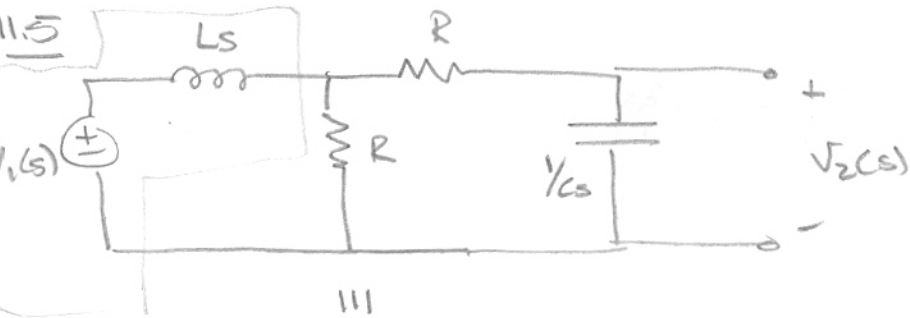
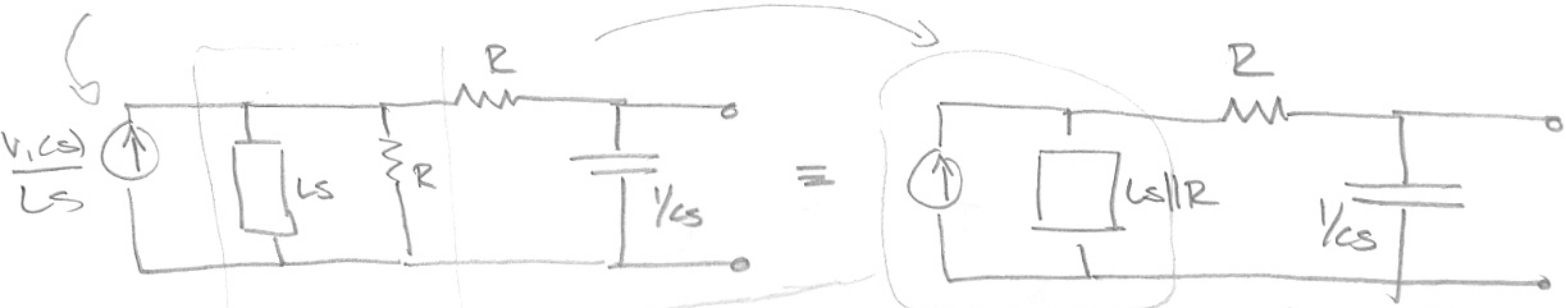
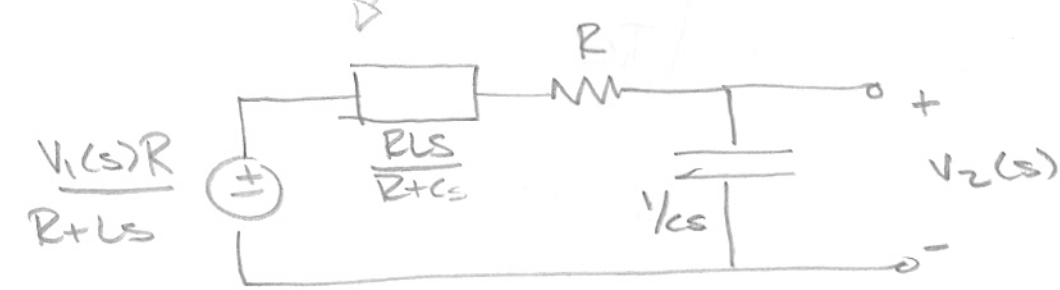
11.5





$$Ls\|R = \left(\frac{1}{Ls} + \frac{1}{R}\right)^{-1} = \frac{RLs}{R+Ls}$$

$\Rightarrow$ 

now we have a voltage divider!

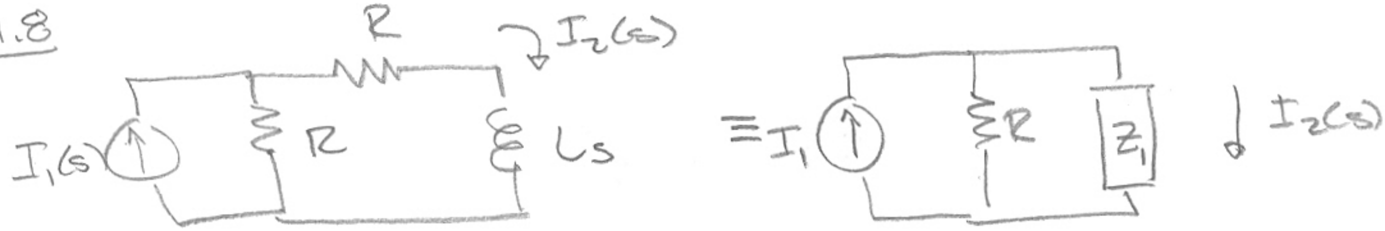
$$V_2(s) = \frac{\frac{1}{Cs}}{\frac{1}{Cs} + R + \frac{RLs}{R+Ls}} \left(\frac{V_1(s)R}{R+Ls}\right)$$

⋮ (a few steps of algebra)

$$V_2(s) = \frac{R}{2RLCs^2 + (L+R^2C)s + R} V_1(s)$$

$$\Rightarrow \boxed{T_v(s) = \frac{V_2(s)}{V_1(s)} = \frac{R}{2RLCs^2 + (L+R^2C)s + R}}$$

11.8



$Z_1 = R + Ls$

current divider:

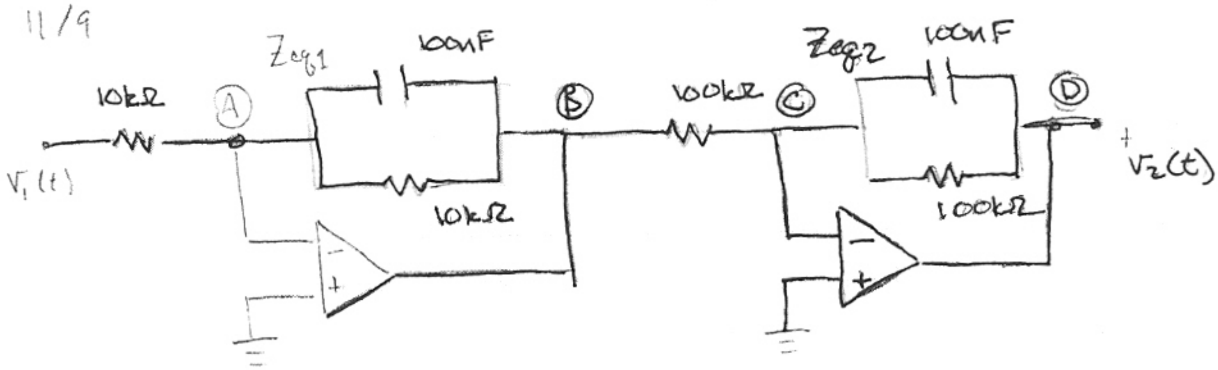
$$I_2(s) = \frac{\frac{1}{R+Ls}}{\frac{1}{R} + \frac{1}{R+Ls}} I_1(s) = \frac{1}{\frac{R+Ls}{R} + 1} I_1(s)$$

$$\Rightarrow I_2(s) = \frac{R}{2R+Ls} I_1(s) \Rightarrow \boxed{T_1(s) = \frac{I_2(s)}{I_1(s)} = \frac{R}{2R+Ls}}$$

the pole $2R + Ls = 0 \Rightarrow s = \frac{-2R}{L} = -500 \frac{rad}{s}$

$\boxed{\text{if let } L = 1 \text{ Henry} \Rightarrow R = 250\ \Omega}$

11/9



10 kΩ ∥ 100nF

$$\Rightarrow Z_{eq1} = \frac{10^4 \times \frac{1}{10^{-7}s}}{10^4 + \frac{1}{10^{-7}s}} = \frac{10^{11}}{10^4 s + 10^7} = \frac{10^7}{s+1000}$$

100kΩ ∥ 100nF

$$\Rightarrow Z_{eq2} = \frac{10^7}{s+100}$$

$V_A = V_C = 0$

KCL @ A :

$$V_1(s) \cdot \frac{1}{10^4} + V_B(s) \times \left(\frac{10^7}{s+1000}\right)^{-1} = 0$$

$$\therefore \quad V_B(s) = -V_1(s) \times \frac{10^3}{s+1000}$$

KCL @ C :

$$\frac{1}{10^5} V_B(s) + V_2(s)\left(\frac{10^7}{s+100}\right)^{-1} = 0$$

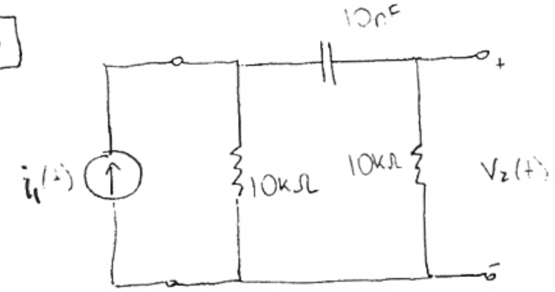
$$\Rightarrow V_2(s) = -\frac{V_B(s)}{10^5} \cdot \frac{10^7}{s+100}$$

$$= V_1(s) \times \underbrace{\frac{10^5}{(s+100)(s+1000)}}_{T(s)}$$

Zeroes : ϕ

Poles : $-100, \ -1000$

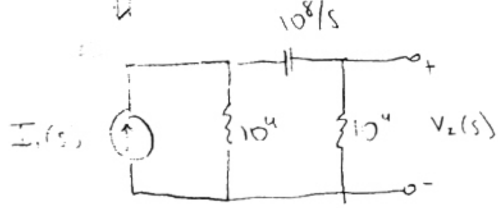
$\boxed{11.30}$



*steady state $\quad i_1(t) = 10\cos 5000t$ mA

(a) Find $V_{2ss}(t)$

$\Downarrow \mathcal{L}$



$$T(s) = \frac{V_2(s)}{I_1(s)} \Rightarrow V_2(s) = \frac{10^4}{10^8/s + 2\times 10^4}(10^4)\,I_1(s) = \frac{10^{8}}{10^{8}/s + 2\times \cancel{10^4}}$$

$$T(s) = \frac{5000\,s}{s + 5000}$$

$$I_1(s) = \mathcal{L}\{i_1(t)\} = \frac{10s}{s^2 + 5000^2} = \frac{10s}{(s - j5000)(s + j5000)}$$

$$\Rightarrow V_2(s) = \underbrace{\left(\frac{5000s}{s + 5000}\right)}_{T(s)}\underbrace{\left(\frac{10s}{(s-j5000)(s+j5000)}\right)}_{F_{ss}(s)}$$

$$\Rightarrow V_2(s) = \frac{K}{s + j5000} + \frac{K^*}{s - j5000} + \frac{K_1}{s + 5000}$$

→ time domain response decays to zero at steady state

$$K = \lim_{s \to j5000} \frac{50000s^2}{(s+5000)(s+j5000)} = \frac{-\cancel{50000}(5000)^2}{j(j+1)(\cancel{5000})(\cancel{10000})} = \frac{-25000}{-1 + j}$$

$$= \frac{\cancel{25000}^{\,12500}(1 + j)}{\cancel{2}} = 12500(1 + j) = 12500\sqrt{2}\,e^{j45°}$$

$$\rightarrow \boxed{V_{2ss}(t) = 25000\sqrt{2}\cos(5000t + 45°)\text{ mV}}$$

From 11.71

$$T(j\omega) = \frac{5000^2 j}{(j+1)\cancel{5000}} = \frac{5000j}{j+1} = \frac{-5000-5000j}{-2}$$

$$= 2500 + 2500j = 2500\sqrt{2}\, e^{j45^\circ}$$

$$\Rightarrow \boxed{V_{2ss}(t) = 2500\sqrt{2}\cos(5000t + 45^\circ)\ \text{mV}}$$

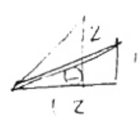
b) what about $i_1(t) = 5\cos 2500t$ mA

$$I_1(s) = \frac{5s}{(s+j2500)(s-j2500)}$$

$$T(j\omega) = \frac{5000(j\omega)}{j\omega + 5000} = \frac{5000\cancel{(2500)}j}{\cancel{2500}(j+2)} = \frac{5000\, e^{j90^\circ}}{\sqrt{5}\, e^{j\tan^{-1}(1/2)}}$$

$$= \frac{-5000-10000j}{-5} = 1000 + 2000j = 1000\sqrt{5}\, e^{j\tan^{-1}(2)}$$

$$\Rightarrow \boxed{V_{2ss}(t) = 5000\sqrt{5}\cos(2500t + \tan^{-1}(2))\ \text{mV}}$$

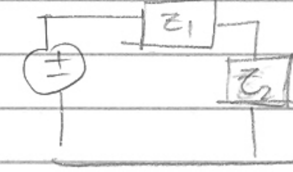


$\tan^{-1}(2)$

11-51 $T_V(S) = \pm \dfrac{2 \times 10^4}{(S+250)(S+2500)}$

$\dfrac{\pm 2 \times 10^4}{(S+250)(S+2500)} = \dfrac{k_1}{S+250} \cdot \dfrac{k_3}{S+2500} \cdot k_2$

RC Divider circuit



$\dfrac{k_1}{S+250} = \dfrac{k_1/s}{1+250/s} = \dfrac{Z_2(s)}{Z_1(S)+Z_2(S)}$

$Z_2(S) = \dfrac{k_1}{S}$

$Z_1(S) = 1 + \dfrac{250-k_1}{S}$

if $k_1 = 250 \Rightarrow Z_2(S) = \dfrac{250}{S}$ $\quad Z_1(S) = 1$

stage 3.

if $k_1 = 250$, $k_2 = 5 \Rightarrow k_3 = 16$.

$\dfrac{k_3}{S+2500} = \dfrac{k_3/s}{1+2500/s} = \dfrac{Z_4(S)}{Z_3(S)+Z_4(S)}$

$Z_4(S) = \dfrac{k_3}{S}$ $\quad Z_3(S) = 1 + \dfrac{2500-k_3}{S}$

if $k_3 = 16$

$Z_4(S) = \dfrac{2500}{S}$ $\quad Z_3(S) = 1 + \dfrac{2484}{S}$

Case (i)

Stage 2: find k_2 $\quad k_1 \cdot k_2 \cdot k_3 = 2 \times 10^4$

$T_V(S) = \dfrac{+2 \times 10^4}{(S+250)(S+2500)}$

using a non-inverting and $k_2 = 5 = \dfrac{R_1 + R_2}{R_1}$

$5R_1 = R_1 + R_2 \Rightarrow R_2 = 4R_1$

$R_1 = 10k$

$R_2 = 40k$

We want $R > 10k$ and $C < 1\mu F$

Stage 1:

$Z_1(S) = 1$

$Z_2(S) = \frac{250}{s} \cdot \frac{1}{1mF\,s}$

$= \frac{1}{4mF\,s}$

stage 2:

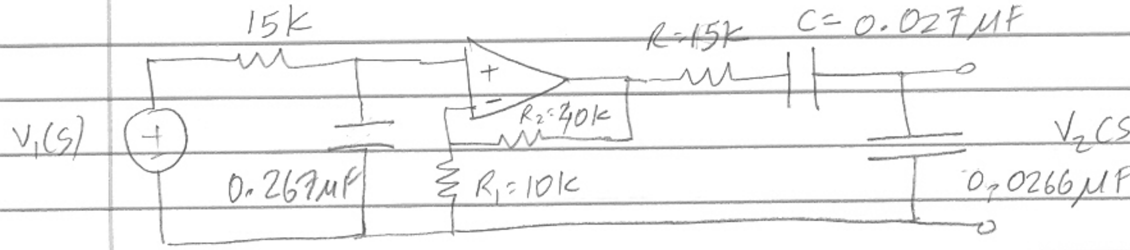
$R_2 = 4R_1$

Stage 3:

$Z_3(S) = 1 + \frac{2484}{s}$

$= 1 + \frac{1}{403\mu F\,s}$

$Z_4(S) = \frac{1}{4\times10^{-4}F\,s}$



stage 1

$k_m = 15K$

$Z_1(s) = 15k$

$C_2 = \frac{4mF}{15k} = 0.267\mu F$

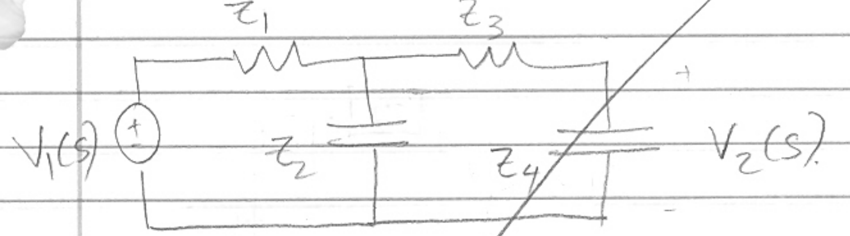
stage 3:

$Z_3(s) = 1 + \frac{1}{403\mu F s}$

$R = 15k$

$C = \frac{403\mu F}{15k} = 0.027\mu F$

$C_4 = \frac{4\times10^{-4}F}{15k} = 0.0266\mu F$



$Z_1 = 1$ $\qquad$ $Z_2(s) = \frac{250}{s} = \frac{1}{0.004s}$

$Z_3 = 1$ $\qquad$ $Z_4 = \frac{2500}{s}$

Use scaling factor $k_m = 10k\Omega$

$Z_1 = R_1 = k_m(1) = 10k\Omega$
$Z_3 = R_3 = k_m(1) = 10k\Omega$

$C_2 = \frac{4mF}{k_m} = \frac{4mF}{10^4} = 4\times10^{-7}F$

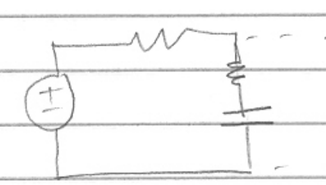
$C_4 = \frac{0.4mF}{k_m} = 4\times10^{-8}F$

11.52 Design a circuit to realize the transfer function below using R, C, and not more than one op-amp. Scale the circuit so that capacitors are exactly 10nF

$T_V(S) = \frac{\pm 100(S+1000)}{(S+250)(S+2000)} = \pm \frac{k_1}{S+250} \quad \frac{K_2(S+1000)}{S+2000}$

$\frac{k_1}{S+250} = \frac{k_1/s}{1+250/s} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)}$

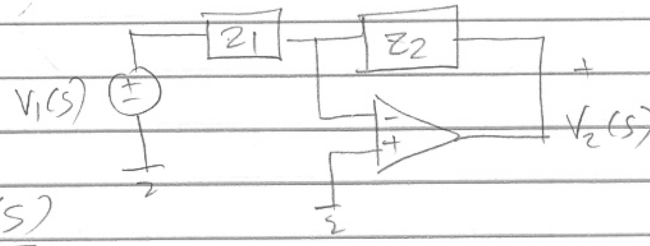
$Z_2(s) = \frac{k_1}{s}$ $\qquad$ $Z_1(s) = 1 + \frac{250-k_1}{s}$



$k_1 = 100 \Rightarrow Z_1 = 1 + \frac{150}{S}$ $\qquad$ $Z_2 = \frac{100}{S}$

$k_2 = 1$

Design 1 $\quad T_2(s) = -\dfrac{(s+1000)}{s+2000}$

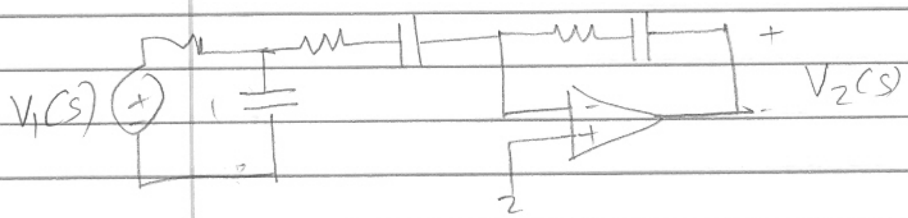


Using inverting op-Amp

$-T(s) = \dfrac{-(s+1000)}{s+2000} = \dfrac{-Z_2(s)}{Z_1(s)}$

$Z_2(s) = 1 + \dfrac{1000}{s} \quad \Rightarrow R = 1 \qquad C = 10^{-3} F$

$Z_1(s) = 1 + \dfrac{2000}{s} \quad \Rightarrow R = 1 \qquad C = 5 \times 10^{-4} F$



Design 2 $\quad$ Using non-inverting op-Amp

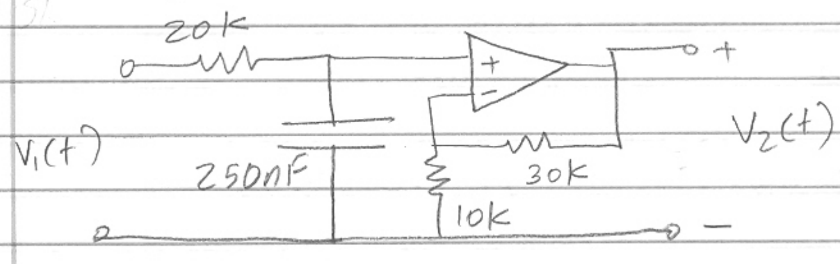
$T(s) = \dfrac{+(s+1000)}{s+2000} = \dfrac{1 + \frac{1000}{s}}{1 + \frac{2000}{s}} = \dfrac{Z_1(s) + Z_2(s)}{Z_1(s)}$

$Z_1(s) = 1 + \dfrac{2000}{s}$

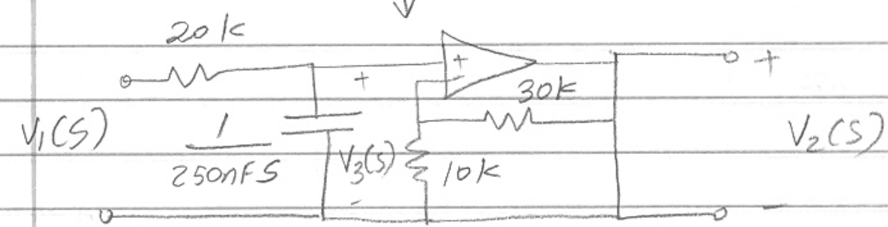
$Z_2(s) = 1 + \dfrac{1000}{s} - 1 - \dfrac{2000}{s} = \dfrac{-1000}{s}$

12.3) Find the transfer function $T_V(S) = V_2(S)/V_1(S)$

a-) Find the dc gain, infinite frequency gain, and cutoff frequency. Identify the type of gain response.



$\updownarrow \mathcal{L}$



$T_V(S) = T_{V_1}(S) \cdot T_{V3}(S)$

$$T_{V1}(S) = \frac{Z_2(S)}{Z_1(S) + Z_2(S)} = \frac{\frac{1}{250nF\,S}}{20k + \frac{1}{250nFS}} = \frac{1}{(20k)(250nF\,s) + 1}$$

$$= \frac{1}{\frac{S}{200} + 1} = \frac{200}{s + 200}$$

$$T_{V3}(S) = \frac{Z_1(S) + Z_2(S)}{Z_1(S)}$$

$$= \frac{10k + 30k}{10k} = \frac{40k}{10k} = 4$$

$$T_V(S) = \frac{4 \times 200}{S + 200} = \boxed{\frac{800}{S + 200}}$$

DC gain $|T(0)| = \boxed{4}$

Infinite frequency gain $|T(\infty)| = \frac{800}{\infty + 200} = \boxed{0}$

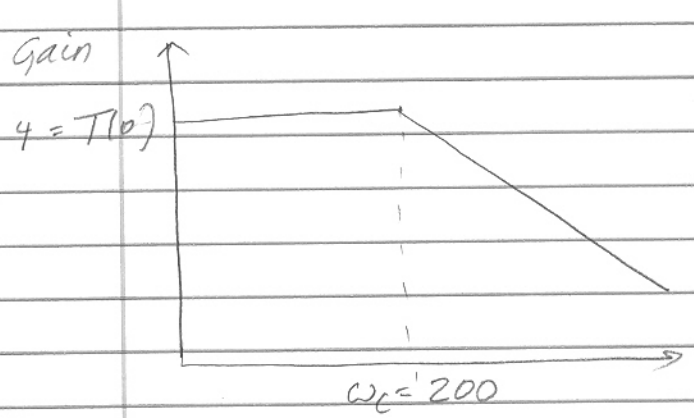
$$T(j\omega) = \frac{800}{j\omega + 200}$$

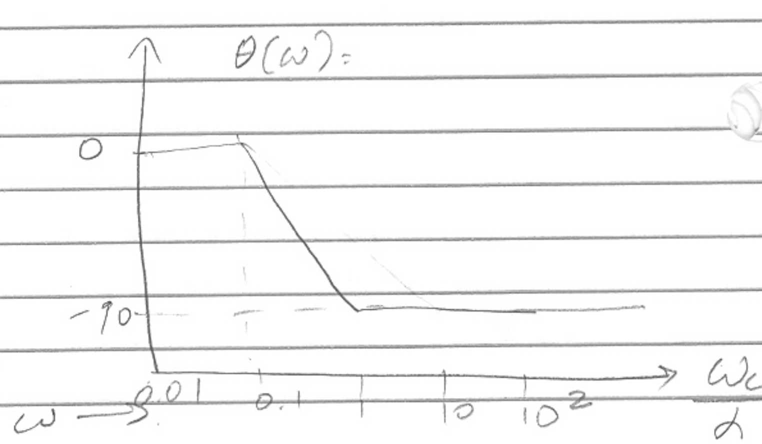
$$|T(j\omega)| = \frac{|800|}{\sqrt{\omega^2 + 200^2}} = \frac{|K|}{\sqrt{\omega^2 + \alpha^2}}$$

$\boxed{|\omega_c| = \frac{1}{200} = (200)\ \text{rad/s}}$ → cut-off frequency

Low-Pass Gain Response — Type of gain response.

b) Sketch a straight-line approximation of the gain response.





c) What element values would you change to increase the passband gain to 10?

$$\frac{K}{\alpha} = 10$$

Change the value of $Z_2(s)$ of the op-Amp (R_2)

$R_2 = 90k$

$$\Rightarrow T_{V3}(s) = \frac{10k + 90k}{10k} = 10$$

$$\Rightarrow T_V(s) = \frac{200(10)}{s + 200} \qquad T_{max} = \frac{K}{\alpha} = \frac{200(10)}{200} = \underline{\underline{10}}$$

12-9,

$$T(s)=\frac{2}{10^{-2}+20/s}=\frac{200s}{s+2000} \Rightarrow T(j\omega)=\frac{200j\omega}{j\omega+2000}$$

→ high pass filter $\Rightarrow |T(j\omega)|=\frac{200\omega}{\sqrt{\omega^2+4\times10^6}}$

→ pass band gain = 200

→ cut off frequency ω_c occurs when $|T(j\omega_c)|=\frac{200}{\sqrt{2}}$

$$\Rightarrow \frac{\omega_c}{\sqrt{\omega_c^2+4\times10^6}}=\frac{1}{\sqrt{2}} \Rightarrow \omega_c=2000 \text{ rad/s}$$

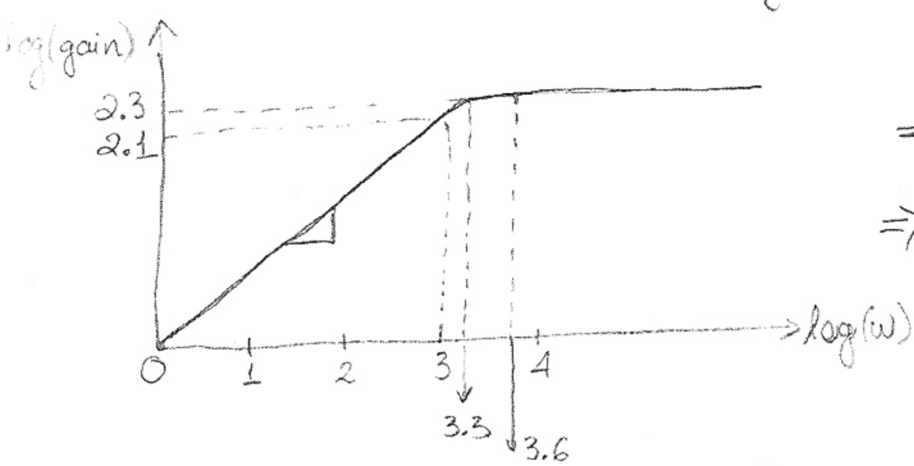
→ Bode-plot in log-log coordinate

→ $\omega_c = 2000$ rad/s $\Rightarrow \log_{10}(\omega_c) = 3.3$

→ gain = 200 $\Rightarrow \log_{10}(\text{gain}) = 2.3$

→ $\omega = 0.5\,\omega_c = 1000$ rad/s $\Rightarrow \log_{10}(\omega) = 3$

→ $\omega = 2\,\omega_c = 4000$ rad/s $\Rightarrow \log_{10}(\omega) = 3.6$



$\Rightarrow$ slope $= 2.3/3.3 = 0.7$

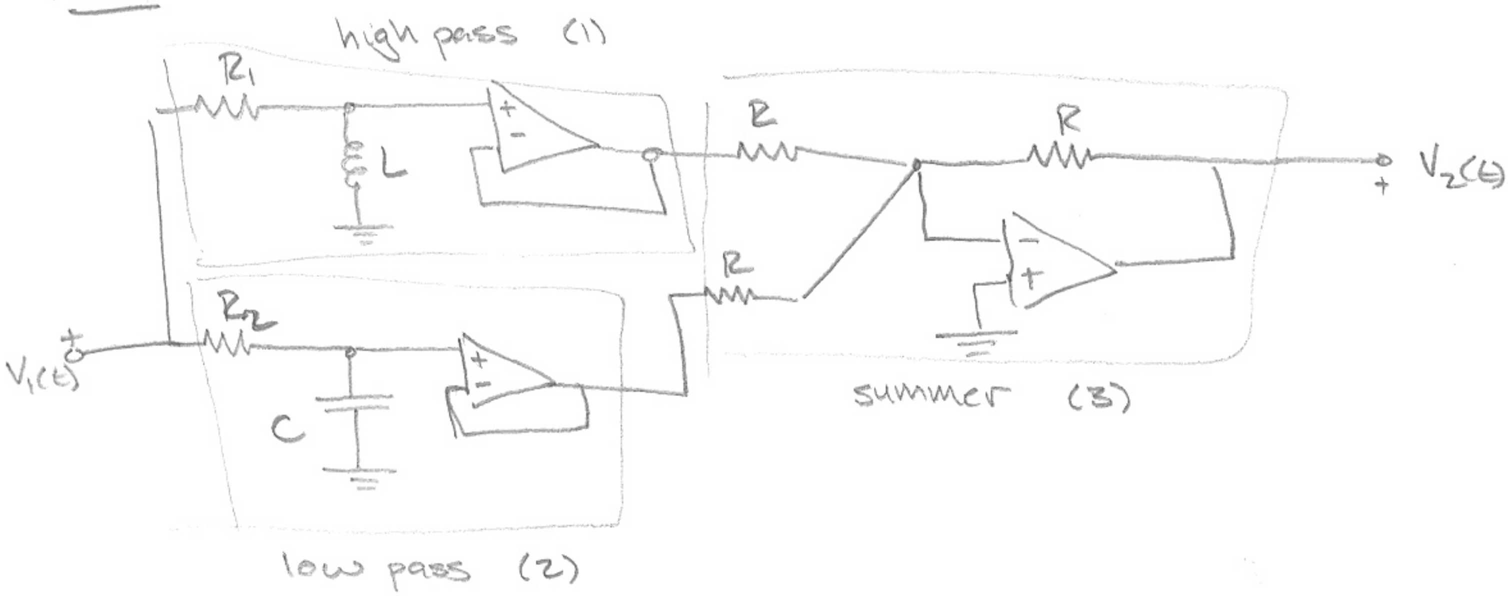
$\Rightarrow \log(\text{gain}) = 0.7 \times \log(\omega)$

→ for $\omega = 0.5\omega_c$, $\log(\text{gain}) = 0.7\times3 = 2.1 \Rightarrow \text{gain} = 125.9$

→ for $\omega = \omega_c$, $\log(\text{gain}) = 2.3 \Rightarrow \text{gain} = 200$

→ for $\omega = 2\omega_c$, $\log(\text{gain}) = 2.3 \Rightarrow \text{gain} = 200$

12.16



(1) $T_1(s) \Rightarrow$ high ω_c $= \dfrac{-Ls}{R_1+Ls} = \dfrac{-s}{s+(R_1/L)}$

$\omega_{c,high} = 500$ rad/s $= R_1/L$

if $L = 100$ mH $\Rightarrow R = 50\,\Omega$

($L = 5$ mH $\Rightarrow R = 100$ kΩ)

(2) $T_2(s) = \dfrac{-1}{R_2Cs+1} = \dfrac{-1/R_2C}{s+1/R_2C}$

$\omega_{c,low} = 10 \frac{rad}{s} = \dfrac{1}{R_2C}$

$\Rightarrow$ if $R_2 = 100$ kΩ

$\Rightarrow C = 1\,\mu F$