Homework VI | Ch. 9: 1, 3, 5, 8, 16, 19, 23, 31, 46, 48

P.9.1 find F(s), poles & zeros for

$$f(t) = A\left[e^{-\alpha t} - 2e^{-\gamma t}\right]u(t)$$

$$\mathcal{L}\{f(t)\} = A\left(\frac{1}{s+\alpha} - \frac{2}{s+\gamma}\right)$$

$$\boxed{F(s) = -\frac{A(s+2\alpha-\gamma)}{(s+\alpha)(s+\gamma)} \qquad \text{poles: } -\alpha, -\gamma \quad \text{zeros: } \gamma - 2\alpha}$$

P.9.3 find F(s), poles & zeros for

$$f(t) = A[1 - 2\cos(\beta t)]u(t)$$

$$\mathcal{L}\{f(t)\} = A\left(\frac{1}{s} - \frac{2s}{s^2+\beta^2}\right)$$

$$\boxed{F(s) = \frac{A(\beta^2 - s^2)}{s(s^2+\beta^2)} \qquad \text{poles: } 0, \pm j\beta \quad \text{zeros: } \pm\beta}$$

P.9.5 find F(s), poles & zeros for

$$f(t) = A\delta(t) - 2A\beta e^{-\beta t}\cos(\beta t)u(t)$$

$$\mathcal{L}\{f(t)\} = A - 2A\beta\frac{s+\beta}{(s+\beta)^2+\beta^2}$$

$$\boxed{F(s) = A\left(\frac{s^2}{(s+\beta)^2+\beta^2}\right) \qquad \text{poles: } -\beta \pm j\beta \quad \text{zeros: } 0}$$

P.9.8 find F(s) and plot the pole-zero diagrams for

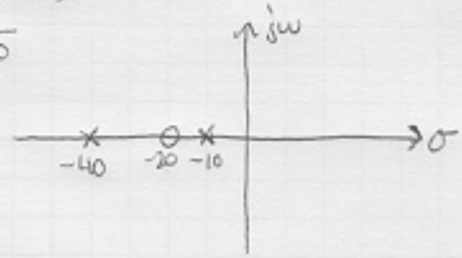
a) $f_1(t) = 3\delta(t) + \left[10e^{-10t} - 40e^{-40t}\right]u(t)$

$$\mathcal{L}\{f_1(t)\} = 3 + 10\frac{1}{s+10} - 40\frac{1}{s+40}$$

$$\boxed{F_1(s) = \frac{3(s+20)^2}{(s+10)(s+40)}}$$

poles: −10, −40

zeros: −20



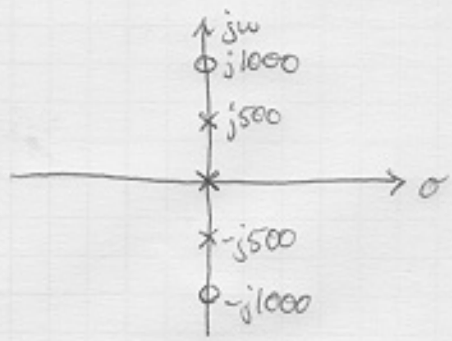
b) $f_2(t) = [20 - 15\cos(500t)]u(t)$

$$\mathcal{L}\{f_2(t)\} = \frac{20}{s} - 15\frac{s}{s^2+500^2}$$

$$\boxed{F_2(s) = \frac{5(s^2+1000000)}{s(s^2+250000)}}$$

poles: $0, \pm j500$

zeros: $\pm j1000$



$\boxed{P.9.16}$ find f(t) for

a) $F_1(s) = \dfrac{s+20}{s(s+10)}$

find form

$$F_1(s) = \frac{k_1}{s} + \frac{k_2}{s+10}$$

cover-up method

$$k_1 = \lim_{s\to 0} s F_1(s) = \lim_{s\to 0} \frac{s+20}{s+10} = 2$$

$$k_2 = \lim_{s\to -10} (s+10) F_1(s) = \lim_{s\to -10} \frac{s+20}{s} = -1$$

$$F_1(s) = \frac{2}{s} - \frac{1}{s+10}$$

$$\mathcal{L}^{-1}\{F_1(s)\} = 2u(t) - e^{-10t}u(t)$$

$$\boxed{f_1(t) = [2 - e^{-10t}]u(t)}$$

b) $F_2(s) = \dfrac{s^2+10s+10}{s(s+10)} = 1 + \dfrac{10}{s(s+10)}$

$$= 1 + \frac{1}{s} - \frac{1}{s+10}$$

$$\mathcal{L}^{-1}\{F_2(s)\} = \delta(t) + u(t) - e^{-10t}u(t)$$

$$\boxed{f_2(t) = \delta(t) + [1 - e^{-10t}]u(t)}$$

$\boxed{P.9.19}$ find f(t) and sketch it for $\beta > 0$

a) $F_1(s) = \dfrac{\beta(s+\beta)}{s(s^2+\beta^2)}$

get to form $\dfrac{k_1}{s} + \dfrac{k_2}{(s-j\beta)} + \dfrac{{}^*k_2}{(s+j\beta)}$

cover up method

$$k_1 = \lim_{s\to 0} s F_1(s) = 1$$

$$k_2 = \lim_{s\to j\beta} (s-j\beta) F_1(s) = -\tfrac{1}{2} - j\tfrac{1}{2}$$

$$F_1(s) = \frac{1}{s} + \frac{-\frac{1}{2} - j\frac{1}{2}}{(s-j\beta)} \frac{(s+j\beta)}{(s+j\beta)} + \frac{-\frac{1}{2} + j\frac{1}{2}}{(s+j\beta)} \frac{(s-j\beta)}{(s-j\beta)}$$

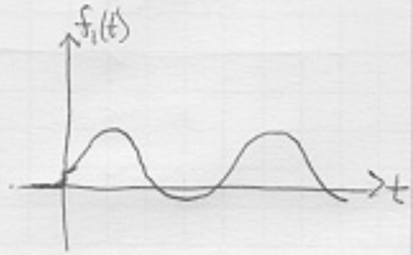
$$= \frac{1}{s} + \frac{1}{2}\frac{-s+\beta}{s^2+\beta^2} + \frac{1}{2}\frac{-s+\beta}{s^2+\beta^2}$$

$$= \frac{1}{s} + \frac{-s+\beta}{s^2+\beta^2}$$

$$F_1(s) = \frac{1}{s} - \frac{s}{s^2+\beta^2} + \frac{\beta}{s^2+\beta^2}$$

$$\mathcal{L}^{-1}\{F_1(s)\} = u(t) - \cos(\beta t)u(t) + \sin(\beta t)u(t)$$

$$f_1(t) = [1 - \cos\beta t + \sin\beta t]u(t)$$



P.9.19 continued

b) $F_2(s) = \dfrac{s(s+\beta)}{s^2+\beta^2} = 1 + \dfrac{\beta(s-\beta)}{(s-j\beta)(s+j\beta)}$

want form $\quad 1 + \dfrac{k}{(s-j\beta)} + \dfrac{{}^{*}k}{(s+j\beta)}$

$$k = \lim_{s\to j\beta} (s-j\beta) F_2(s) = \frac{\beta}{2}(1+j)$$

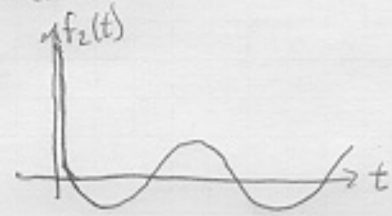
$$F_2(s) = 1 + \frac{\beta}{2}\frac{(1+j)}{(s-j\beta)}\frac{(s+j\beta)}{(s+j\beta)} + \frac{\beta}{2}\frac{(1-j)}{(s+j\beta)}\frac{(s-j\beta)}{(s-j\beta)}$$

$$= 1 + \frac{\beta}{2}\left(\frac{s-\beta+j(s+\beta)}{s^2+\beta^2} + \frac{s-\beta-j(s+\beta)}{s^2+\beta^2}\right)$$

$$= 1 + \frac{\beta(s-\beta)}{s^2+\beta^2} = 1 + \frac{\beta s}{s^2+\beta^2} - \frac{\beta^2}{s^2+\beta^2}$$

$$\mathcal{L}^{-1}\{F_2(s)\} = \delta(t) + \beta\cos(\beta t)\,u(t) - \beta\sin(\beta t)\,u(t)$$

$$f_2(t) = \delta(t) + \beta[\cos\beta t - \sin\beta t]\,u(t)$$



$\boxed{\text{P.9.23}}$ find f(t) of the following

a) $F_1(s) = \dfrac{(s+4)(s+8)}{s(s+2)(s+6)} = \dfrac{k_1}{s} + \dfrac{k_2}{s+2} + \dfrac{k_3}{s+6}$

cover-up method

$$k_1 = \lim_{s\to 0} s\,F_1(s) = 8/3$$

$$k_2 = \lim_{s\to -2} (s+2)F_1(s) = -3/2$$

$$k_3 = \lim_{s\to -6} (s+6)F_1(s) = -1/6$$

$$F_1(s) = \frac{8}{3}\frac{1}{s} - \frac{3}{2}\frac{1}{s+2} - \frac{1}{6}\frac{1}{s+6}$$

$$\mathcal{L}^{-1}\{F_1(s)\} = \frac{8}{3}u(t) - \frac{3}{2}e^{-2t}u(t) - \frac{1}{6}e^{-6t}u(t)$$

$$\boxed{f_1(t) = \left[\frac{8}{3} - \frac{3}{2}e^{-2t} - \frac{1}{6}e^{-6t}\right]u(t)}$$

b) $F_2(s) = \dfrac{3s^4+10s^2+4}{s(s^2+1)(s^2+4)} = \dfrac{3s^4+10s^2+4}{s(s-j)(s+j)(s-j2)(s+j2)}$

$$= \frac{k_1}{s} + \frac{k_2}{s-j} + \frac{{}^{*}k_2}{s+j} + \frac{k_3}{s-j2} + \frac{{}^{*}k_3}{s+j2}$$

$$k_1 = \lim_{s\to 0} s\,F_2(s) = 1$$

$$k_2 = \lim_{s\to j} (s-j)F_2(s) = \frac{1}{2}$$

$$k_3 = \lim_{s\to 2j} (s-2j)F_2(s) = \frac{1}{2}$$

P.9.23.b continued

$$F_2(s) = \frac{1}{s} + \frac{1}{2}\frac{1}{s-j}\frac{(s+j)}{(s+j)} + \frac{1}{2}\frac{1}{s+j}\frac{(s-j)}{(s-j)} + \frac{1}{2}\frac{1}{s-j2}\frac{(s+j2)}{(s+j2)} + \frac{1}{2}\frac{1}{s+j2}\frac{(s-j2)}{(s-j2)}$$

$$= \frac{1}{s} + \frac{s}{s^2+1} + \frac{s}{s^2+4}$$

$$\mathcal{L}^{-1}\{F_2(s)\} = u(t) + \cos(t)\,u(t) + \cos(2t)\,u(t)$$

$$\boxed{f_2(t) = [1 + \cos t + \cos 2t]\,u(t)}$$

P.9.31 Use Laplace transformation to find $y(t)$ that satisfies the following

a) $50\frac{dy}{dt} + 250y = 0$ with $y(0-) = 10$

$$\mathcal{L}\{50\frac{dy}{dt} + 250y = 0\} = 50(sY(s) - y(0-)) + 250Y(s) = 0$$

$$\Downarrow$$

$$50sY(s) - 500 + 250Y(s) = 0$$

$$(50s + 250)Y(s) = 500$$

$$Y(s) = \frac{500}{50s+250}$$

$$= \frac{10}{s+5}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\}$$

$$\boxed{y(t) = 10e^{-5t}u(t)}$$

b) $\frac{dy}{dt} + 20y = 40u(t)$

$$\mathcal{L}\{\frac{dy}{dt} + 20y = 40u(t)\} = sY(s) - y(0-) + 20Y(s) = \frac{40}{s}$$

$$\Downarrow$$

$$(s+20)Y(s) = \frac{40}{s} - 10$$

$$Y(s) = \frac{(\frac{40}{s} - 10)}{s+20}$$

$$= \frac{10(4-s)}{s(s+20)} = \frac{k_1}{s} + \frac{k_2}{s+20}$$

$$k_1 = \lim_{s\to 0} sY(s) = 2$$

$$k_2 = \lim_{s\to -20} (s+20)Y(s) = -12$$

$$y(t) = \mathcal{L}^{-1}\{\frac{2}{s} - \frac{12}{s+20}\}$$

$$\boxed{y(t) = [2 - 12e^{-20t}]\,u(t)}$$

$\boxed{\text{P.9.46}}$ Use initial & final value properties to find initial & final values of f(t)s from P.9.23

a) $F_1(s) = \dfrac{(s+4)(s+8)}{s(s+2)(s+6)}$

initial $\rightarrow \lim_{t \to 0^+} f(t) = \lim_{s \to \infty} sF_1(s) = 1$

final $\rightarrow \lim_{t \to \infty} f(t) = \lim_{s \to 0} sF_1(s) = 8/3$

b) $F_2(s) = \dfrac{3s^4 + 10s^2 + 4}{s(s^2+1)(s^2+4)}$

initial $\rightarrow \lim_{t \to 0^+} f(t) = \lim_{s \to \infty} sF_1(s) = 3$

final $\rightarrow$ not applicable because $F_2(s)$ has poles on the j-axis.

$\boxed{\text{P.9.48}}$ Do the same for the following

a) $F_1(s) = \dfrac{s(s+5)}{s^2+6s+9}$

initial $\rightarrow$ not applicable because $F_1(s)$ is not a proper rational function.

final $\rightarrow \lim_{t \to \infty} f(t) = \lim_{s \to 0} sF_1(s) = 0$

b) $F_2(s) = \dfrac{10(s^2+10s-20)}{s(s^2+100)}$

initial $\rightarrow \lim_{t \to 0^+} f(t) = \lim_{s \to \infty} sF_2(s) = 1$

final $\rightarrow$ not applicable because $F_2(s)$ has poles on the j-axis.