## MAE 140 - Linear Circuits - Fall 2008

## Final

## Instructions

1) This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities.
2) You have 170 minutes.
3) On the questions for which we have given the answers, please provide detailed derivations.

## Guestion 1 - Equivalent Circuits



Figure 1: Circuit for Question 1 (i)


Figure 2: Circuit for Question 1 (ii)

Part (i) [5 marks] Assuming zero initial conditions, find the impedance equivalent to the circuit in Figure 1 as seen from terminals A and B. The answer should be given as a ratio of two polynomials.

Part (ii) [5 marks] Assuming that the initial condition of the inductor is as indicated in the diagram, redraw the circuit shown in Figure 2 in the $s$-domain. Then use source transformations to find the $s$-domain Thevenin equivalent of this circuit as seen from terminals A and B.

## Question 2 - Laplace Domain Circuit Analysis



Figure 3: RC circuit for Laplace analysis.

Part (i) [3 marks] Consider the circuit depicted in Figure 3. The voltage source is constant. The switch is kept in position $A$ for a very long time. At $t=0$ it is moved to position $B$. Show that the initial capacitor voltage is given by

$$
v_{C}\left(0^{-}\right)=6 V .
$$

[Show your work.]
Part (ii) [2 marks] Use this initial condition to transform the right side of the circuit (the one that contains the capacitor) into the $s$-domain for $t \geq 0$. Use an equivalent model for the capacitor in which the initial condition appears as a voltage source. [Show your work.]

Part (iii) [3 marks] Use $s$-domain circuit analysis and inverse Laplace transforms to show that the capacitor voltage satisfies,

$$
v_{C}(t)=6 e^{-t /(R C)} u(t)
$$

[Show your work.]
Part (iv) [2 marks +2 bonus marks] Using what you know about $s$-domain analysis, compute the voltage and current across the inductor (in the left side of the circuit) when the switch is moved from $A$ to $B$. Is this a safe circuit?

## Question 3 - Active Filter Analysis and Design



Figure 4: Filter for analysis

Part (i) [2 marks] Assuming zero initial conditions, transform the circuit in Figure 4 into the $s$-domain and compute the transfer function from $V_{i}(s)$ to $V_{o}(s)$.

Part (ii) [3 marks] Showing your reasoning, determine the nature of this filter's frequency response. Further, determine the gain of the filter and its cut-off frequency.

Part (iii) [2 marks] If $C_{1}=C_{2}=100 \mathrm{nF}$, find the value of $R$ so that the cutoff frequency is 5 KHz .
Part (iv) [3 marks] Using $C_{1}=C_{2}=100 \mathrm{nF}$ and the value of $R$ you computed in the previous item, design a second stage op-amp circuit that if connected to $V_{o}(t)$ will make the overal filter gain (from $V_{i}(s)$ to the output of the second stage) be equal to 1 .
[Attention: the gain equal to -1 is not correct!]

## Question 4 - Nodal and Mesh Analysis



Figure 5: Nodal and Mesh Analysis Circuit

Part (i) [5 marks] Formulate node-voltage equations in the $s$-domain for the circuit in Figure 5. Use the reference node and other labels as shown in the figure. Do not assume zero initial conditions! Transform initial conditions on the capacitor and on the inductor into current sources.

Part (ii) [5 marks] Formulate mesh-current equations in the $s$-domain for the circuit in Figure 5 . Use the currents shown in the figure. Do not assume zero initial conditions! Transform initial conditions on the capacitor and on the inductor into voltage sources.

## Guestion 5 - Op-Amp Analysis and Application



Figure 6: Op-Amp Circuit

Part (i) [5 marks] Using the fundamental op-amp relationships, find the transfer function $T(s)$ from $V_{i}(s)$ to $V_{o}(s)$ in Figure 6. Assume zero initial conditions.

Part (ii) [5 marks] Compute the magnitude and phase of the transfer function $T(j \omega)$, that is $|T(j \omega)|$ and $\angle T(j \omega)$. Describe in words what is the function of this circuit.
[Hint: Use the frequency response of the circuit to see what happens to an input of the form $V_{i}(t)=A \cos (\omega t)$.]

Part (iii) [2 bonus marks] Can you describe a possible application for this circuit?

