MAE 140 – Linear Circuits – Summer 2007
Final

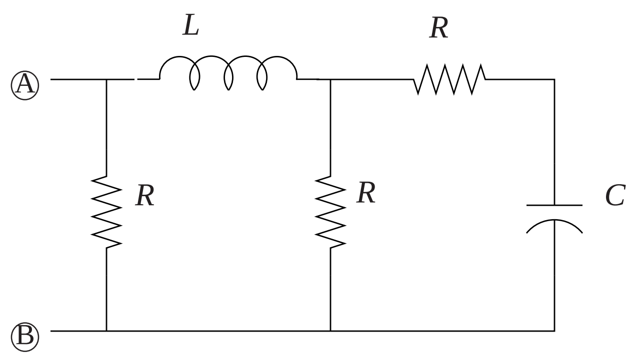
Instructions

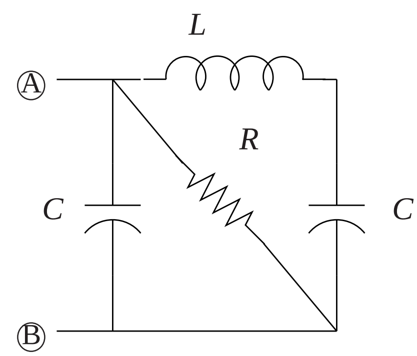
1) This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities.

2) You have 170 minutes.

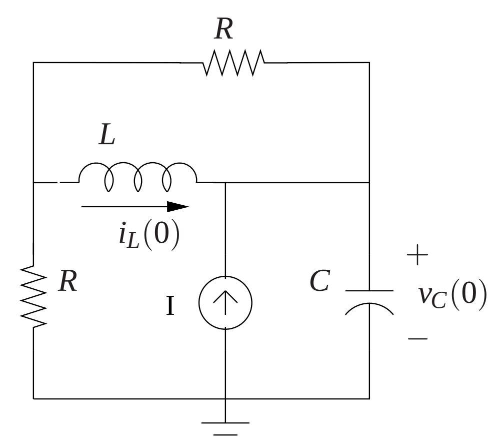
3) On the questions for which I have given the answers, please provide detailed derivations.

Question 1 [Equivalent Circiuts]
[4 marks] Find the equivalent impedance between terminals Ⓐ and Ⓑ in the following circuits.

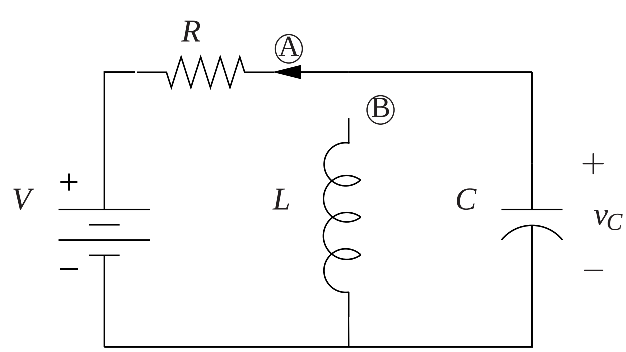




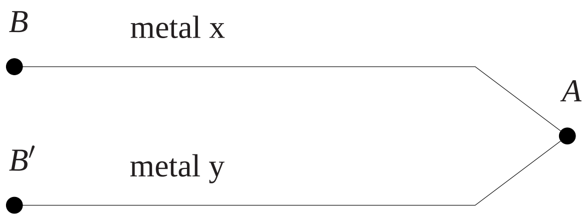
Question 2 [Nodal Analysis in the s-Domain]
[6 marks] Transform the following circuit to the s-domain and formulate node-voltage equations. Assume initial conditions and reference ground as indicated in the figure. The current source is constant.



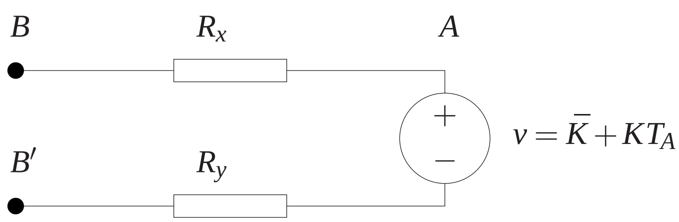
Question 3 [Transient Analysis in the s-Domain]
[6 marks] The switch in the next circuit has been left in position Ⓐ for a long time and is moved to position Ⓑ at $t = 0$. Find $v_c(t)$ for $t \geq 0$. The voltage source is constant.



Question 4 [Circuit Variables and OpAmp Circuit Design]
When two *different* metal wires are placed in contact (creating a junction) a voltage appears that is proportional to the junction temperature and the material properties. A pair of wires made with different materials connected at one end as in the next figure is known as a *thermocouple*, and is a very popular temperature sensor. No voltage appears on junctions made of same materials because of temperature. The points B and B' are at the same temperature T_B.

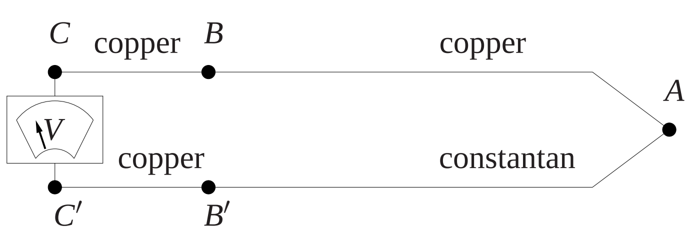


A good model for the thermocouple junction is as a voltage source with voltage $v = \bar{K} + KT$, where $\bar{K}$ and K are constants that depend only on the material used in the junction and T is the junction temperature. A simple circuit model for the above thermocouple is given in the next diagram, where R_x and R_y represent the resistance of the wires, which are essentially functions of the cross section area and length of the thermocouple. It is fair to assume that $R_x \approx R_y$.



A thermocouple made with 'metal x' being copper and 'metal y' being constantan can measure temperatures in the range -200 ^{o}C to 350 ^{o}C with $K = 43\mu$ V/^{o}C. The voltage v is measured from the copper terminal $(+)$ to the constantan terminal $(-)$. As you will see soon, the value of $\bar{K}$ is not important.

a) [3 marks] A friend of yours suggested that you can measure the temperature of point A (T_A) by simply connecting a voltmeter with copper leads to the points B and B' and measure the resulting voltage in C and C' (internal to the voltmeter), as in the next figure. The points B and B' are at the same temperature T_B. The points C and C' are at the same temperature T_C. Draw the circuit diagram corresponding to this setup and show that he/she is not correct: this setup can only measure $V_C - V_{C'} = K(T_A - T_B)$.
(Hint: remember that a voltage appear on all junctions made with different materials!)



One way of overcomming the above problem is to let the temperature T_B be known. A popular approach is to have the junction B' be immersed in a bath of water and ice, in which the temperature is exactly 0.01 ^{o}C (known as the triple point of water) so that $T_A = T_B + (V_C - V_{C'})/K \approx (V_C - V_{C'})/K$.

b) [4 marks] Assume that T_B is in a cold bath at the triple point of water (assume $T_B \approx 0$) and design an OpAmp circuit to be connected at C-C' that outputs a voltage $v_0 = \alpha T_A$, where $\alpha = 10$ m V/^{o}C. Note that this circuit should makes the measurement independent of the wire resistance. If the OpAmp is powered with $+10V$ and $-10V$ what is the temperature range that you can measure accurately with your circuit?

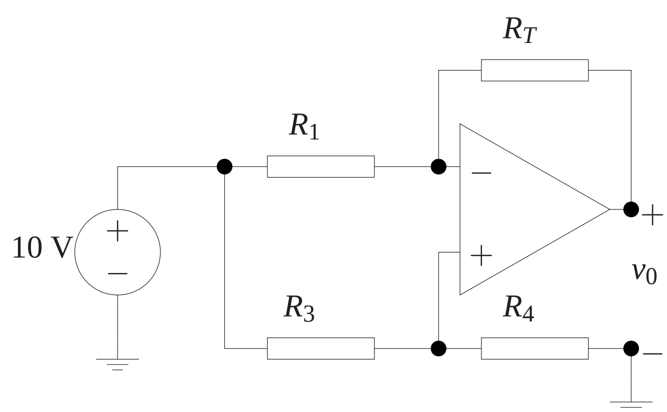
Another way of overcomming the temperature reference problem is to directly measure the temperature T_B. The justification for this is that T_B is the temperature of a controlled environment, say your workbench, while T_A may be an extreme temperature you're trying to measure. Therefore, you could use a temperature sensor to measure T_B that is less expensive or perhaps acurate only on ambient temperature. One such device is called a *termistor*, which is a resistor whose resitance varies with the temperature. Termistors are typically accurate and approximately linear from 0 ^{o}C to a dozen degrees above ambient temperature.

c) [2 marks] You have a linear termistor with a resistance of $30K\Omega$ at 0 ^{o}C and a resistance of $10K\Omega$ at 20 ^{o}C. Show that the relationship between the termistor resistance (R_T) and the termistor temperature (T_B) is

$$R_T = (30 - T_B) \times 10^3\,\Omega.$$

Up to what temperature do you think this termistor is acurate (or at least linear)? Why?

d) [Bonus: 4 marks] Using the above relationship between the termistor resistance and temperature find values for the components R_1, R_3 and R_4 so that the following circuit produces $v_0 = \alpha T_B$, where $\alpha = 10$ mV/^{o}C and T_B is the temperature of the termistor and the junction B'.



e) [Bonus: 4 marks] Design an OpAmp circuit that has as output voltage $v_0 = \alpha(T_A - T_B)$, where $\alpha = 10$ mV/^{o}C and T_B is measured using the termistor as in item d).
(Hint: use the circuit you designed in item d)!)