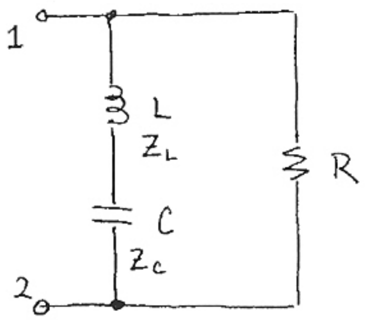
10/3




$1^{st}$ Transform circuit to s-domain

$$Z_L = sL$$

$$Z_c = \frac{1}{sC}$$

(No IC for an equivalent impedance problem)

$2^{nd}$ Apply methods for resistive networks

$$Z_{eq} = \left( \frac{1}{Z_L + Z_c} + \frac{1}{R} \right)^{-1}$$

$$= \left( \frac{1}{sL + \frac{1}{sC}} + \frac{1}{R} \right)^{-1}$$

$$= \left( \frac{sC}{s^2LC + 1} + \frac{1}{R} \right)^{-1}$$

$$= \left( \frac{sRC + s^2LC + 1}{s^2RLC + R} \right)^{-1} \qquad \text{where} \left\{ \begin{array}{l} R = 4 \times 10^3\ \Omega \\ L = .4\ H \\ C = 10^{-7}\ F \end{array} \right.$$

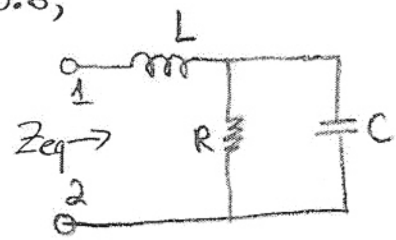
$$= R \times \frac{s^2 + 1/(LC)}{s^2 + s(R/L) + (LC)^{-1}}$$

Use the following MatLab command:

```
>> R = 4e3; L =.4; C= 1e-7; syms s
>> eval ( solve ('s^2 + 1/(L*C)', 's'))
>> eval ( solve ('s^2 + s*(R/L) + 1/(L*C)', 's'))
```

$$\boxed{\begin{array}{l} \text{Zeroes: } \pm\ 5000j \text{ rad/s} \\ \text{pole: } -5000 \text{ rad/s} \\ Z_{eq} = 4 \times 10^3 \dfrac{(s + 5000j)(s - 5000j)}{(s + 5000)^2} \end{array}}$$

10.6,

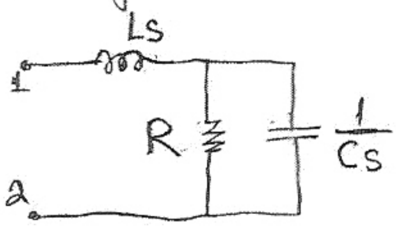



$R = 1k\Omega$

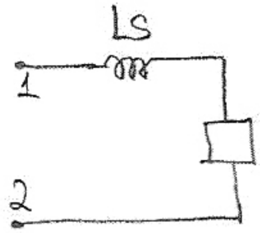
$L = 1H$

$C = 500nF$

→ transform cct into s-domain:

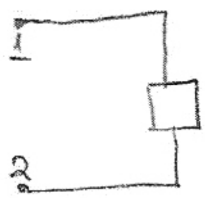



⟹




$$\frac{(R)\left(\frac{1}{Cs}\right)}{R+\frac{1}{Cs}} = \frac{R}{RCs+1}$$

⟹




$$Z_{eq} = Ls + \frac{R}{RCs+1} = \boxed{\frac{RLCs^2+Ls+R}{RCs+1}}$$

→ sub in numbers ⟹ $Z_{eq} = \dfrac{s^2+2\times10^3 s + 2\times10^6}{s+2\times10^3} = \dfrac{(s+10^3)^2+10^6}{s+2\times10^3}$

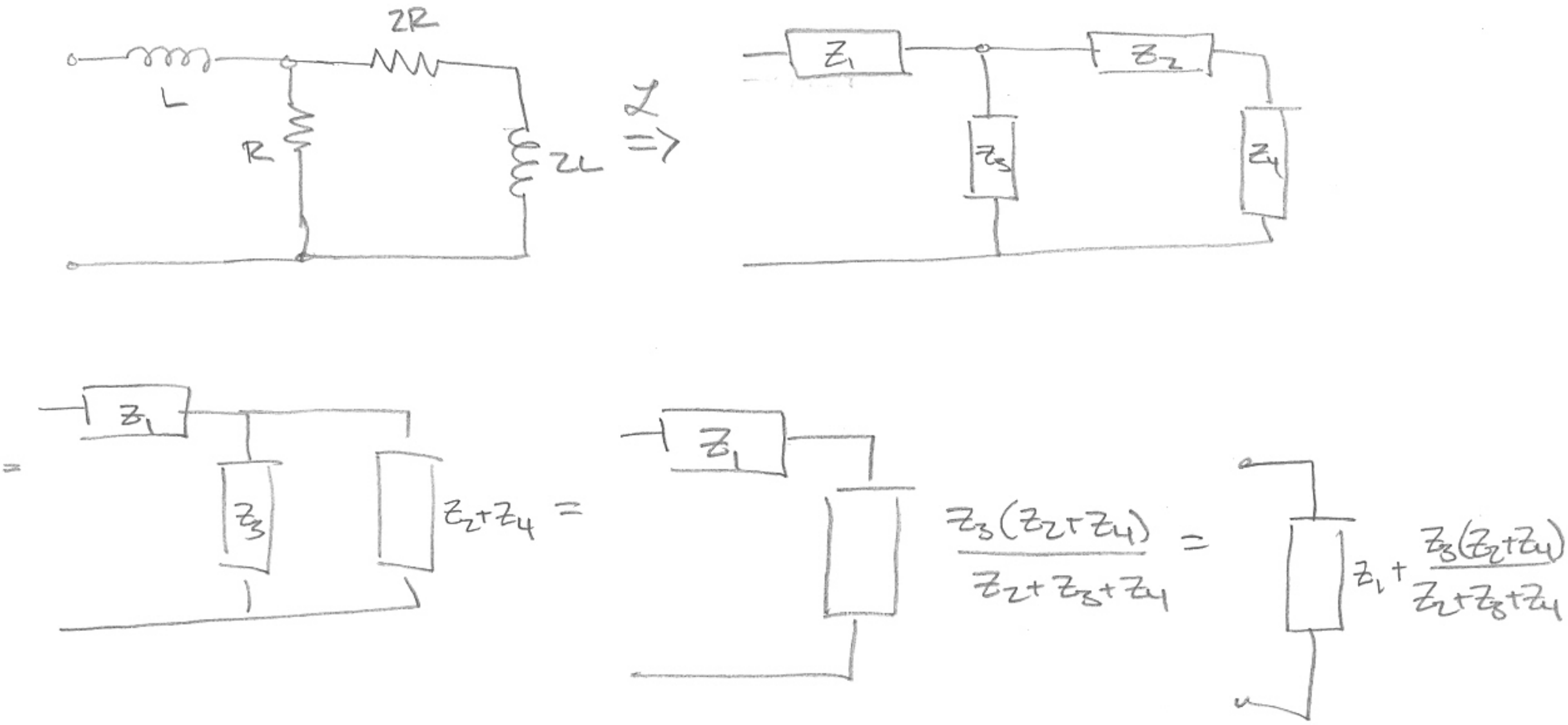
→ zeros: → $(s+10^3)^2+10^6 = 0 \Rightarrow (s+10^3)^2 = -10^6$

$\Rightarrow s+10^3 = \pm 10^3 j$

$\Rightarrow \boxed{s = -10^3 \pm 10^3 j}$

→ poles: → $s + 2\times10^3 = 0 \Rightarrow \boxed{s = -2\times10^3}$

10.9

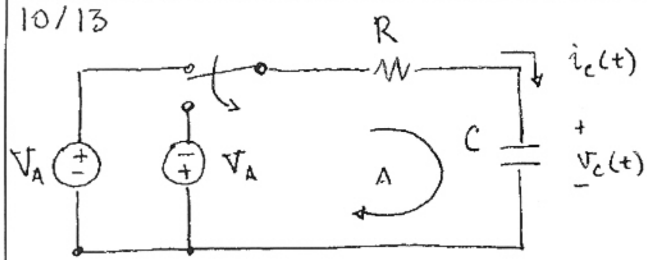



$$\Rightarrow Z_{eq} = Ls + \frac{R(2R+2Ls)}{2R+R+2Ls} = Ls + \frac{2R^2+2RLs}{3R+2Ls} = \frac{3RLs+2L^2s^2+2R^2+2RLs}{2Ls+3R}$$

$$= \frac{2L^2s^2+5RLs+2R^2}{2Ls+3R} = \frac{s^2+\frac{5R}{2L}s+\left(\frac{R}{L}\right)^2}{\frac{1}{L}\left(s+\frac{3R}{2L}\right)} = \boxed{\frac{\left(2s+\frac{R}{L}\right)\left(s+\frac{2R}{L}\right)}{\frac{1}{L}\left(s+\frac{3}{2}\frac{R}{L}\right)}}$$

$\Rightarrow$ $\boxed{\text{zeros @ } -\frac{R}{2L},\ -\frac{2R}{L} \text{ and poles @: } -\frac{3}{2}\frac{R}{L}}$

10/13




$1^{st}$ Transform circuit to s-domain

$$i_c(t) = C\,\frac{dv_c}{dt}$$

$$\Rightarrow \; i_c(s) = sC\,v_c(s) - C\,v_c(0)$$

where $v_c(0) = V_A$

closing switch $\Rightarrow V_A\,u(t) \rightarrow V_A/s$

$2^{nd}$ Apply KCL or KVL (KVL is slightly easier)

note: $i_c(s) = sC\,v_c(s) - C\,v_c(0)$

$$i_c(s) + C\,v_c(0) = sC\,v_c(s)$$

$$v_c(s) = (sC)^{-1}\,i_c(s) + s^{-1}\,v_c(0)$$

KVL @ A: $s^{-1}V_A + i_c(s)\,R + (sC)^{-1}\,i_c(s) + s^{-1}\,v_c(0) = 0$

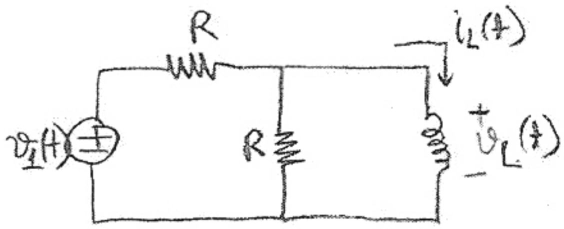
$$CV_A + i_c(s)\,sRC + i_c(s) + CV_A = 0$$

$$i_c(s) = -2CV_A \times \frac{1}{sRC+1}$$

$$\boxed{\begin{gathered} i_c(s) = -\frac{2V_A}{R} \times \frac{1}{s + (RC)^{-1}} \\ i_c(t) = -\frac{2V_A}{R} \times e^{-t/RC}\,u_t \end{gathered}}$$

0/10

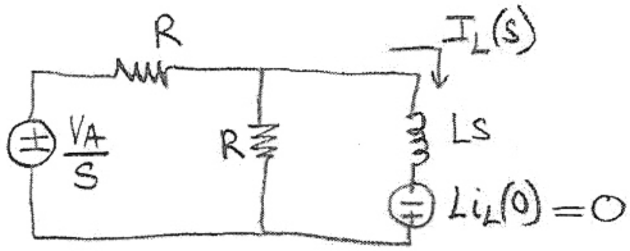
10.15,



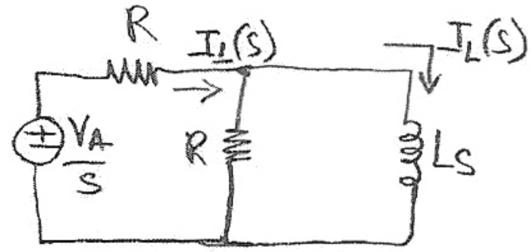

→ no initial energy

→ $v_1(t) = V_A\, u(t)$

$\overset{\mathcal{L}}{\Longrightarrow}$




⇒




$$\rightarrow I_1(s) = \frac{V_A/s}{Z_{eq}} = \frac{V_A/s}{R + \left(\frac{RLs}{R+Ls}\right)} = \frac{V_A(R+Ls)}{s(R^2+2RLs)}$$

→ use current divider to find $I_L(s)$

$$\Rightarrow I_L(s) = \frac{R}{R+Ls} I_1(s) = \frac{R}{R+Ls}\,\frac{V_A(R+Ls)}{s(R^2+2RLs)}$$

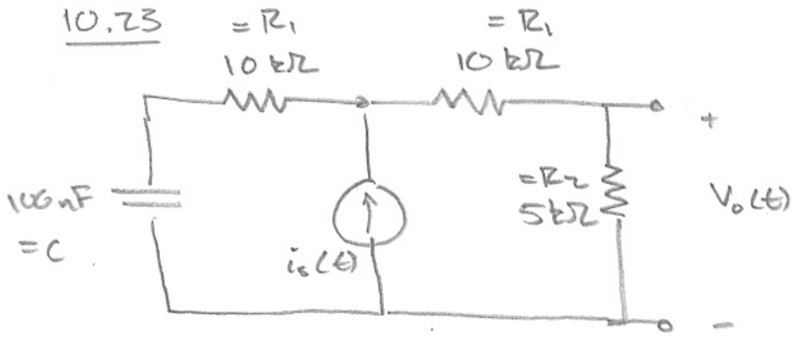
$$\Rightarrow \boxed{I_L(s) = \frac{V_A}{s(R+2Ls)}}$$

$$\Rightarrow I_L(s) = \frac{V_A/R}{s} + \frac{-V_A/R}{s + R/2L}$$

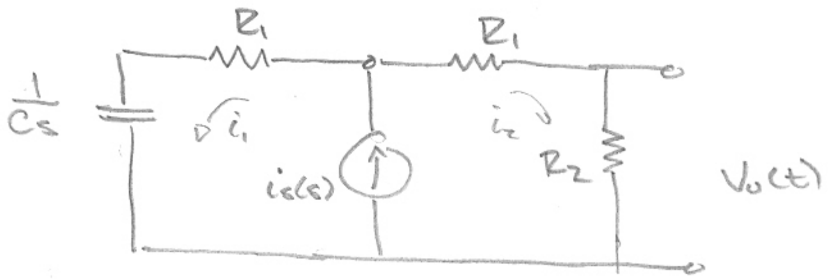
$$\Rightarrow i_L(t) = \frac{V_A}{R} u(t) - \frac{V_A}{R} e^{-\frac{R}{2L}t} u(t)$$

$$\Rightarrow \boxed{i_L(t) = \frac{V_A}{R}\left[1 - e^{-\frac{R}{2L}t}\right] u(t)}$$

10.23




$\Downarrow \mathcal{L}$




current divider $\Rightarrow$ $i_2(s) = \dfrac{\frac{1}{R_1+R_2}}{\frac{1}{R_1+R_2} + \frac{1}{R_1 + 1/Cs}} \left( i_s(s) \right)$

$V = iR$

$V_o = i_2 R_2$

$$= \frac{1}{1 + \frac{R_1+R_2}{R_1 + 1/Cs}}\, i_s(s) = \frac{R_1 + 1/Cs}{2R_1 + R_2 + 1/Cs}\, i_s(s)$$

$$= \frac{R_1Cs + 1}{(2R_1+R_2)Cs + 1}\, i_s(s) \Rightarrow V_o = R_2 \left( \frac{R_1Cs+1}{(2R_1+R_2)Cs+1} \right) i_s(s)$$

$i_s(t) = 2.5e^{-100t}u(t)$ mA $\Rightarrow$ $i_s(s) = \dfrac{(2.5\times10^{-3})}{s+100}$

$$V_o = (5000)\left( \frac{(1\times10^{-3})s + 1}{0.0025s + 1} \right)\left( \frac{2.5\times10^{-3}}{s+100} \right) = (5k)\left( \frac{(1\times10^{-3})s+1}{s+400} \right)\left( \frac{1}{s+100} \right)$$

$$= \frac{A}{s+400} + \frac{B}{s+100}$$

$A: (5k)\left( \dfrac{.001s+1}{s+100} \right)\Big|_{s=-400} = -10$ $\qquad$ $B: (5000)\left( \dfrac{.001s+1}{s+400} \right)\Big|_{s=-100} = 15$

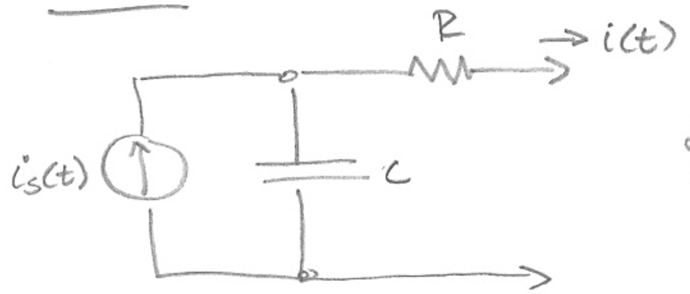
$$\Rightarrow V_o(s) = \frac{-10}{s+400} + \frac{15}{s+100}$$

$$\Rightarrow \boxed{V_o(t) = \left[-10e^{-400t} + 15e^{-100t}\right]u(t)}$$

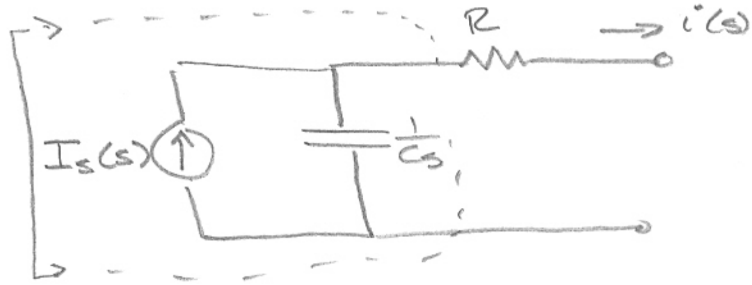
where a forced pole is at $-100$ rad/s

and an unforced pole at $-400$ rad/s

10.26




$\overset{\mathcal{L}}{\Rightarrow}$

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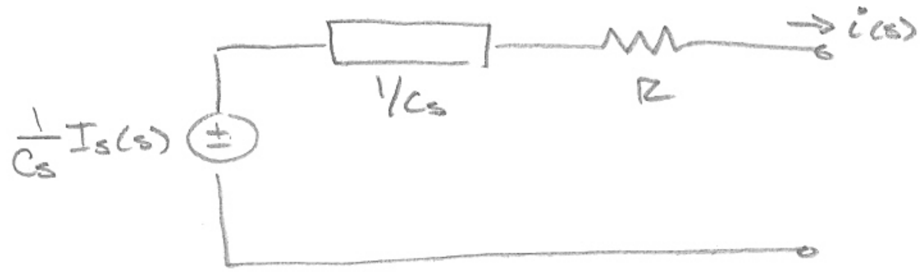


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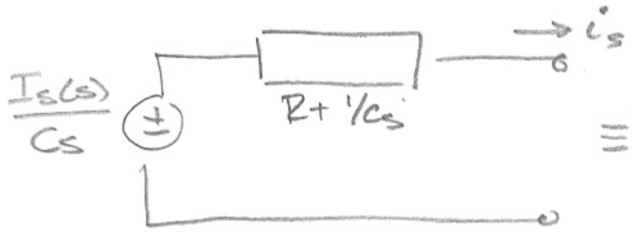


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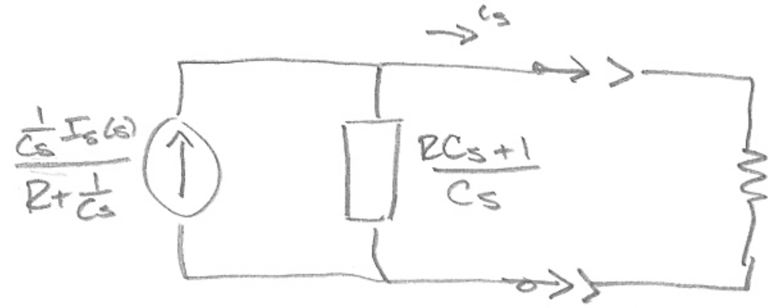



using a current divider w/ the —WW— replaced

$$\Rightarrow i_s(s) = \left(\frac{\frac{1}{R}}{\frac{1}{R} + \frac{Cs}{RCs+1}}\right)\left(\frac{\frac{1}{Cs}}{R+\frac{1}{Cs}}\right) I_s(s)$$

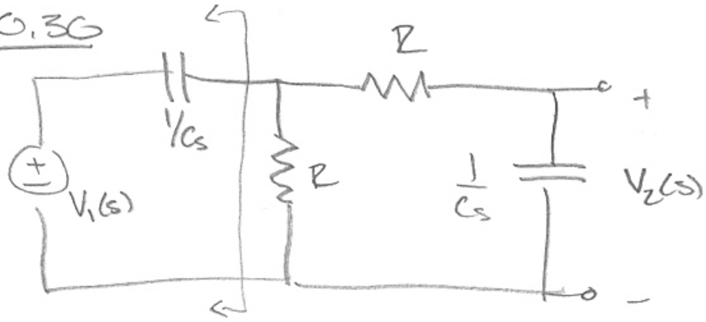
$$= \left(\frac{1}{RCs+1}\right)\left(\frac{\frac{1}{R}}{\frac{1}{R} + \frac{Cs}{RCs+1}}\right) I_s(s)$$

$$= \left(\frac{1}{RCs+1}\right)\left(\frac{1}{1+\frac{RCs}{RCs+1}}\right) I_s(s)$$

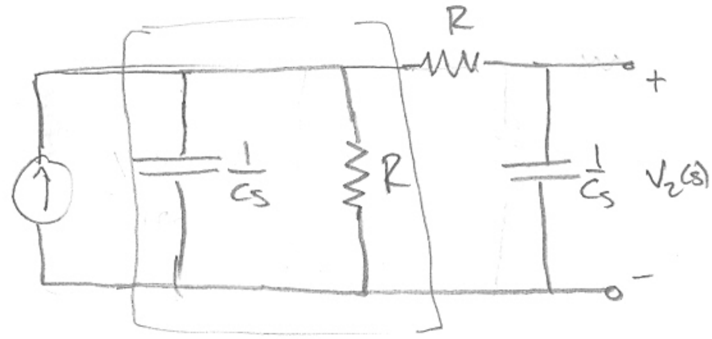
$$= \left(\frac{1}{\cancel{RCs+1}}\right)\left(\frac{\cancel{RCs+1}}{2RCs+1}\right) I_s(s)$$

$$\Rightarrow \boxed{i_s(s) = \left(\frac{1}{2RCs+1}\right) I_s(s)}$$

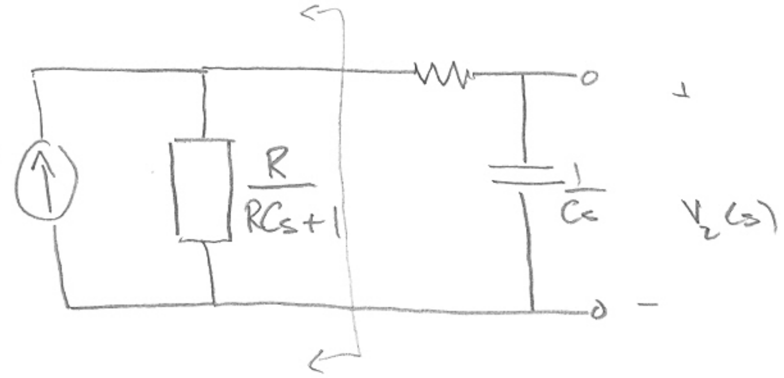
10.36



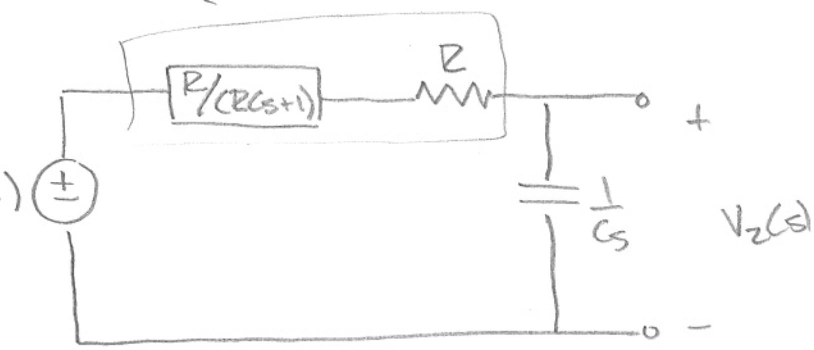

$\equiv \frac{V_1(s)}{1/Cs}$



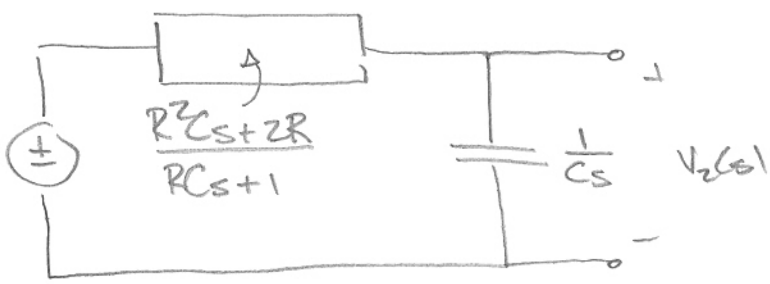

$\equiv CsV_1(s)$




$\equiv \frac{RCs}{RCs+1}V_1(s)$



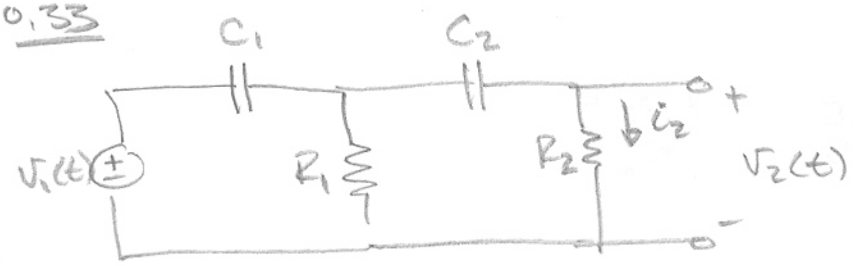

$\equiv \frac{RCs}{RCs+1}V_1(s)$



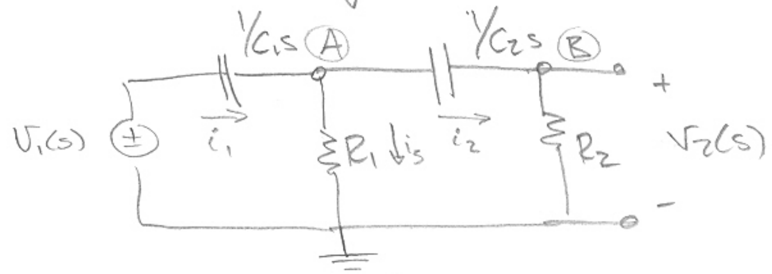

Voltage divider: $$V_2(s) = \frac{\frac{1}{Cs}}{\frac{1}{Cs} + \frac{R^2Cs+2R}{RCs+1}}\left(\frac{RCs}{RCs+1}\right)V_1(s)$$

$$\Rightarrow \boxed{V_2(s) = \frac{RCs}{R^2C^2s^2+3RCs+1}V_1(s)}$$

10.33




$\mathcal{L} \Downarrow$




(a)

@A: $i_1 = i_2 + i_3$

$$\frac{V_1 - V_A}{1/C_1 s} = \frac{V_A - V_B}{1/C_2 s} + \frac{V_A}{R_1}$$ note $V_B = V_2$!

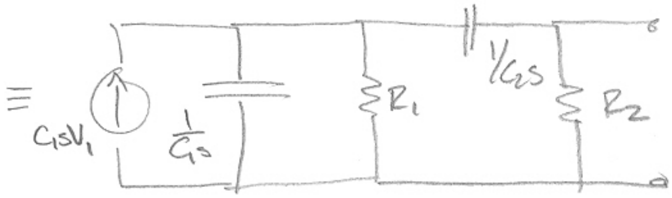
$$V_1 C_1 s - V_A C_1 s = V_A C_2 s - V_2 C_2 s + \frac{1}{R_1} V_A$$

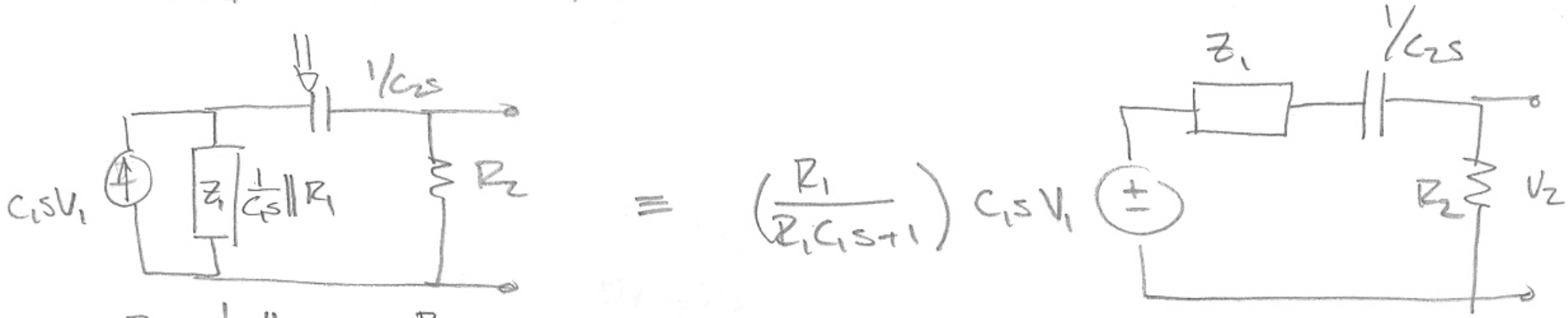
$$\underline{V_1(s) C_1 s = \left(C_1 s + C_2 s + \frac{1}{R_1}\right) V_A - (C_2 s) V_2(s)}$$

@B: $$\frac{V_A - V_B}{1/C_2 s} = \frac{V_B}{R_2}$$

$$(C_2 s) V_A - (C_2 s) V_2(s) = \frac{1}{R_2} V_2(s) \Rightarrow \underline{\left(C_2 s + \frac{1}{R_2}\right) V_2(s) - (C_2 s) V_A(s) = 0}$$

$$\Rightarrow \boxed{\begin{bmatrix} C_1 s + C_2 s + \frac{1}{R_1} & -C_2 s \\ -C_2 s & C_2 s + \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} V_A(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} V_1(s) C_1 s \\ 0 \end{bmatrix}}$$




$$Z_1 = \frac{1}{C_1 s} \| R_1 = \frac{R_1}{R_1 C_1 s + 1}$$

now we have a voltage divider

$$V_2(s) = \frac{R_2}{Z_1 + \frac{1}{C_2 s} + R_2} \left(\frac{R_1}{R_1 C_1 s+1}\right) C_1 s V_1 = \frac{R_2}{\frac{R_1}{R_1 C_1 s+1} + \frac{1}{C_2 s} + R_2} \left(\frac{R_1}{R_1 C_1 s+1}\right) C_1 s V_1$$

$$= \frac{R_1 R_2 C_1 V_1(s) s}{R_1 + \frac{(R_1 C_1 s+1)}{C_2 s} + R_2 (R_1 C_1 s+1)}$$

$$= \frac{R_1 R_2 C_1 C_2 V_1(s) s^2}{R_1 C_2 s + R_1 C_1 s + 1 + R_2 (R_1 C_1 s + 1) C_2 s}$$

$$\boxed{V_2(s) = \frac{R_1 R_2 C_1 C_2 s^2 V_1(s)}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2) s + 1}}$$

(c) $V_1(t) = 30u(t) \Rightarrow V_1(s) = \left(\frac{30}{s}\right)$

$R_1 = 10\text{ k}\Omega$

$R_2 = 20\text{ k}\Omega$

$C_1 = 1\,\mu F$

$C_2 = 0.5\,\mu F$

$$V_2(s) = \frac{(10\text{ k}\Omega)(20\text{ k}\Omega)(1\,\mu F)(0.5\,\mu F)s^2}{(10\text{ k}\Omega)(20\text{ k}\Omega)(1\mu F)(0.5\mu F)s^2 + \left[(10k)(1\mu) + (20k)(0.5\mu) + (10k)(0.5\mu)\right]s + 1}\left(\frac{30}{s}\right)$$

$$= \frac{(1\times10^{-4})s^2}{(1\times10^{-4})s^2 + (0.025)s + 1}\left(\frac{30}{s}\right)$$

$$= \frac{(.003)s}{(0.0001)s^2 + (0.025)s + 1} = \frac{30s}{s^2 + 250s + 10000}$$

$$= \frac{30s}{(s+50)(s+200)}$$

$$= \frac{K_1}{s+50} + \frac{K_2}{s+200}$$

$$K_1 = \frac{30s}{(s+50)(s+200)}(s+50)\Big|_{s=-50} = \underline{-10}$$

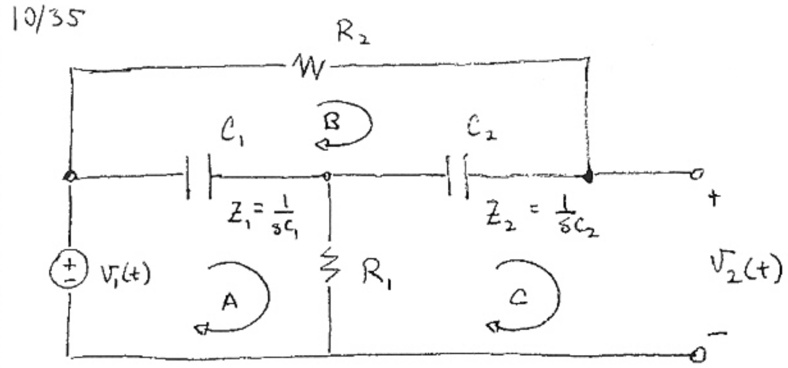
$$K_2 = \frac{30s}{(s+50)(s+200)}(s+200)\Big|_{s=-200} = 40$$

$$\Rightarrow V_2(s) = \frac{-10}{s+50} + \frac{40}{s+200}$$

$$\mathcal{L}^{-1} \Rightarrow -10e^{-50t}u(t) + 40e^{-200t}u(t)$$

$$\Rightarrow \boxed{V_2(t) = \left[40e^{-200t} - 10e^{-50t}\right]u(t)}$$

10/35




No initial energy $\Rightarrow$ $v_{c1}(0) = v_{c2}(0) = 0$

**Answer:**

$$\frac{V_2(s)}{V_1(s)} = \frac{s^2R_1R_2C_1C_2 + s(R_1C_2 + R_1C_1) + 1}{s^2R_1R_2C_1C_2 + s(R_1C_1 + R_1C_2 + R_2C_2) + 1}$$

(a) In the s-domain

$$Z_{C1} = \frac{1}{sC_1} \Leftrightarrow i_{c_1}(s) = sC_1 V_{c_1}(s)$$

$$Z_{C2} = \frac{1}{sC_2} \Leftrightarrow i_{c2}(s) = sC_2\, v_{c2}(s)$$

Mesh equations:

$$\left.\begin{aligned}
A&: \frac{1}{sC_1}(i_A - i_B) + R_1(i_A - i_C) = V_1(s)\\
B&: \frac{1}{sC_1}(i_B - i_A) + R_2(i_B) + \frac{1}{sC_2}(i_B - i_C) = 0\\
C&: R_1(i_c - i_A) + \frac{1}{sC_2}(i_c - i_B) = -V_2(s)
\end{aligned}\right\} \quad (1)$$

noting that $i_c(s) = 0$

$$(1) \Rightarrow \boxed{\begin{aligned}
&\left(R_1 + \frac{1}{sC_1}\right) i_A + \left(-\frac{1}{sC_1}\right) i_B = V_1(s) && (2)\\
&\left(-\frac{1}{sC_1}\right) i_A + \left(R_2 + \frac{1}{sC_1} + \frac{1}{sC_2}\right) i_B = 0 && (3)\\
&(-R_1)\, i_A + \left(-\frac{1}{sC_2}\right) i_B = -V_2(s) && (4)
\end{aligned}}$$

(b) $3 \Rightarrow \quad i_A = i_B\left(sC_1R_2 + 1 + \frac{C_1}{C_2}\right)$

$2 \Rightarrow$

$$\left[\left(R_1 + \frac{1}{sC_1}\right)\left(sC_1R_2 + 1 + \frac{C_1}{C_2}\right) - \frac{1}{sC_1}\right] i_B = V_1(s)$$

$$1 \Rightarrow \left[-R_1\left(sC_1R_2 + 1 + \frac{C_1}{C_2}\right) - \frac{1}{sC_2}\right] i_B = -V_2(s)$$

$$V_2(s)\left[sR_1R_2C_1 + R_1 + \frac{R_1C_1}{C_2} + \frac{1}{sC_2}\right]^{-1} = i_B(s)$$

$$\left[sR_1R_2C_1 + R_1 + \frac{C_1R_1}{C_2} + R_2 + \cancel{\frac{1}{sC_1}} + \frac{1}{sC_2} - \cancel{\frac{1}{sC_1}}\right]\left(sR_1R_2C_1 + R_1 + \frac{R_1C_1}{C_2} + \frac{1}{sC_2}\right)^{-1} = \boxed{\frac{V_1}{V_2}}$$

answer @ top