HW #6 | Ch. 9: 1, 4, 7, 9, 17, 18, 24, 32, 45, 47

1) find F(s), poles & zeros

$$f(t) = A\left[e^{-\alpha t} - 2e^{-\gamma t}\right]u(t)$$

$$F(s) = A\left(\frac{1}{s+\alpha} - \frac{2}{s+\gamma}\right)$$

$$= -A\frac{(s+2\alpha-\gamma)}{(s+\alpha)(s+\gamma)}$$

poles: $-\alpha, -\gamma$

zeros: $-2\alpha+\gamma$

4) find F(s), poles & zeros

$$f(t) = A\left[\cos(\beta t) - \sin(\beta t)\right]u(t)$$

$$F(s) = A\left(\frac{s}{s^2+\beta^2} - \frac{\beta}{s^2+\beta^2}\right)$$

$$= \frac{A(s-\beta)}{s^2+\beta^2}$$

poles: $\pm j\beta$

zeros: β

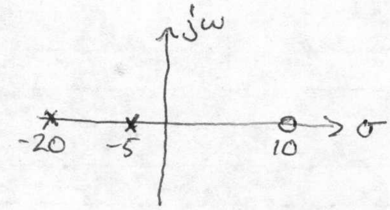
7) find F(s) & plot pole-zero diagrams

a) $f_1(t) = \left[5e^{-5t} - 10e^{-20t}\right]u(t)$

$$F_1(s) = \frac{5}{s+5} - \frac{10}{s+20}$$

$$= \frac{-5(s-10)}{(s+5)(s+20)}$$

(graph not to scale)



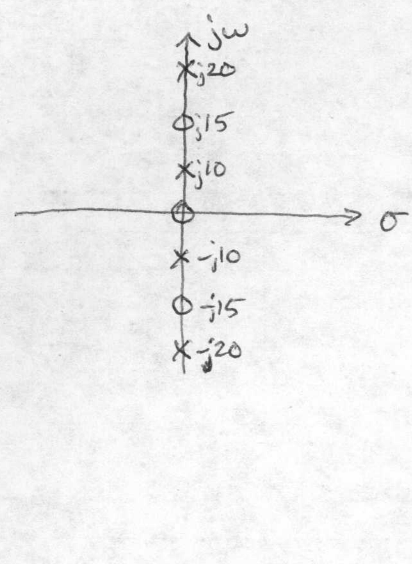
b) $f_2(t) = \left[5\cos(10t) + 7\cos(20t)\right]u(t)$

$$F_2(s) = \frac{5s}{s^2+100} + \frac{7s}{s^2+400}$$

$$= \frac{5s^3+2000s+7s^3+700s}{(s^2+100)(s^2+400)}$$

$$= \frac{12s^3+2700s}{(s^2+100)(s^2+400)}$$

$$= \frac{12s(s^2+15^2)}{(s^2+100)(s^2+400)}$$



9) Find $F(s)$, poles & zeros

a) $f_1(t) = \delta(t) - (625\, t\, e^{-50t})u(t)$

$$F_1(s) = 1 - \frac{625}{(s+50)^2}$$

$$= \frac{s^2 + 100s + 1875}{(s+50)^2}$$

$$= \frac{(s+25)(s+75)}{(s+50)^2}$$

poles: $-50, -50$
zeros: $-25, -75$

b) $f_2(t) = [5 + e^{-20t} - 6\cos(10t) + 2\sin(10t)]\, u(t)$

$$F_2(s) = \frac{5}{s} + \frac{1}{s+20} - \frac{6s}{s^2+10^2} + \frac{20}{s^2+10^2}$$

$$= \frac{5(s+20)(s^2+100) + s(s^2+100) + (-6s+20)s(s+20)}{s(s+20)(s^2+100)}$$

$$= \frac{5s^3 + 500s + 100s^2 + 100^2 + s^3 + 100s - 6s^3 - 120s^2 + 20s^2 + 400s}{s(s+20)(s^2+100)}$$

$$= \frac{1000(s+10)}{s(s+20)(s^2+100)}$$

poles: $0, -20, \pm j10$
zeros: -10

17) find $f(t)$

a) $F_1(s) = \dfrac{2(s+5)}{s(s+10)} = \dfrac{k_1}{s} + \dfrac{k_2}{s+10}$

$$k_1 = \lim_{s\to 0} s\,F_1(s) = 1$$

$$k_2 = \lim_{s\to -10} (s+10)F_1(s) = 1$$

$$F_1(s) = \frac{1}{s} + \frac{1}{s+10}$$

$$f_1(t) = [1 + e^{-10t}]\,u(t)$$

b) $F_2(s) = \dfrac{s^2}{(s+5)(s+10)}$ improper rational

$$\begin{array}{r|l} & \quad 1 \\ s^2 + 15s + 50 & \quad s^2 \\ & -(s^2+15s+50) \\ \hline & \quad -15s - 50 \end{array}$$

17) b) continued

now $F_2(s) = 1 - \frac{15s+50}{(s+5)(s+10)} = 1 - \frac{k_1}{s+5} - \frac{k_2}{s+10}$

$k_1 = \lim_{s \to -5} (s+5)\frac{15s+50}{(s+5)(s+10)} = -5$

$k_2 = \lim_{s \to -10} (s+10)\frac{15s+50}{(s+5)(s+10)} = 20$

$F_2(s) = 1 + \frac{5}{s+5} - \frac{20}{s+10}$

$f_2(t) = \delta(t) + [5e^{-5t} - 20e^{-10t}]u(t)$

18) find $f(t)$

a) $F_1(s) = \frac{20(s+20)}{(s+10)^2+400}$ poles: $-10 \pm j20$

$= \frac{20(s+20)}{(s+10+j20)(s+10-j20)} = \frac{k_1}{s+10+j20} + \frac{k_1^*}{s+10-j20}$

$k_1 = \lim_{s \to -10-j20} (s+10+j20)F_1(s) = \frac{20(-10-j20+20)}{(-10-j20+10-j20)}$

$= \frac{20-j40}{-j4}$

$= 10+j5$

$F_1(s) = \frac{10+j5}{s+10+j20} + \frac{10-j5}{s+10-j20}$

use table of transform pairs

	$f(t)$	$F(s)$
simple complex poles	$[2\lvert k\rvert e^{-\alpha t}\cos(\beta t + \angle k)]u(t)$	$\frac{k}{s+\alpha-j\beta} + \frac{k^*}{s+\alpha+j\beta}$

for $F_1(s)$ $k = 10-j5$ $\lvert k\rvert = 5\sqrt{5}$

$\angle k = \tan^{-1}(-5/10)$

$f_1(t) = [10\sqrt{5}\, e^{-10t}\cos(20t + \tan^{-1}(-1/2))]u(t)$

18) b) $F_2(s) = \dfrac{20(s-20)}{(s+10)^2+400} = \dfrac{k_1}{s+10-j20} + \dfrac{k_1^*}{s+10+j20}$

$k_1 = \lim_{s \to -10+j20} (s+10-j20)\, F_2(s) = \dfrac{-60+j40}{j4} = 10+j15$

$k_1^* = 10-j15$

simple complex pole $\quad k = 10+j15 \quad |k| = 5\sqrt{13} \quad \angle k = \tan^{-1}\left(\frac{15}{10}\right)$

$f_2(t) = \left[10\sqrt{13}\, e^{-10t} \cos\left(20t + \tan^{-1}\left(3/2\right)\right)\right] u(t)$

24) find $f(t)$

a) $F_1(s) = \dfrac{30(s+2)}{s(s^2+4s+5)} = \dfrac{k_1}{s} + \dfrac{k_2}{s+2-j} + \dfrac{k_2^*}{s+2+j}$

$k_1 = \lim_{s \to 0} s F_1(s) = 12$

$k_2 = \lim_{s \to -2+j} (s+2-j) F_1(s) = \dfrac{(j30)}{(-2+j)(j2)} = \dfrac{15}{-2+j} \dfrac{(-2-j)}{(-2-j)} = \dfrac{-30-j15}{5} = -6-j3$

$k_2^* = -6+j3$

step function with simple complex pole $\quad k = -6-j3 \quad |k| = 3\sqrt{5} \quad \angle k = \pi + \tan^{-1}\left(3/6\right)$

$f_1(t) = \left[12 + 6\sqrt{5}\, e^{-2t} \cos\left(t + \pi + \tan^{-1}\left(1/2\right)\right)\right] u(t)$

b) $F_2(s) = \dfrac{2s}{(s+4)(s^2+4s+8)} = \dfrac{k_1}{s+4} + \dfrac{k_2}{s+2-j2} + \dfrac{k_2^*}{s+2+j2}$

$k_1 = \lim_{s \to -4} (s+4) F_2(s) = -1$

$k_2 = \lim_{s \to -2+j2} (s+2-j2) F_2(s) = \dfrac{2(-2+j2)}{(2+j2)(j4)} = \dfrac{(-1+j)}{(1+j)(j2)} = \dfrac{1}{2}$

$k_2^* = 1/2 \qquad |k| = 1/2 \qquad \angle k = 0$

$f_2(t) = \left[-e^{-4t} + e^{-2t} \cos(2t)\right] u(t)$

32) Use Laplace transformation to find $y(t)$

$\dfrac{dy}{dt} + 500y = \left[2500\, e^{-250t}\right] u(t) \quad$ with $y(0-) = 0$

$\mathcal{L}\left\{\dfrac{dy}{dt} + 500y = \left[2500\, e^{-250t}\right] u(t)\right\} \Rightarrow sY(s) - y(0-) + 500Y(s) = \dfrac{2500}{s+250}$

$(s+500)Y(s) = \dfrac{2500}{s+250}$

$Y(s) = \dfrac{2500}{(s+500)(s+250)} = \dfrac{k_1}{s+500} + \dfrac{k_2}{s+250}$

$k_1 = \lim_{s \to -500} (s+500) Y(s) = -10$

$k_2 = \lim_{s \to -250} (s+250) Y(s) = 10$

32) continued

$$Y(s) = -\frac{10}{s+500} + \frac{10}{s+250}$$

$$\mathcal{L}^{-1}\{Y(s)\} = y(t) = 10e^{-250t} - 10e^{-500t}$$

45) Use initial and final value properties to find the initial and final values of the waveform corresponding to the transforms

a) $F_1(s) = \dfrac{16}{(s+2)(s^2+12s+36)}$

$$\lim_{t\to 0+} f_1(t) = \lim_{s\to\infty} sF_1(s) = 0$$

$$\lim_{t\to\infty} f_1(t) = \lim_{s\to 0} sF_1(s) = 0$$

b) $F_2(s) = \dfrac{2(s^2+2)}{s(s^2+4)}$

$$\lim_{t\to 0+} f_2(t) = \lim_{s\to\infty} sF_2(s) = 2$$

$$\lim_{t\to\infty} f_2(t) = \lim_{s\to 0} sF_2(s)$$ N/A because of j-axis poles

47)

a) $F_1(s) = \dfrac{2(s^2+5s+6)}{(s+2)(s+6)(s+12)}$

$$\lim_{t\to 0+} f_1(t) = \lim_{s\to\infty} sF_1(s) = 2$$

$$\lim_{t\to\infty} f_1(t) = \lim_{s\to 0} sF_1(s) = 0$$

b) $F_2(s) = \dfrac{10(s^2+10s+40)}{s(s^2-100)}$

$$\lim_{t\to 0+} f_2(t) = \lim_{s\to\infty} sF_2(s) = 10$$

$$\lim_{t\to\infty} f_2(t) = \lim_{s\to 0} sF_2(s)$$ N/A because of pole in right side of plane.