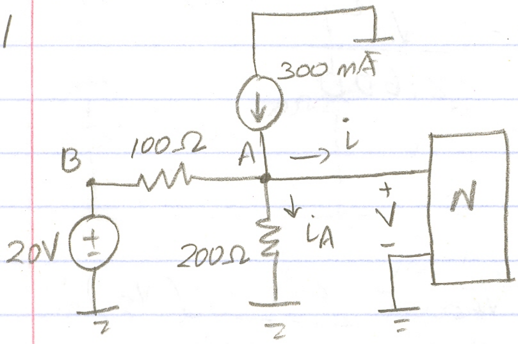
MAE 140
HW # 3 SOLUTIONS

3.71



$i = (5\times10^{-3}\,V - 0.5)$

$V = V_A$

At A

$$0.3 = i + \frac{V_A - 20}{100} + \frac{V_A}{200}$$

$$0.3 = (0.005V_A - 0.5) + 0.01V_A - 0.2 + V_A(0.005)$$

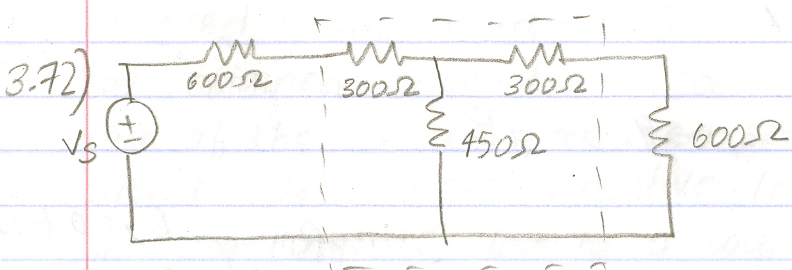
$$0.3 = 0.01V_A + 0.01V_A - 0.7$$

$$1 = 0.02V_A$$

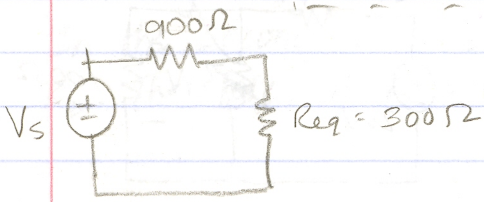
$$V_A = \frac{100}{2} = 50V$$

$$V = V_A = 50V$$

3.72)



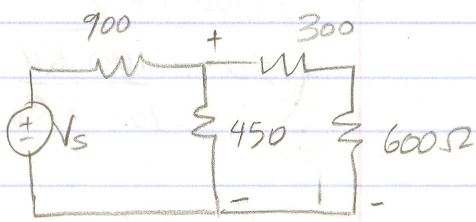
$$P = \frac{V^2}{R}$$



$$\frac{1}{R_{eq}} = \frac{1}{450} + \frac{1}{900}$$

$$R_{eq} = \frac{900(450)}{1350}$$

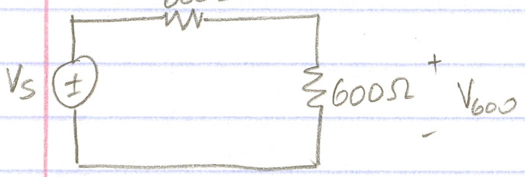
$$V_{300} = \frac{300}{1200}V_s = \frac{V_s}{4}$$



$$V_{600} = \frac{\overset{2}{\cancel{600}}}{\underset{3}{\cancel{900}}}\left(\frac{V_s}{\cancel{4}_2}\right) = \frac{V_s}{6}$$

$$P = \frac{V_{600}^2}{R} = \frac{V_s^2}{36 \times 600} = \frac{V_s^2}{21600}$$

Without Attenuator



$$V_{600} = \frac{600}{600+600}(V_s)$$

$$= \frac{600}{1200}(V_s) = \frac{V_s}{2}$$

$$P = \frac{V_{600}^2}{R} = \frac{V_s^2}{4(600)} = \frac{V_s^2}{2400}$$

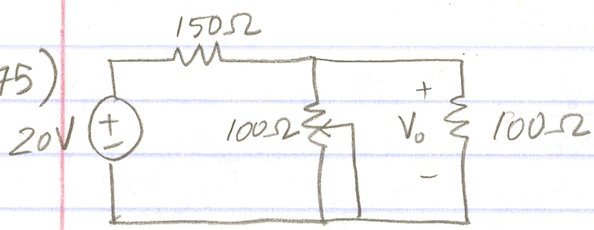
$$\frac{P_{W/O\ attenuator}}{P_{W/\ attenuator}} = \frac{\cancel{V_s^2} \times 216\cancel{00}}{24\cancel{00} \times \cancel{V_s^2}}$$

$$= 9$$

$$\frac{P_{W/\ attenuator}}{P_{W/O\ attenuator}} = \frac{1}{9}$$

$$\Rightarrow 10 \log\left(\frac{1}{9}\right) = -9.54\ dB$$

3.75)



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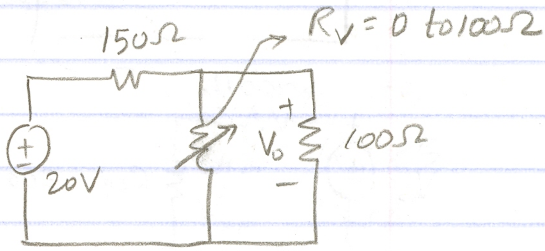
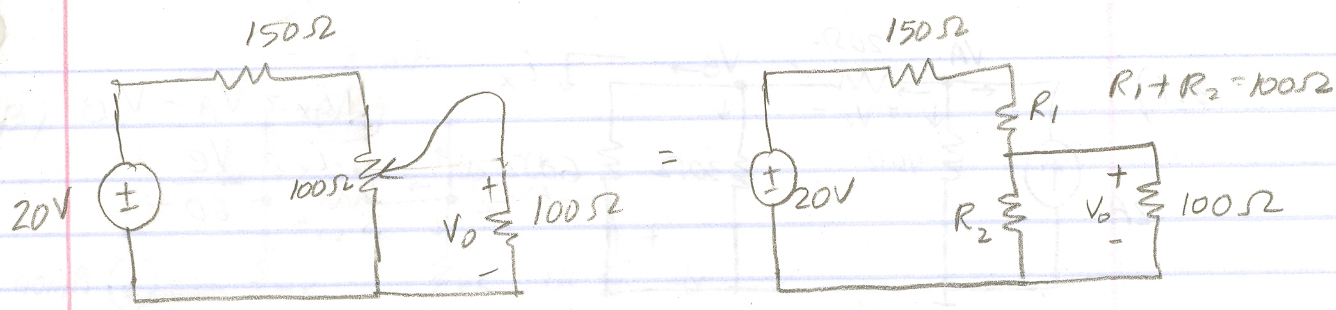


$$V_0 = \left(\frac{\frac{100\,R_V}{100+R_V}}{150 + \frac{100\,R_V}{100+R_V}}\right)(20V) = 20\left(\frac{100\,R_V}{15000+250R_V}\right)$$



$$V_0 = 20\left(\frac{\dfrac{100R_2}{100+R_2}}{150+R_1+\dfrac{100R_2}{100+R_2}}\right)$$

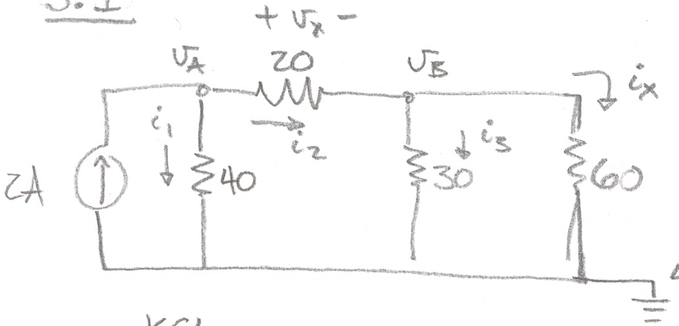
$$= 20\left(\frac{100R_2}{(150+R_1)(100+R_2)+100R_2}\right)$$

$$R_1 + R_2 = 100$$

$$V_0 = 20\left(\frac{100R_2}{25000+250R_2-R_2^2}\right)$$

We need to find a circuit in which the output voltage can be obtained close to 0. Looking at V_0 equations for each of the circuit, we can see that in the first circuit V_0 is more sensitive to the changes in R_V. Thus, getting V_0 close to 0 could be difficult. In the second circuit, V_0 is less sensitive to changes in R_2. Therefore, it is more desirable.

3.1



I) Select a reference node

KCL

@A: $2 = i_1 + i_2$

@B: $i_2 = i_3 + i_4$

Ohm's Law: $v = iR \Rightarrow i = \frac{v}{R}$

@A: $2 = \left(\frac{1}{40}\right)v_A + \left(\frac{1}{20}\right)(v_A - v_B)$

@B: $\left(\frac{1}{20}\right)(v_A - v_B) = \left(\frac{1}{30}\right)v_B + \left(\frac{1}{60}\right)v_B$

$$\Rightarrow \begin{cases} \left(\frac{1}{40}+\frac{1}{20}\right)v_A - \left(\frac{1}{20}\right)v_B = 2 \\ \left(-\frac{1}{20}\right)v_A + \left(\frac{1}{20}+\frac{1}{30}+\frac{1}{60}\right)v_B = 0 \end{cases}$$

or

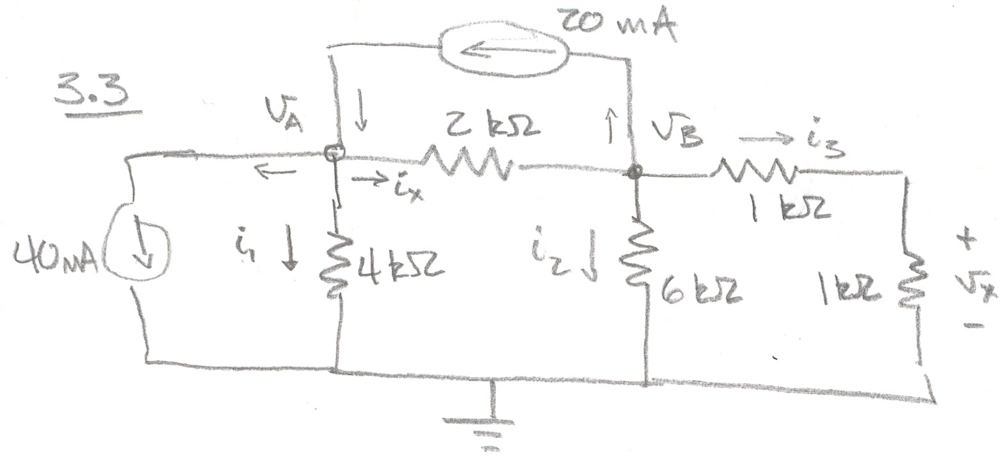
$$\begin{bmatrix} \left(\frac{1}{40}+\frac{1}{20}\right) & -\left(\frac{1}{20}\right) \\ -\left(\frac{1}{20}\right) & \left(\frac{1}{20}+\frac{1}{30}+\frac{1}{60}\right) \end{bmatrix} \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

(b) Find v_x and i_x:

$$\begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{40}+\frac{1}{20}\right) & -\frac{1}{20} \\ -\frac{1}{20} & \left(\frac{1}{20}+\frac{1}{30}+\frac{1}{60}\right) \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} 40 \\ 20 \end{bmatrix}$$

from our layout: $v_A - v_B = v_x \Rightarrow (40-20) = \boxed{v_x = 20\ \Omega}$

and $v_B = i_x(60) \Rightarrow 20 = i_x(60) \Rightarrow i_x = \frac{20}{60} = \boxed{\frac{1}{3}\ A = i_x}$



KCL @ A/B

A: $20\,mA - 40\,mA - i_1 - i_x = 0$

B: $i_x - 20\,mA - i_2 - i_3 = 0$

w/ Ohms Law: $v = iR \Rightarrow i = v/R$

$i_1 = \left(\frac{1}{4000}\right)v_A \qquad i_x = \left(\frac{1}{2000}\right)(v_A - v_B) \qquad i_2 = \left(\frac{1}{6000}\right)v_B$

$i_3 = \left(\frac{1}{1000+1000}\right)v_B$

A: $-20\,mA - \left(\frac{1}{4000}\right)v_A - \left(\frac{1}{2000}\right)(v_A - v_B) = 0$

B: $\left(\frac{1}{2000}\right)(v_A - v_B) - \left(\frac{1}{6000}\right)v_B - \left(\frac{1}{2000}\right)v_B - 20\,mA = 0$

$\Downarrow$

A: $\left(\frac{1}{4000} + \frac{1}{2000}\right)v_A - \left(\frac{1}{2000}\right)v_B = -20\,mA$

B: $\left(\frac{1}{2000} + \frac{1}{6000} + \frac{1}{2000}\right)v_B - \left(\frac{1}{2000}\right)v_A = -20\,mA$

or

$$\begin{bmatrix} \left(\frac{1}{4000} + \frac{1}{2000}\right) & -\left(\frac{1}{2000}\right) \\ -\left(\frac{1}{2000}\right) & \left(\frac{1}{2000} + \frac{1}{6000} + \frac{1}{2000}\right) \end{bmatrix} \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} -20\times10^{-3} \\ -20\times10^{-3} \end{bmatrix}$$

3.3 cont

which, when solved

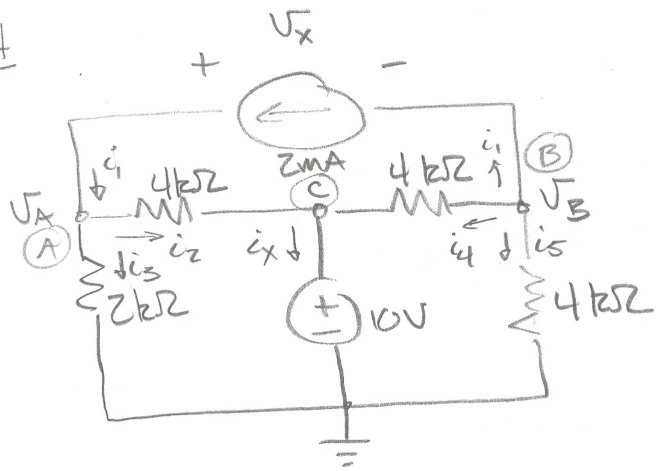
$$\Rightarrow \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} -160/3 \\ -40 \end{bmatrix}$$

voltage divider:

$$v_x = \left(\frac{1000}{2000}\right) v_B = \left(\frac{1}{2}\right)(-40) = \boxed{-20\,V}$$

$$i_x = \frac{(v_A - v_B)}{2000} = \frac{-160/3 + 40}{2000} = \frac{-40/3}{2000} = \frac{-40}{6000} = \boxed{\frac{-20}{3000}\,A \text{ or } i_x = -\frac{20}{3}\,mA}$$

3.4



things we know: $i_1 = 2mA$, $v_c = 10\,V$

Ohm's law for each element:

$$i_2 = \frac{v_A - v_c}{4000} \Rightarrow \frac{v_A - 10}{4000}$$

$$i_3 = \frac{v_A}{2000}$$

$$i_4 = \frac{v_B - v_c}{4000} \Rightarrow \frac{v_B - 10}{4000}$$

$$i_5 = \frac{v_B}{4000}$$

KCL @ A: $i_1 - i_2 - i_3 = 0$

@ B: $-i_1 - i_4 - i_5 = 0$

KCL + elements:

$$\Rightarrow @A: \frac{2}{1000} - \frac{v_A - 10}{4000} - \frac{v_A}{2000} = 0 \Rightarrow \left(\frac{1}{4000} + \frac{1}{2000}\right) v_A = \frac{9}{2000}$$

$$@B: -\frac{2}{1000} - \frac{v_B - 10}{4000} - \frac{v_B}{4000} = 0 \Rightarrow \left(\frac{1}{4000} + \frac{1}{4000}\right) v_B = \frac{1}{2000}$$

or

$$\begin{bmatrix} \left(\frac{1}{4000} + \frac{1}{2000}\right) & 0 \\ 0 & \left(\frac{1}{4000} + \frac{1}{4000}\right) \end{bmatrix} \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} \frac{9}{2000} \\ \frac{1}{2000} \end{bmatrix}$$

3.4 cont.

solving this system $X = A^{-1}B$

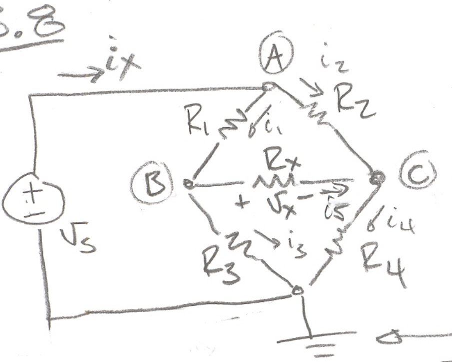
$$\Rightarrow \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

and we know that

$$@C: i_x = i_2 + i_4 = \frac{v_A - 10}{4000} + \frac{v_B - 10}{4000} = \frac{-4}{4000} + \frac{-9}{4000} = \boxed{\frac{-13}{4000} \text{ A} \text{ or } \frac{-13}{4} \text{ mA} = i_x}$$

and $v_x = v_A - v_B = 6 - 1 = \boxed{5 \text{ V} = v_x}$

3.8



← add a reference!

element laws:

$$i_1 = \frac{v_A - v_B}{R_1}, \quad i_2 = \frac{v_A - v_C}{R_2}$$

$$i_3 = \frac{v_B}{R_3}, \quad i_4 = \frac{v_C}{R_4}, \quad i_5 = \frac{v_B - v_C}{R_x}$$

things we know:

$v_A = v_s$!

KCL equations

A: no need: $v_s = v_A$!

B: $i_1 - i_5 - i_3 = 0$

C: $i_2 + i_x - i_4 = 0$

(plug element laws)

$$\Rightarrow B: \frac{v_A - v_B}{R_1} - \frac{v_B - v_C}{R_x} - \frac{v_B}{R_3} = 0$$

$$C: \frac{v_A - v_C}{R_2} + \frac{v_B - v_C}{R_x} - \frac{v_C}{R_4} = 0$$

3.8 cont

simplifying the pevious equations

note $v_s = v_A$ = constant

$$B: \frac{v_A}{R_1} - \frac{v_B}{R_1} - \frac{v_B}{R_x} + \frac{v_c}{R_x} - \frac{v_B}{R_3} = 0 \Rightarrow \left(\frac{1}{R_1} + \frac{1}{R_x} + \frac{1}{R_3}\right) v_B - \left(\frac{1}{R_x}\right) v_c = \left(\frac{1}{R_1}\right) v_s$$

$$C: \frac{v_A}{R_2} - \frac{v_B}{R_2} + \frac{v_B}{R_x} - \frac{v_c}{R_x} - \frac{v_c}{R_4} = 0 \Rightarrow \left(\frac{1}{R_2} + \frac{1}{R_x} + \frac{1}{R_4}\right) v_c - \left(\frac{1}{R_x}\right) v_B = \left(\frac{1}{R_2}\right) v_s$$

or

$$\begin{bmatrix} \left(\frac{1}{R_1} + \frac{1}{R_x} + \frac{1}{R_3}\right) & -\left(\frac{1}{R_x}\right) \\ -\left(\frac{1}{R_x}\right) & \left(\frac{1}{R_2} + \frac{1}{R_x} + \frac{1}{R_4}\right) \end{bmatrix} \begin{bmatrix} v_B \\ v_c \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{R_1}\right) v_s \\ \left(\frac{1}{R_2}\right) v_s \end{bmatrix}$$

(b) given $R_1 = R_4 = 1000\,\Omega$
$R_2 = R_3 = 250\,\Omega$
$R_x = 500\,\Omega$
$v_s = 15$ V

find v_x and i_x

$$\Rightarrow \begin{bmatrix} \left(\frac{1}{1000} + \frac{1}{500} + \frac{1}{250}\right) & -\frac{1}{500} \\ -\frac{1}{500} & \left(\frac{1}{250} + \frac{1}{500} + \frac{1}{1000}\right) \end{bmatrix} \begin{bmatrix} v_B \\ v_c \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{1000}\right)(15) \\ \left(\frac{1}{250}\right)(15) \end{bmatrix}$$

$$\parallel$$

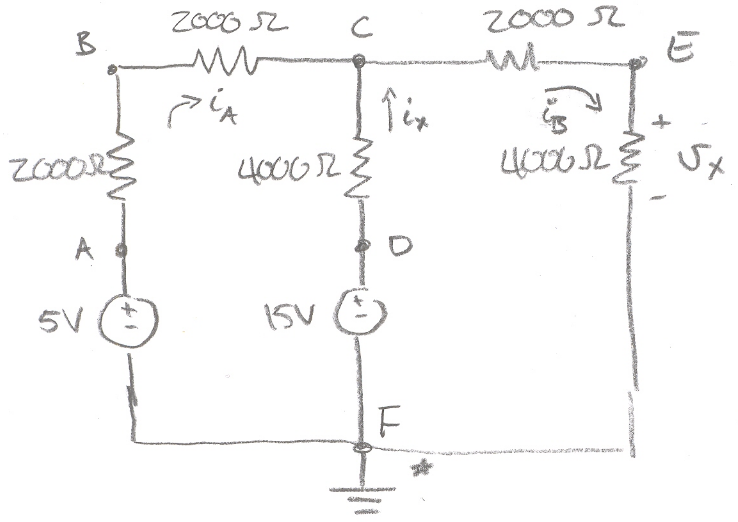
$$\begin{bmatrix} \frac{7}{1000} & -\frac{1}{500} \\ -\frac{1}{500} & \frac{7}{1000} \end{bmatrix} \begin{bmatrix} v_B \\ v_c \end{bmatrix} = \begin{bmatrix} \frac{3}{200} \\ \frac{3}{50} \end{bmatrix} \Rightarrow \begin{bmatrix} v_B \\ v_c \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix} \text{V}$$

and note $v_x = v_B - v_c = 10 - 5 = \boxed{5\text{ V}}$

$$@A: \; i_x = i_1 + i_2 = \frac{v_A - v_B}{R_1} + \frac{v_A - v_c}{R_2} = \frac{v_s - v_B}{R_1} + \frac{v_s - v_c}{R_2}$$

$$= \frac{15-5}{1000} + \frac{15-10}{250} = \frac{10}{1000} + \frac{5}{250} = \boxed{\frac{30}{1000}\text{ A} \text{ or } 30\text{ mA}}$$

3.16



We need to deal w/ (±)'s, choose option 2 and add a ground (⏚)*. @ F

$\Rightarrow \underline{V_A = 5\,V}$ and $\underline{V_D = 15\,V}$

By analysis inspection:

$$@B: \left(\frac{1}{2000}+\frac{1}{2000}\right)V_B - \left(\frac{1}{2000}\right)V_A - \left(\frac{1}{2000}\right)V_C = 0$$

$$@C: \left(\frac{1}{2000}+\frac{1}{4000}+\frac{1}{2000}\right)V_C - \left(\frac{1}{2000}\right)V_B - \left(\frac{1}{4000}\right)V_D - \left(\frac{1}{2000}\right)V_E = 0$$

$$@E: \left(\frac{1}{2000}+\frac{1}{4000}\right)V_E - \left(\frac{1}{2000}\right)V_C - \cancel{\left(\frac{1}{4000}\right)V_F} = 0$$

$\rightarrow V_F = 0$ b/c F is now ⏚

remember V_A and V_D are constants so we move them to the "B" matrix

$$\begin{bmatrix} \left(\frac{1}{2000}+\frac{1}{2000}\right) & -\left(\frac{1}{2000}\right) & 0 \\ -\left(\frac{1}{2000}\right) & \left(\frac{1}{2000}+\frac{1}{4000}+\frac{1}{2000}\right) & -\left(\frac{1}{2000}\right) \\ 0 & -\left(\frac{1}{2000}\right) & \left(\frac{1}{2000}+\frac{1}{4000}\right) \end{bmatrix} \begin{bmatrix} V_B \\ V_C \\ V_E \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{2000}\right)V_A \\ \left(\frac{1}{4000}\right)V_D \\ 0 \end{bmatrix}$$

$$\equiv$$

$$\begin{bmatrix} \frac{1}{1000} & -\frac{1}{2000} & 0 \\ -\frac{1}{2000} & \frac{1}{800} & -\frac{1}{2000} \\ 0 & -\frac{1}{2000} & \frac{3}{4000} \end{bmatrix} \begin{bmatrix} V_B \\ V_C \\ V_E \end{bmatrix} = \begin{bmatrix} \frac{5}{2000} \\ \frac{15}{4000} \\ 0 \end{bmatrix}$$

solving this system:

$$\begin{bmatrix} v_B \\ v_C \\ v_E \end{bmatrix} = \begin{bmatrix} 6.25 \\ 7.5 \\ 5 \end{bmatrix}$$

B/c we used a gnd. at F $\Rightarrow$ $v_x = v_E - v_F = v_E$

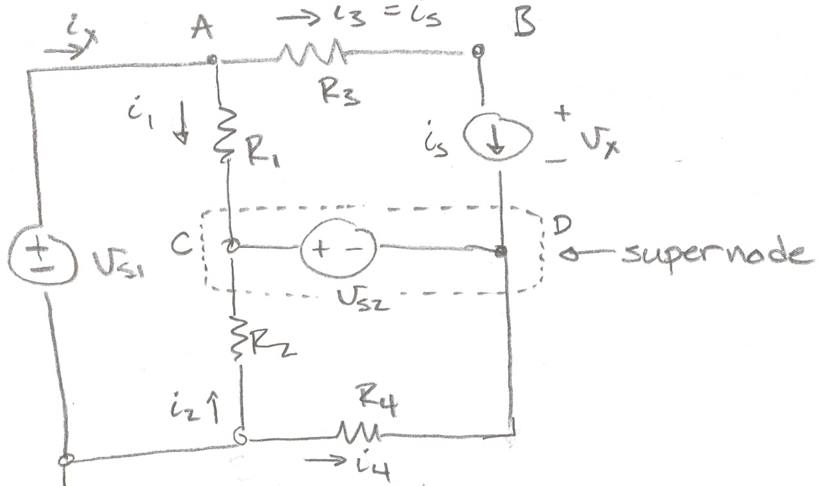
$\Rightarrow \boxed{v_x = 5\text{ V}}$

Consider Ohm's Law for the middle 4 kΩ Resistor:

$v = iR \Rightarrow (v_D - v_C) = i_x (4000)$

$$i_x = \frac{15 - 7.5}{4000} = \frac{7.5}{4000} = \boxed{\frac{15}{8}\text{ mA}}$$

3.13



$\Rightarrow \quad v_A = v_{s1}$

supernode equation: $i_1 + i_2 + i_s + i_4 = 0$

$$= \frac{1}{R_1}(v_A - v_C) + \frac{1}{R_2}(0 - v_C) + i_s + \frac{1}{R_4}(0 - v_D) = 0$$

and $v_C - v_D = v_{s2}$

KCL @ B: $\frac{1}{R_3}(v_A - v_B) = i_s$

which gives us 3 equations:

$$\frac{1}{R_1}v_A - \left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_C - \frac{1}{R_4}v_D = -i_s$$

$$v_C - v_D = v_{s2}$$

$$\frac{1}{R_3}v_A - \frac{1}{R_3}v_B = i_s$$

and, b/c of the $\frac{|}{=}$, $v_A = v_{s1}$

$$\boxed{\begin{aligned} -\left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_C - \frac{1}{R_4}v_D &= -i_s - \frac{1}{R_1}v_{s1} \\ v_C - v_D &= v_{s2} \\ \frac{1}{R_3}v_B &= \frac{1}{R_3}v_{s1} - i_s \end{aligned}}$$

which gives us 3 eq for 3 unknowns (V_B, V_C, V_D) and can be written as

$$\boxed{\begin{bmatrix} 0 & -\left(\frac{1}{R_1}+\frac{1}{R_2}\right) & -\frac{1}{R_4} \\ 0 & 1 & -1 \\ \frac{1}{R_3} & 0 & 0 \end{bmatrix} \begin{bmatrix} V_B \\ V_C \\ V_D \end{bmatrix} = \begin{bmatrix} -i_s - \frac{1}{R_1} V_{S1} \\ V_{S2} \\ \frac{1}{R_3} V_{S1} - i_s \end{bmatrix}}$$

(b) when $R_1 = R_2 = 10{,}000\,\Omega$

$R_3 = 2{,}000\,\Omega$

$R_4 = 1{,}000\,\Omega$

$i_s = 2.5 \times 10^{-3}\,A$

$V_{S1} = 12\,V$

$V_{S2} = 0.5\,V$

$$\Rightarrow V_B = V_{S1} - R_3 i_s = 12 - (2.5)(2) = 12 - 5 = \boxed{7\,V = V_B}$$

and

$$\begin{bmatrix} -\frac{2}{10} & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} V_C \\ V_D \end{bmatrix} = \begin{bmatrix} -3.7 \\ 0.5 \end{bmatrix}$$

$$\boxed{\Rightarrow \begin{bmatrix} V_C \\ V_D \end{bmatrix} = \begin{bmatrix} 3.5 \\ 3 \end{bmatrix}}$$

(b)

$$\Rightarrow V_x = V_B - V_D = 7 - 3 \Rightarrow \boxed{V_x = 4\,V}$$

KCL @ A: $i_x = i_1 + i_s = \frac{1}{R_1}(V_A - V_C) + i_s = \frac{1}{10000}(12 - 3.5) + 2.5 \times 10^{-3}$

$$\Rightarrow \boxed{i_x = 3.35\,mA}$$

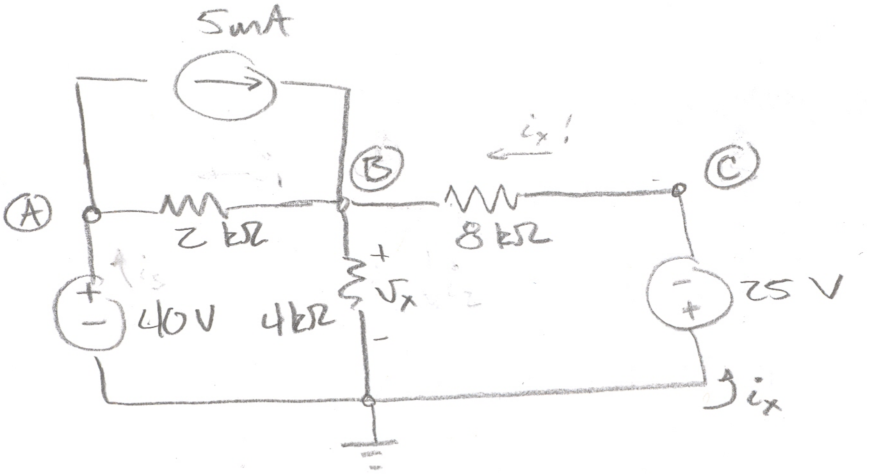
3.13 cont.

(c) the power supplied by v_{s1}

$P = vi \Rightarrow P = (v_{s1})(i_x) = (12\,V)(3.35 \times 10^{-3})$

$$\boxed{\begin{aligned} &= 0.0402 W \\ P &= 40.2\, mW \end{aligned}}$$

3.15



w/ ⏚:

$\Rightarrow V_A = 40\text{ V}$

$V_C = -25\text{ V}$

(Method 2)

Need only write 1 nodal equation

$$\boxed{\left(\frac{1}{2000} + \frac{1}{4000} + \frac{1}{8000}\right)V_B - \left(\frac{1}{2000}\right)V_A - \left(\frac{1}{8000}\right)V_C = (5\times10^{-3})}$$

$$\left(\frac{7}{8000}\right)V_B = \left(\frac{1}{2000}\right)(40) + \left(\frac{1}{8000}\right)(-25) + (5\times10^{-3}) \qquad \frac{5}{1000}$$

$$V_B\left(\frac{7}{8000}\right) = \frac{160}{8000} - \frac{25}{8000} + \frac{40}{8000}$$

$$\Rightarrow V_B = \frac{175}{7} \Rightarrow \boxed{V_B = 25}\text{ V}$$

(b)

$$V_x = V_B = 0 \Rightarrow \boxed{V_x = V_B = 25\text{ V}}$$

$$i_x = \frac{V_C - V_B}{8000} = \frac{-25-25}{8000} = \frac{-50}{8000} = \boxed{-\frac{25}{4}\text{ mA}}$$

(c) $P = \frac{V^2}{R}$ $\qquad P_T = \sum_{n=1}^{3} P_n$

$$P_T = \frac{(V_B - V_A)^2}{2000} + \frac{(V_B - V_C)^2}{8000} + \frac{(V_B - 0)^2}{4000} = \frac{225}{2000} + \frac{2500}{8000} + \frac{225}{4000}$$

$$\boxed{= 581.25\text{ mV or } 0.581\text{ W}}$$