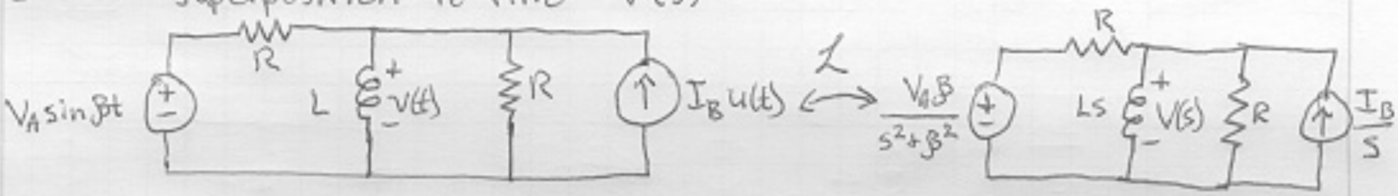
$$R_T = \frac{V_T}{I_N} = \frac{RCs+1}{Cs}$$



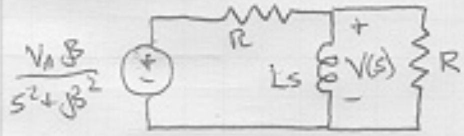
$$I(s) = \frac{V_T}{R_T+R} = \frac{I_s(s)}{Cs} \frac{1}{\frac{RCs+1}{Cs}+R}$$

$$\boxed{I(s) = \frac{I_s(s)}{2RCs+1}}$$

P.10.28 With zero initial energy stored in the circuit, use superposition to find $V(s)$

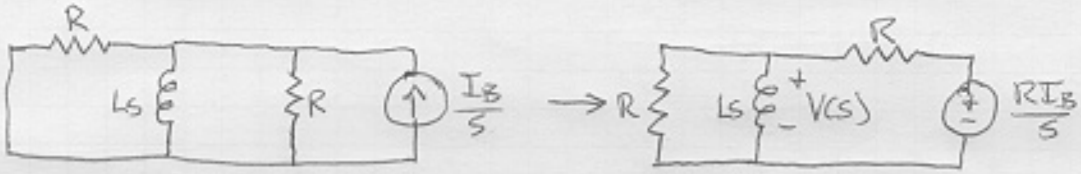


Superposition



$$V(s) = \frac{\left(\frac{RLs}{R+Ls}\right)}{R + \left(\frac{RLs}{R+Ls}\right)} \frac{V_A \beta}{s^2+\beta^2}$$

$$V(s) = \frac{V_A \beta R L s}{(2RLs + R^2)(s^2+\beta^2)} \checkmark$$



$$V(s) = \frac{RLs}{R^2+2RLs} \frac{RI_B}{s} \checkmark$$

total

$$V(s) = \frac{V_A \beta R L s}{(R^2+2RLs)(s^2+\beta^2)} + \frac{I_B R^2 L s}{s(R^2+2RLs)}$$

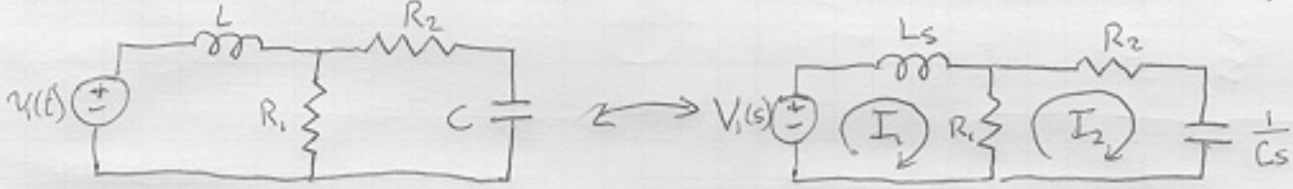
$$\boxed{V(s) = \frac{L(V_A \beta s + I_B R(s^2+\beta^2))}{(2Ls+R)(s^2+\beta^2)}}$$

Identify the forced & natural poles

poles:	$\pm j\beta$	forced
	$-\frac{R}{2L}$	natural

$\boxed{P.10.31}$ With no initial energy stored

a) Transform the circuit to the s-domain & formulate mesh-currents eqns.



mesh 1 $\quad -V_1(s) + Ls\,I_1 + R_1(I_1 - I_2) = 0$

mesh 2 $\quad R_1(I_2 - I_1) + \left(R_2 + \frac{1}{Cs}\right) I_2 = 0$

b) Solve these eqns for $I_2(s)$

$$I_1 = \frac{R_1 + R_2 + 1/Cs}{R_1}\, I_2$$

$$(R_1 + Ls)\frac{R_1 + R_2 + 1/Cs}{R_1}\, I_2 - R_1 I_2 = V_1(s)$$

$$I_2 = \frac{V_1(s)\, R_1 C s}{(R_1 + Ls)(Cs(R_1 + R_2) + 1) - R_1^2 Cs}$$

$$\boxed{I_2 = \frac{V_1(s)\, R_1 C s}{(R_1 + R_2) LC s^2 + (L + R_1 R_2 C) s + R_1}}$$

c) Find $i_2(t)$ when $v_1(t) = 100\,u(t)\,V$, $R_1 = 1\,k\Omega$, $R_2 = 2\,k\Omega$, $L = 4H$, & $C = 500\,nF$

$$I_2(s) = \frac{(25/3)}{s^2 + \frac{2500}{3} s + \frac{500000}{3}}$$

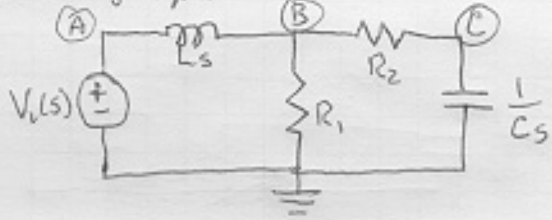
$$= \frac{(25/3)}{(s + 500)(s + 1000/3)}$$

$$= -\frac{0.05}{(s + 500)} + \frac{0.05}{(s + \frac{1000}{3})}$$

$$\boxed{i_2(t) = 50\left[e^{-\frac{1000t}{3}} - e^{-500t}\right] u(t)\ mA}$$

$\boxed{\text{P.10.32}}$ for the same circuit as in P.10.31

a) transform to the s-domain & develop node-voltage eqns



A $\quad V_A = V_1(s)$

B $\quad -\frac{1}{Ls}V_A + \left(\frac{1}{Ls}+\frac{1}{R_1}+\frac{1}{R_2}\right)V_B - \frac{1}{R_2}V_C = 0$

C $\quad -\frac{1}{R_2}V_B + \left(\frac{1}{R_2} + Cs\right)V_C = 0$

b) Solve these eqns for $V_2 = V_C$

$$V_B = (R_2Cs+1)V_C$$

$$\frac{(R_1R_2 + R_2Ls + R_1Ls)(R_2Cs+1)}{R_1R_2Ls}V_C - \frac{1}{R_2}V_C = \frac{V_1(s)}{Ls}$$

$$V_C = \frac{V_1(s)R_1}{Cs(R_1R_2+R_2Ls+R_1Ls)+R_1+Ls}$$

$$= \frac{V_1(s)R_1}{((R_1+R_2)LC)s^2 + (R_1R_2C+L)s + R_1}$$

c) find $v_2(t)$ when $v_1(t) = 10u(t)\,V$, $R_1 = R_2 = 500\Omega$, $L = 0.5\,H$, $C = 2\mu F$

$$V_2(s) = \frac{5000}{s(0.001s^2+s+500)}$$

$$= \frac{5000000}{s(s^2+1000s+500000)} \qquad \text{poles: } 0,\ -500 \pm j500$$

want form $\quad V_2(s) = \frac{k_1}{s} + \frac{k_2}{s+500-j500} + \frac{k_2^*}{s+500+j500}$

$$k_1 = \lim_{s\to 0}\frac{5000000}{s^2+1000s+500000} = 10$$

$$k_2 = \lim_{s\to -500+j500}\frac{5000000}{s(s+500+j500)} = -\frac{10}{1+j} = -5+j5$$

$$k_2^* = -5-j5$$

$$V_2(s) = \frac{10}{s} + \frac{(-5+j5)}{s+500-j500} + \frac{(-5-j5)}{s+500+j500}$$

$$\boxed{\begin{aligned} v_2(t) &= \left[10 + (-5+j5)e^{(-500+j500)t} + (-5-j5)e^{(-500-j500)t}\right]u(t) \\ v_2(t) &= 10\left[1+\sqrt{2}\,e^{-500t}\cos\left(500t+\tfrac{3\pi}{4}\right)\right]u(t) \\ v_2(t) &= 10\left[1+e^{-500t}(\sin 500t - \cos 500t)\right]u(t) \end{aligned}}$$