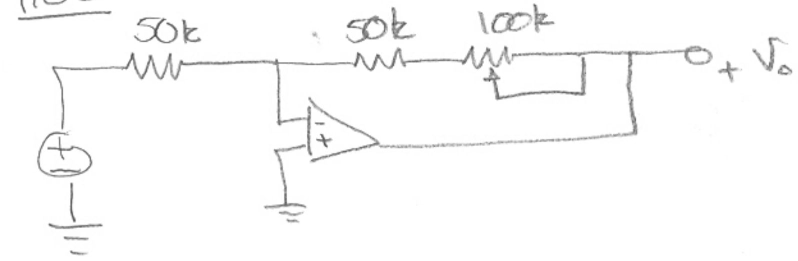
HW #5 Solutions

4.22



find range of gain V_o/V_s:

$\equiv$ inverting amp w/ $R_1 = 50\,k\Omega$

$R_2 = 50\,k\Omega + X$

where

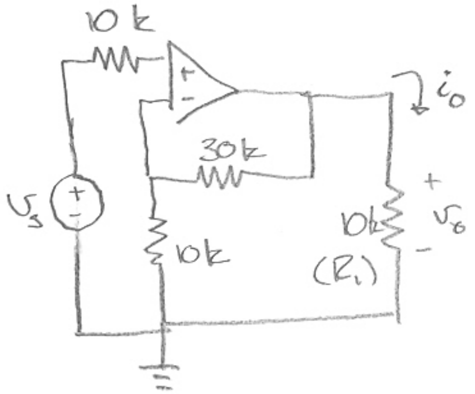
$0 < X < 100\,k\Omega$

$$\therefore \frac{V_o}{V_s} = K_v = -\left(\frac{R_2}{R_1}\right) = -\left(\frac{50k + X}{50}\right)$$

$X = 0 \Rightarrow K_v = -1$, $X = 100\,k\Omega \Rightarrow K_v = -3$

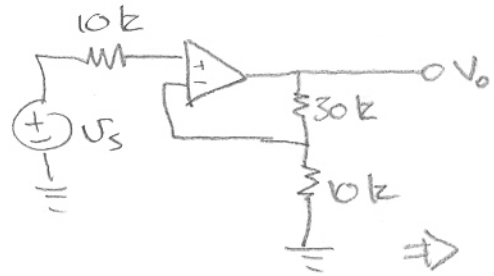
$$\Rightarrow \boxed{-3 \leq K_v \leq -1}$$

4.23



$\equiv$

(a) find V_o in terms of V_s



(a non-inverting amplifier!)

$$\Rightarrow \frac{V_o}{V_s} = \left(\frac{30k + 10k}{10k}\right) = 4$$

$$\therefore \boxed{V_o = 4V_s}$$

(b) find i_o for $V_s = 1.5\,V$

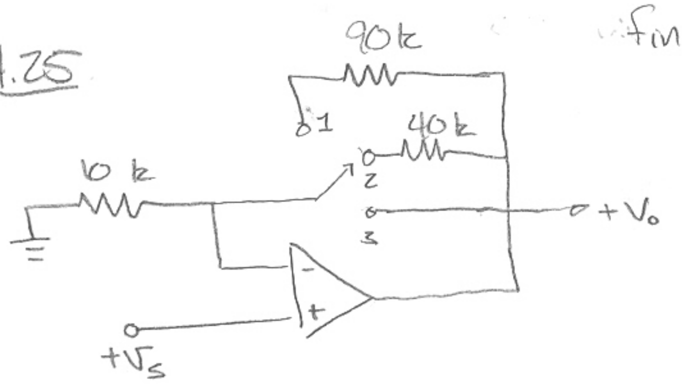
$\Rightarrow V_o = 4(1.5\,V) = 6V$

ohm's law over $R_1 = 10k$

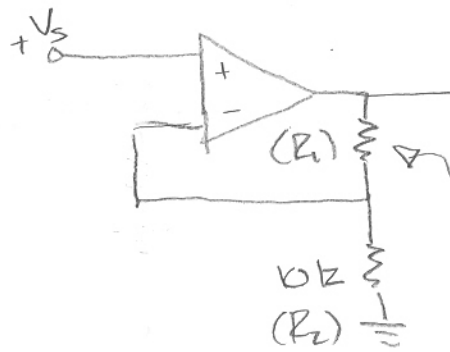
$(V_o - 0) = i_o(10k) \Rightarrow i_o = \frac{6}{10k} \Rightarrow \boxed{i_o = 0.6\,mA}$

4.25

find gain at each switch position:



$\equiv$



where this resistor is:
position 1: 90 kΩ
position 2: 40 kΩ
position 3: 0 kΩ

This 'equivalent' circuit is a non-inverting amp:

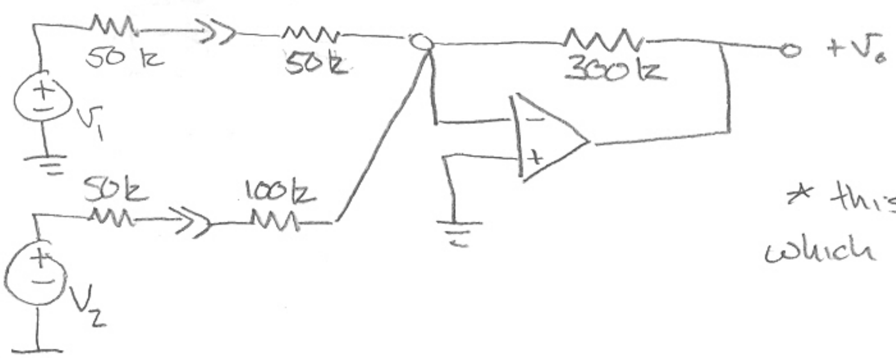
$$\frac{v_o}{v_s} = K = \frac{R_1 + R_2}{R_2}$$ where $R_2 = 10\,k\Omega$ and R_1 varies

position 1: $K = \frac{100}{10} = 10$

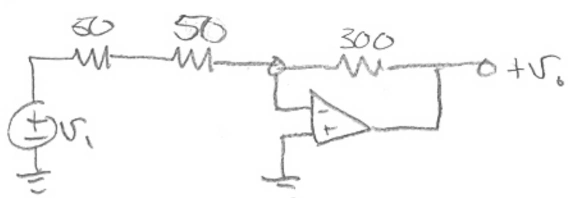
position 2: $K = \frac{10+40}{10} = 5$

position 3: $K = \frac{10+0}{10} = 1$

4.26 find v_o in terms of v_1 and v_2



* this is a summer circuit which we solve w/ linearity:



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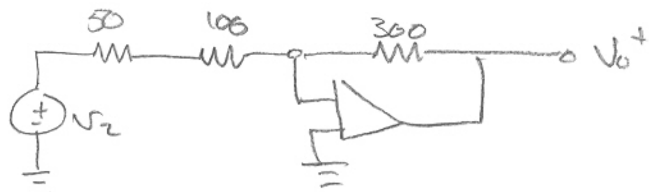


$K_1 = \frac{V_o}{V_1} = -\left(\frac{300}{100}\right) = -3$

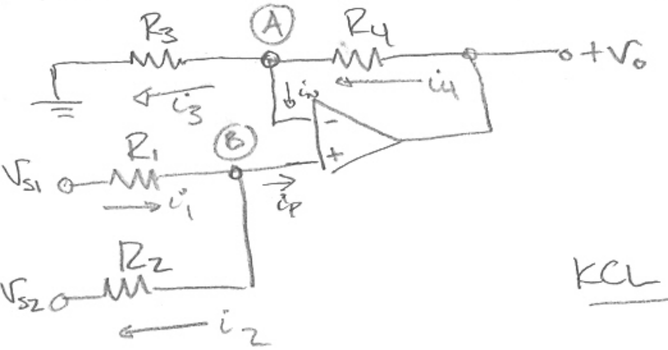
$V_o = -3V_1$

$K_2 = \frac{V_o}{V_2} = -\left(\frac{300}{150}\right) = -2$

$V_o = -2V_2$

$\Rightarrow \boxed{V_{o_{TOTAL}} = -3v_1 - 2v_2}$

4.36 Find V_o in terms of V_s



op amp rules:

$$\underbrace{V_N = V_P}_{} \quad ; \quad i_P = i_N = 0$$

$V_N = V_A$, $V_P = V_B$

$\Rightarrow V_A = V_B$

KCL @ B: $i_1 = \cancel{i_N} + i_2 = \quad i_1 = i_2$

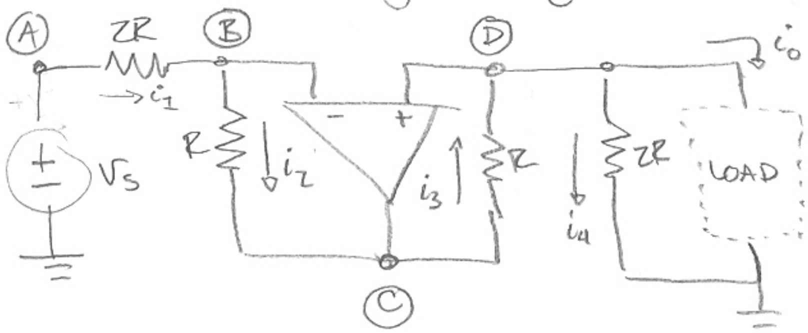
$$\Rightarrow \frac{V_{S1} - V_B}{R_1} = \frac{V_B - V_{S2}}{R_2} \Rightarrow V_B = \frac{R_2 V_{S1} + R_1 V_{S2}}{R_1 + R_2} = V_A$$

KCL @ A: $i_4 = \cancel{i_N} + i_3 \Rightarrow i_4 = i_3$

$$\frac{V_o - V_A}{R_4} = \frac{V_A - 0}{R_3} \Rightarrow V_o = \left(\frac{R_4}{R_3} + 1\right) V_A = \left(\frac{R_4}{R_3} + 1\right) V_B$$

$$\Rightarrow \boxed{V_o = \left(\frac{R_4}{R_3} + 1\right)\left(\frac{R_2 V_{S1} + R_1 V_{S2}}{R_1 + R_2}\right)}$$

4.34 Use node voltage analysis to show $i_o = -V_s/2R$ regardless of load.



NODE A: $\Rightarrow V_A = V_s$!

node B: $i_1 - i_2 = 0$

$$\frac{V_A - V_B}{2R} - \frac{V_B - V_C}{R} = 0 \Rightarrow \left(\frac{1}{2R}\right)V_A - \left(\frac{1}{2R} + \frac{1}{R}\right)V_B + \left(\frac{1}{R}\right)V_C = 0$$

$$\Rightarrow \left(\frac{1}{2R}\right)V_A = \left(\frac{3}{2R}\right)V_B - \left(\frac{1}{R}\right)V_C \qquad (1)$$

node D: $i_3 - i_4 - i_o = 0$

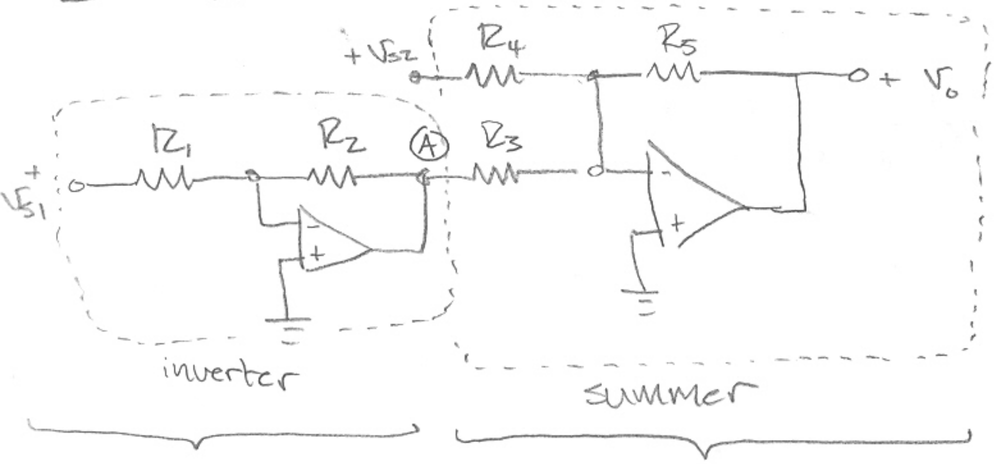
$$\frac{V_C - V_D}{R} - \frac{V_D}{2R} - i_o = 0 \Rightarrow -\frac{1}{R}V_C + \left(\frac{3}{2R}\right)V_D = -i_o \qquad (2)$$

$$(1)\ \left(\frac{3}{2R}\right)V_B - \left(\frac{1}{R}\right)V_C = \frac{V_A}{2R}$$

$$-\left[(2)\ \left(\frac{3}{2R}\right)V_B - \left(\frac{1}{R}\right)V_C = -i_o\right]$$

$$= \qquad 0 = \frac{V_A}{2R} + i_o \Rightarrow i_o = -\frac{V_A}{2R} = \boxed{-\frac{V_s}{2R} = i_o}$$

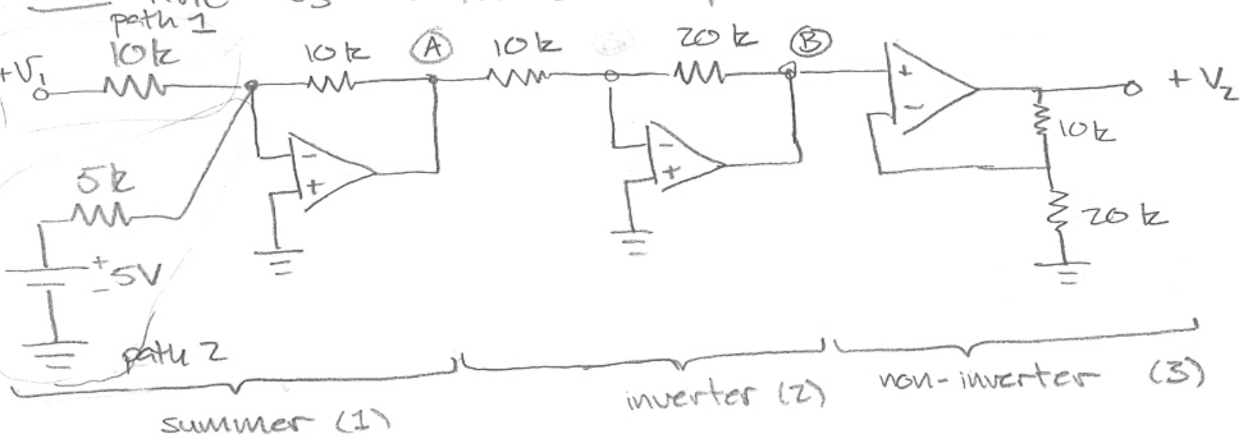
4.36 find v_o in terms of v_{s1} & v_{s2}



$$\frac{V_A}{V_{S1}} = -\left(\frac{R_2}{R_1}\right) \qquad V_o = -\left(\frac{R_5}{R_4}\right)V_{S2} - \left(\frac{R_5}{R_3}\right)V_A$$

$$\Rightarrow \boxed{v_o = -\left(\frac{R_5}{R_4}\right)v_{s2} + \left(\frac{R_5 R_2}{R_3 R_1}\right)v_{s1}}$$

4.37 find v_s in terms of v_1



(1): $V_A = -\left(\frac{10k}{10k}\right)V_1$ from path 1

$V_A = -\left(\frac{10k}{5k}\right)(+5V)$ from path 2

$\Rightarrow V_A = (-V_1 - 10)$

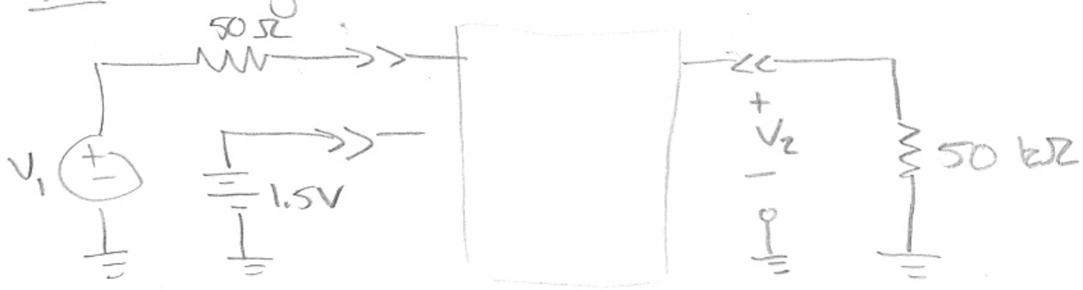
(2): $v_B = -\left(\frac{20}{10}\right)v_A = -2V_A$

(3): $V_2 = \left(\frac{10+20}{10}\right)v_B = \left(\frac{3}{2}\right)V_B$

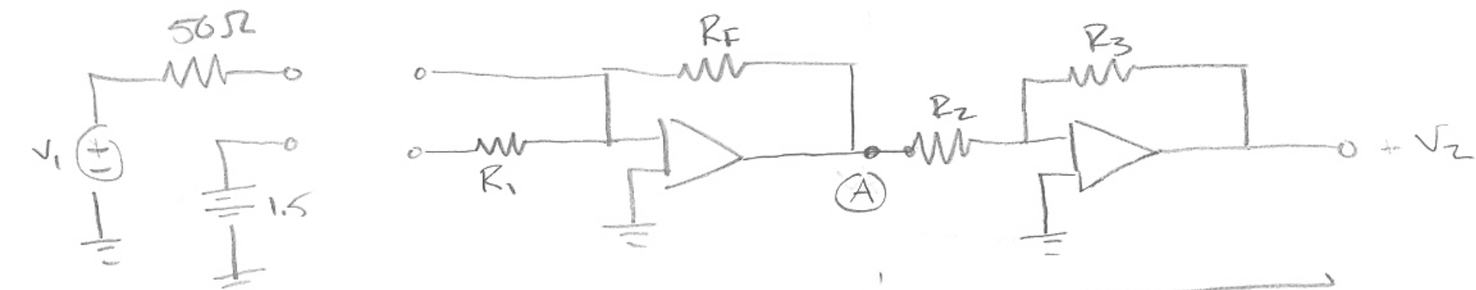
$\Rightarrow V_2 = \left(\frac{3}{2}\right)(-2)(-V_1 - 10)$

$\therefore \boxed{V_2 = 30 + 3V_1 \text{ volts}}$

4.47 Design interface so that $v_2 = (50v_1 + 3)V$



$v_2 = 50v_1 + 3$ indicates a summer.
positive values of a sum indicate an inverter (see 4.26)



summer

$$V_A = -\left(\frac{R_F}{50}\right)v_1 - \left(\frac{R_F}{R_1}\right)(1.5)$$

inverter

$$v_2 = -\left(\frac{R_3}{R_2}\right)V_A$$

$$\Rightarrow \boxed{v_2 = -\left(\frac{R_3}{R_2}\right)\left(-\left(\frac{R_F}{50}\right)v_1 - \left(\frac{R_F}{R_1}\right)(1.5)\right) = 50v_1 + 3}$$

$$\Rightarrow \boxed{\frac{R_3 R_F}{50 R_2} = 50 \quad \text{and} \quad \frac{R_3 R_F}{R_1 R_2}(1.5) = 3}$$

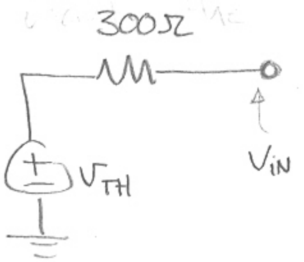
which we can solve w/ any feasible choice

ex: (a possible solution)

$$\frac{R_3}{R_2} = 1, \quad \underbrace{\frac{R_F}{50} = 50}_{R_F = 2500\,\Omega}, \quad \underbrace{\frac{R_F}{R_1} = 2}_{R_1 = 1250\,\Omega}$$

$R_3 = R_2 = 1\,\Omega$

4.55 Consider the transducer modled a thevenin equivalent.



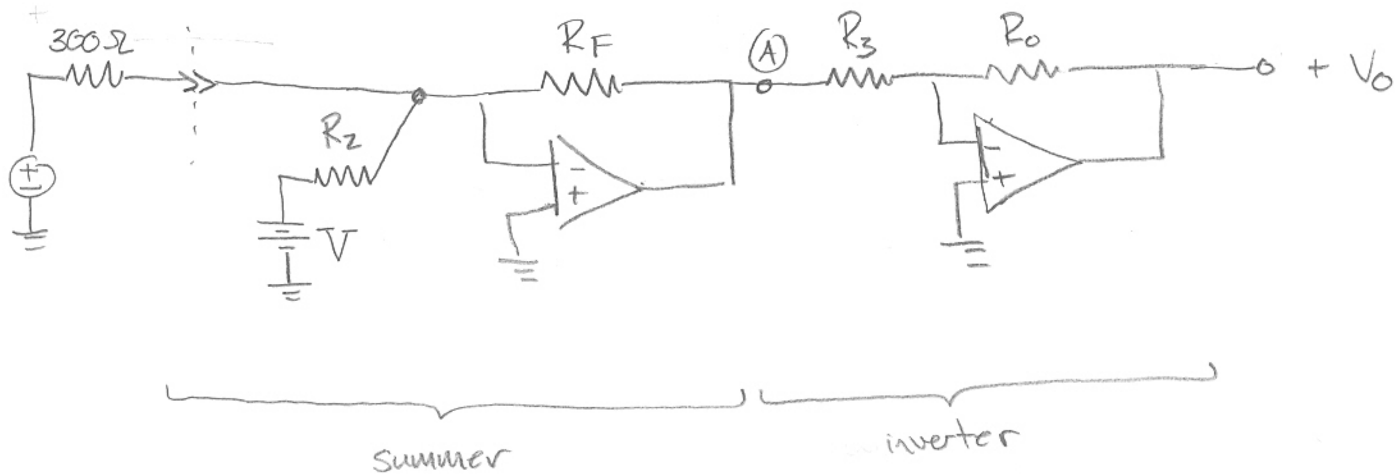
$$V_{TH} = (0.05P - 0.75 \text{ mV})$$

$P = 20$ mmHg $\Rightarrow$ 0.25 mV

$P = 200$ mm Hg $\Rightarrow$ 9.25 mV

∴ We need to add 0.75 mV to the measurement (i.e. add a constant)

⇒ Note a summer output is negative, so we'll need an inverter



$$V_A = -\left(\frac{R_F}{300}\right) V_{TH} - \left(\frac{R_F}{R_2}\right) V \qquad V_o = -\left(\frac{R_o}{R_3}\right) V_A$$

$$\Rightarrow V_o = -\frac{R_o}{R_3}\left(-\frac{R_F}{300} V_{TH} - \frac{R_F}{R_2} V\right)$$ where V is an un-chose battery voltage.

ex sol'n

if $\boxed{R_F = 300}$ $\Rightarrow V_o = \frac{R_o}{R_3} V_{TH} + \frac{300}{R_2} V$

let $\boxed{R_o = R_3}$ $\Rightarrow V_o = V_{TH} + \frac{300}{R_2} V$

$\frac{300}{R_2} = 0.75 \Rightarrow \boxed{R_2 = 400\,\Omega}$ for $\boxed{V = +1 \text{ volt}}$