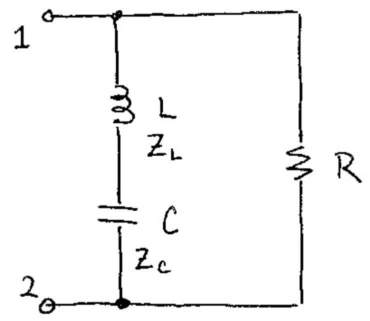
10/3



1^{st} Transform circuit to s-domain

$$Z_L = sL$$

$$Z_c = \frac{1}{sC}$$

(No IC for an equivalent impedance problem)

2^{nd} Apply methods for resistive networks

$$Z_{eq} = \left(\frac{1}{Z_L + Z_c} + \frac{1}{R} \right)^{-1}$$

$$= \left(\frac{1}{sL + \frac{1}{sC}} + \frac{1}{R} \right)^{-1}$$

$$= \left(\frac{sC}{s^2LC + 1} + \frac{1}{R} \right)^{-1}$$

$$= \left(\frac{sRC + s^2LC + 1}{s^2RLC + R} \right)^{-1} \qquad \text{where} \begin{cases} R = 4\times10^3\ \Omega \\ L = .4\ H \\ C = 10^{-7}\ F \end{cases}$$

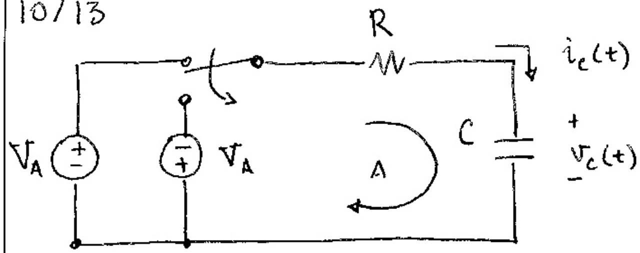
$$= R \times \frac{s^2 + 1/(LC)}{s^2 + s(R/L) + (LC)^{-1}}$$

Use the following MatLab command:

```
>> R= 4e3; L =.4; C= 1e-7; syms s
>> eval(solve('s^2 + 1/(L*C)', 's'))
>> eval(solve('s^2 + s*(R/L) + 1/(L*C)', 's'))
```

$$\boxed{\begin{array}{l} \text{Zeroes: } \pm 5000j \text{ rad/s} \\ \text{pole: } -5000 \text{ rad/s} \\ Z_{eq} = 4\times10^3 \dfrac{(s+5000j)(s-5000j)}{(s+5000)^2} \end{array}}$$

10/13



1^{st} Transform circuit to s-domain

$i_c(t) = C \frac{dv_c}{dt}$

$\Rightarrow \quad i_c(s) = sC\, v_c(s) - C\, v_c(0)$

where $v_c(0) = V_A$

closing switch $\Rightarrow V_A\, u(t) \rightarrow V_A / s$

2^{nd} Apply KCL or <u>KVL</u> (KVL is slightly easier)

note : $i_c(s) = sC\, v_c(s) - C\, v_c(0)$

$i_c(s) + C\, v_c(0) = sC\, v_c(s)$

$v_c(s) = (sC)^{-1}\, i_c(s) + s^{-1}\, v_c(0)$

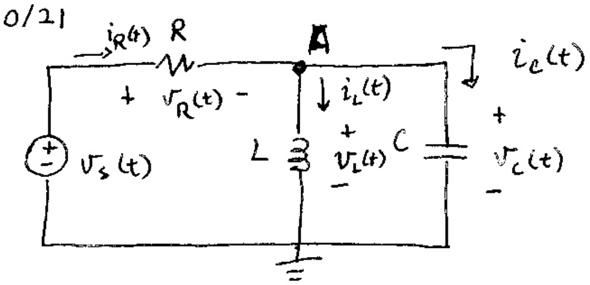
<u>KVL @ A</u>: $s^{-1} V_A + i_c(s)\, R + (sC)^{-1}\, i_c(s) + s^{-1}\, v_c(0) = 0$

$C V_A + i_c(s)\, sRC + i_c(s) + C V_A = 0$

$i_c(s) = -2CV_A \times \frac{1}{sRC + 1}$

$$\boxed{\begin{aligned} i_c(s) &= -\frac{2V_A}{R} \times \frac{1}{s + (RC)^{-1}} \\ i_c(t) &= -\frac{2V_A}{R} \times e^{-t/RC}\, u_t \end{aligned}}$$

10/21



$v_c(0) = V_0$

$i_L(0) = 0$

In the s-domain:

$$i_c(s) = sC\, v_c(s) - C\, v_c(0) \Leftrightarrow v_c(s) = (sC)^{-1} i_c(s) + s^{-1} v_c(0)$$

$$v_L(s) = sL\, i_L(s) - L\, i_L(0) \Leftrightarrow i_L(s) = (sL)^{-1} v_L(s) + s^{-1} i_L(0)$$

<u>Zero-state</u>: $v_c(0) = 0$, $i_L(0) = 0$

KCL @ A: $i_R - i_L - i_c = 0$

$$v_A = v_L = v_c$$

$$\frac{1}{R}(v_s - v_A) - (sL)^{-1} v_L(s) - sC\, v_c(s) = 0$$

$$\frac{1}{R}(v_s(s) - v_A(s)) - (sL)^{-1} v_A(s) - sC\, v_A(s) = 0$$

$$v_A(s)\left(-\frac{1}{R} - \frac{1}{sL} - sC\right) = -\frac{v_s(s)}{R}$$

$$v_A(s)(sL + R + s^2 RLC) = sL\, v_s(s)$$

$$v_A(s) = \boxed{\frac{sL}{s^2 RLC + sL + R} v_s(s) = v_c(s)} \quad \text{(for zero state)}$$

<u>Zero-input</u>: $v_s(t) = 0$, $v_c(0) = V_0$, $i_L(0) = 0$

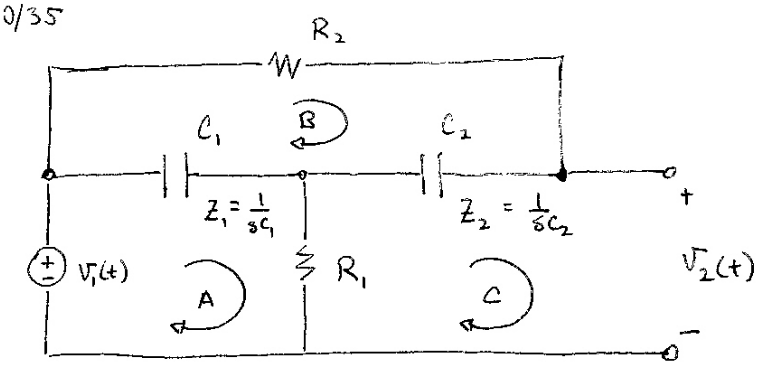
KCL@A: $i_R - i_L - i_c = 0$

$$\frac{1}{R}(-v_c(s)) - (sL)^{-1} v_c(s) - \cancel{s^{-1} i_L(0)} - sC\, v_c(s) + C\, v_c(0) = 0$$

$$v_c(s)\left(-\frac{1}{R} - \frac{1}{sL} - sC\right) = -CV_0$$

$$\boxed{v_c(s) = \frac{sRLC\, V_0}{s^2 RLC + sL + R}} \quad \text{(for zero input)}$$

10/35



No initial energy $\Rightarrow$ $v_{c1}(0) = v_{c2}(0) = 0$

(a) In the s-domain

$$Z_{c1} = \frac{1}{sC_1} \Leftrightarrow i_{c_1}(s) = sC_1 V_{C_1}(s)$$

$$Z_{c2} = \frac{1}{sC_2} \Leftrightarrow i_{c2}(s) = sC_2 v_{c2}(s)$$

Answer:

$$\frac{V_2(s)}{V_1(s)} = \frac{s^2 R_1 R_2 C_1 C_2 + s(R_1 C_2 + R_2 C_1) + 1}{s^2 R_1 R_2 C_1 C_2 + s(R_2 C_1 + R_1 C_2 + R_1 C_2) + 1}$$

Mesh equations:

$$\left.\begin{aligned} A&: \frac{1}{sC_1}(i_A - i_B) + R_1(i_A - i_c) = v_1(s) \\ B&: \frac{1}{sC_1}(i_B - i_A) + R_2(i_B) + \frac{1}{sC_2}(i_B - i_c) = 0 \\ C&: R_1(i_c - i_A) + \frac{1}{sC_2}(i_c - i_B) = -v_2(s) \end{aligned}\right\} \quad (1)$$

noting that $i_c(s) = 0$

$$(1) \Rightarrow \boxed{\begin{aligned} &\left(R_1 + \frac{1}{sC_1}\right) i_A + \left(-\frac{1}{sC_1}\right) i_B = v_1(s) && (2) \\ &\left(-\frac{1}{sC_1}\right) i_A + \left(R_2 + \frac{1}{sC_1} + \frac{1}{sC_2}\right) i_B = 0 && (3) \\ &(-R_1)\, i_A + \left(-\frac{1}{sC_2}\right) i_B = -v_2(s) && (4) \end{aligned}}$$

(b) $3 \Rightarrow \quad i_A = i_B\left(sC_1 R_2 + 1 + \frac{C_1}{C_2}\right)$

$2 \Rightarrow$

$$\left[\left(R_1 + \frac{1}{sC_1}\right)\left(sC_1 R_2 + 1 + \frac{C_1}{C_2}\right) - \frac{1}{sC_1}\right] i_B = v_1(s)$$

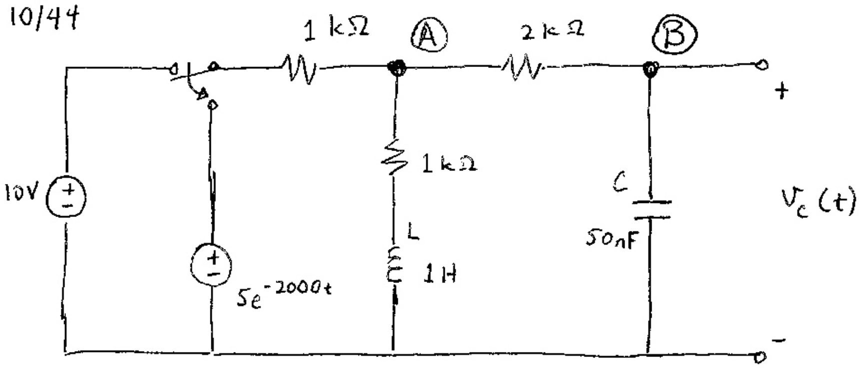
$$1 \Rightarrow \left[-R_1\left(sC_1 R_2 + 1 + \frac{C_1}{C_2}\right) - \frac{1}{sC_2}\right] i_B = -v_2(s)$$

$$v_2(s)\left[sR_1 R_2 C_1 + R_1 + \frac{R_1 C_1}{C_2} + \frac{1}{sC_2}\right]^{-1} = i_B(s)$$

$$\left[sR_1R_2C_1 + R_1 + \frac{C_1 R_1}{C_2} + R_2 + \cancel{\frac{1}{sC_1}} + \frac{1}{sC_2} - \cancel{\frac{1}{sC_1}}\right]\left(sR_1R_2C_1 + R_1 + \frac{R_1C_1}{C_2} + \frac{1}{sC_2}\right)^{-1} = \frac{V_1}{V_2}$$

answer @ top

10/44



Initial conditions:

$$i_L(0) = \frac{10\text{ V}}{2\text{k}\Omega} = 5\times10^{-3}\text{ A}$$

$$v_C(0) = 10\text{ V} \times \frac{1\text{ k}\Omega}{1+1\text{ k}\Omega} = 5\text{ V}$$

Answer:

$$v_c(t) = 50e^{-2000t} + 5.3e^{6925t} - 50e^{-1925t}$$

After the switch is closed:

KCL @ A: $\cancel{\left(\frac{5}{s+2000} - v_A\right)\times\frac{1}{1\text{k}\Omega}} +$

$$\frac{1}{1\text{k}\Omega}\left(v_A - \frac{5}{s+2000}\right) + i_L + \frac{v_A - v_B}{2\text{k}\Omega} = 0$$

$$sL\, i_L - L i_L(0) = v_L$$

$$i_L = (sL)^{-1} v_L + s^{-1}(5\times10^{-3})$$

$$v_L = v_A - i_L(1000\,\Omega)$$

$$i_L\left(1 + 1000/(sL)\right) = (sL)^{-1} v_A + s^{-1}(5\times10^{-3})$$

$$\Rightarrow\quad 10^{-3}\left(v_A - \frac{5}{s+2000}\right) + \left(v_A + 5\times10^{-3}\right)/(s+1000) + \frac{v_A - v_B}{2000} = 0$$

KCL @ B: $\frac{v_A - v_B}{2000} - i_C = 0$

$$i_C = sC\,v_c - C\,v_c(0)$$

$$= sC\,v_c - 5C$$

$$= 5\times10^{-8}\, s\, v_c - 2.5\times10^{-7}$$

$$\frac{v_A - v_B}{2000} - 2.5\times10^{-7} + 5\times10^{-8}\, s\, v_B = 0$$

Use MatLab
$$\begin{cases} v_A = \left(s^2 + 10^3 s + 2.2\times10^7\right)/\left(-9\times10^{+3}s^2 - 7\times10^7 s - 8\times10^{10} + 3s^3\right) \\ v_B = 5\left(3\times10^7 + 1.1\times10^4 s + 3s^2\right)/\left(-9\times10^3 s^2 - 7\times10^7 s - 8\times10^{10} + 3s^3\right) \end{cases}$$