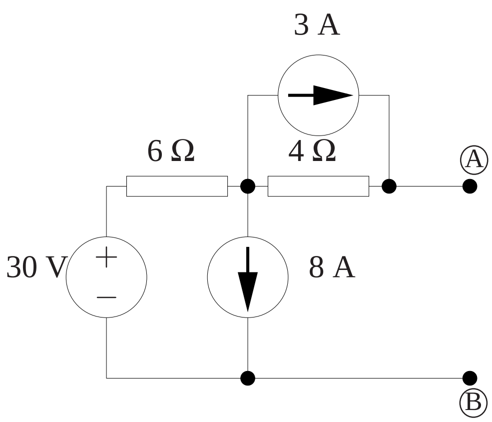
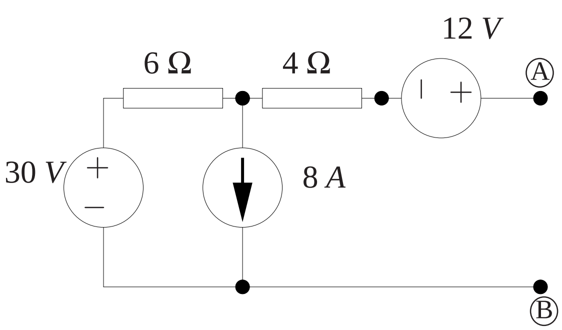
MAE 140 – Linear Circuits – Fall 2007
Midterm – Solution

Question 1 [Thevenin equivalent]
Regarding the following circuit:

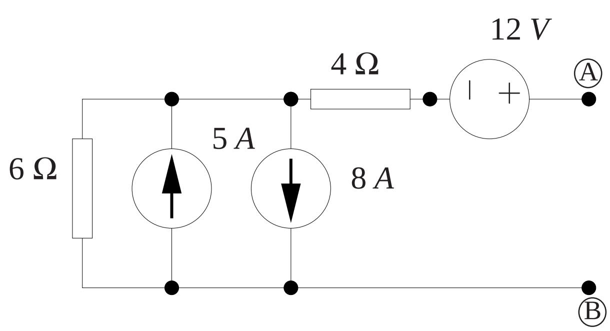


a) [4 marks] Determine the Thevenin equivalent as seen from terminals Ⓐ and Ⓑ using source transformations only.

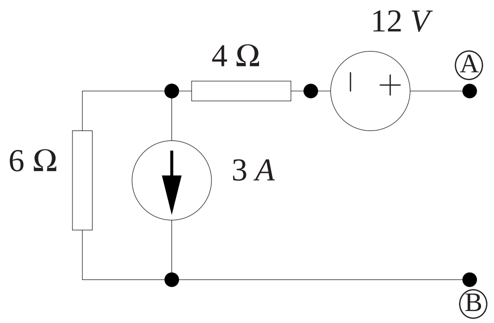
Transform the 3 *A current source in parallel with the* 4 Ω *resistor.*



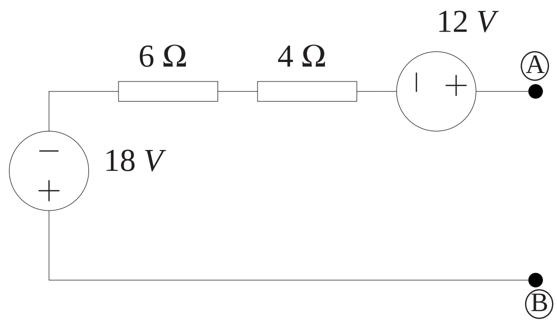
Transform the 30 *V voltage source in series with the* 6 Ω *resistor.*



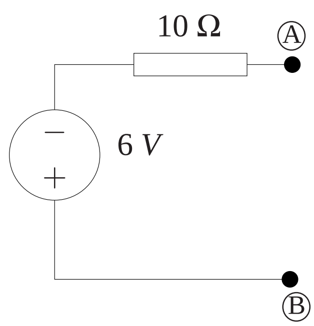
Combine the 5 *A and the* 8 *A sources of current into a single* 3 *A source of current.*



Transform the 3 *A current source in parallel with the* 6 Ω *resistor.*



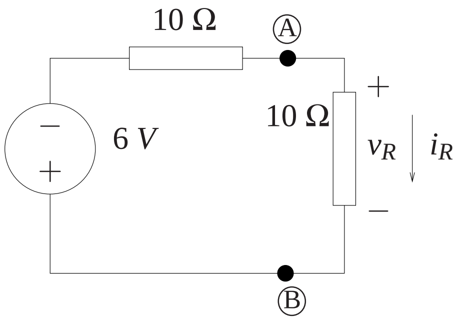
Combine the 12 *V and the* 18 *V sources of voltage into a single* 6 *V source of voltage and associate the resistor to obtain the Thevenin equivalent circuit.*



REMARK: You get 1 mark per correct transformation up to a max of 4 marks.

b) [2 marks] How much power would be absorbed by a 10 Ω resistor connected between terminals Ⓐ and Ⓑ?

Plug the resistor in the Thevenin equivalent.

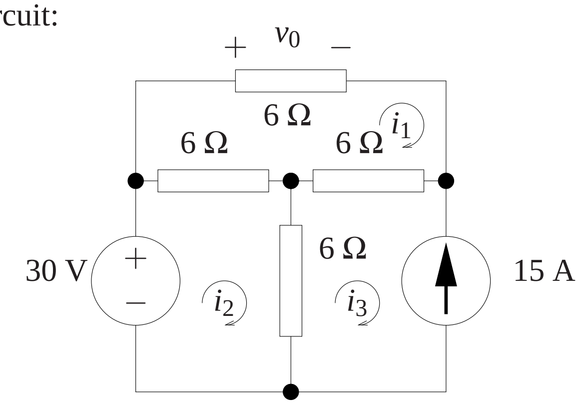


Use the voltage divider formula to conclude that

$$v_R = \frac{10}{10+10}(-6) = -3 \text{ V}, \qquad i_R = \frac{v_R}{10} = -\frac{3}{10} \text{ A}$$

so that the power absorbed by the resistor is $p_R = v_R\, i_R = 9/10$ *W.*

Question 2 [Superposition / Mesh analysis]
Regarding the following circuit:



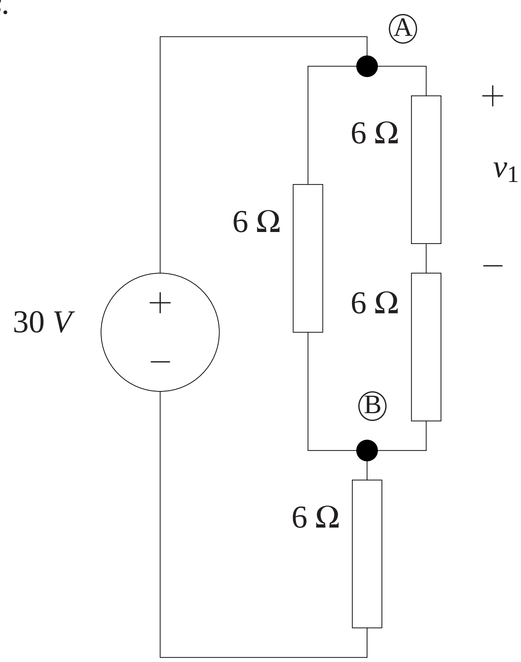
a) [4 marks] Find v_0 using superposition.

We will use superposition to compute

$$v_0 = v_1 + v_2$$

where v_1 is the solution when the current source is zero and v_2 is he solution when the voltage source is zero.

First case: Zero the current source, that is, replace the current source by an open circuit, and redraw the circuit as follows.



Voltage v_1 can now be found using the voltage divider formula

$$v_1 = \frac{6}{6+6}(v_A - v_B) = \frac{1}{2}(v_A - v_B)$$

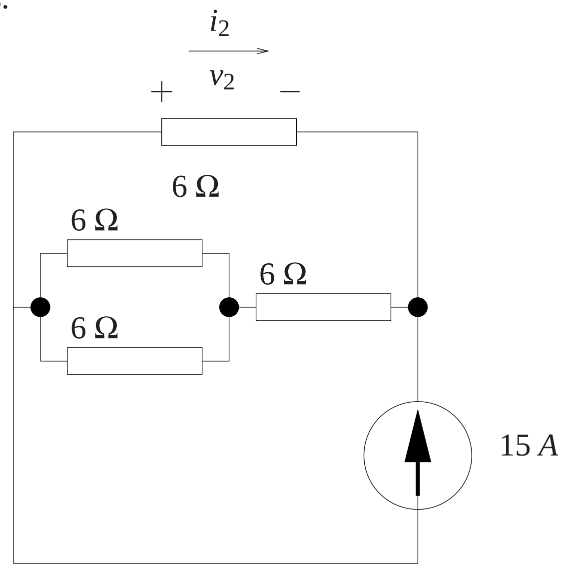
and

$$v_A - v_B = \frac{6//(6+6)}{6+[6//(6+6)]}30 = \frac{4}{6+4}30 = 12 \text{ V}$$

where we used the fact that $6//(6+6) = 6 \times 12/(6+12) = 4$. *That is*

$$v_1 = \frac{1}{2}(v_A - v_B) = 6 \text{ V}.$$

Second case: Zero the voltage source, that is, replace the voltage source by a short circuit, and redraw the circuit as follows.



Voltage v_2 *is*

$$v_2 = 6\ i_2$$

where can now be found using the current divider formula. Indeed

$$i_2 = \frac{\frac{1}{6}}{\frac{1}{6} + \frac{1}{6+(6//6)}}(-15) \text{ A} = -\frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{9}}15 \text{ A} = -\frac{3}{5}15 \text{ A} = -9 \text{ A}$$

where we used the fact that $6 + (6//6) = 6 + 3 = 9$. *That is*

$$v_2 = 6\ i_2 = -54V.$$

The final result is then

$$v_0 = v_1 + v_2 = 6 - 54 = -48 \text{ V}.$$

REMARK: Two marks per case: one for correct setup, another for correct answer.

b) [4 marks] Formulate mesh-current equations for the circuit. Clearly indicate the equations to be solved and the unknowns. Use the mesh currents indicated in the drawing.

Because the 15 *A source of current is on a single mesh we have*

$$i_3 = -15 \text{ A}. \tag{1}$$

Writting the mesh-current equations by inspection for meshes 1 *and* 2 *we have*

$$\begin{bmatrix} 6+6+6 & -6 & -6 \\ -6 & 6+6 & -6 \end{bmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 30 \end{pmatrix}$$

Using (1) *we obtain the final form of the mesh-current equations*

$$\begin{bmatrix} 18 & -6 \\ -6 & 12 \end{bmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} -90 \\ -60 \end{pmatrix}, \qquad i_3 = -15 \text{ A}.$$

REMARK: You get 1 mark if you stated eq. (1) *correctly. 1 mark if you stated mesh-current equation for mesh 1 correctly. One more if mesh 2 is correct. One more mark if indicated the final equations to be solved, either by substituting i_3 or clearly indicating that meshes 1 and 2 must be solved in the presence of this constraint.*

c) [2 marks] Use the mesh-current equations to find v_0.

First note that

$$v_0 = 6\, i_1. \tag{2}$$

Then determine i_1 by solving

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \frac{1}{6} \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}^{-1} \begin{pmatrix} -90 \\ -60 \end{pmatrix} = \frac{1}{30} \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{pmatrix} -90 \\ -60 \end{pmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{pmatrix} -3 \\ -2 \end{pmatrix} = \begin{pmatrix} -8 \\ -9 \end{pmatrix}.$$

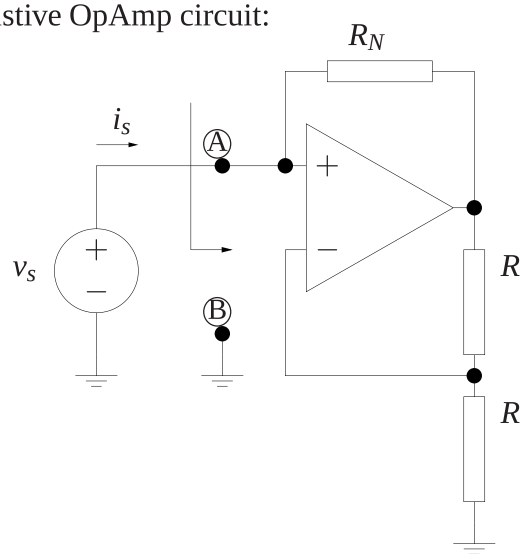
Hence

$$v_0 = 6\, i_1 = -48 \text{ V}.$$

REMARK: You get 1 mark if you stated eq. (2) *correctly. 1 more mark if you solved for i_1 correctly.*

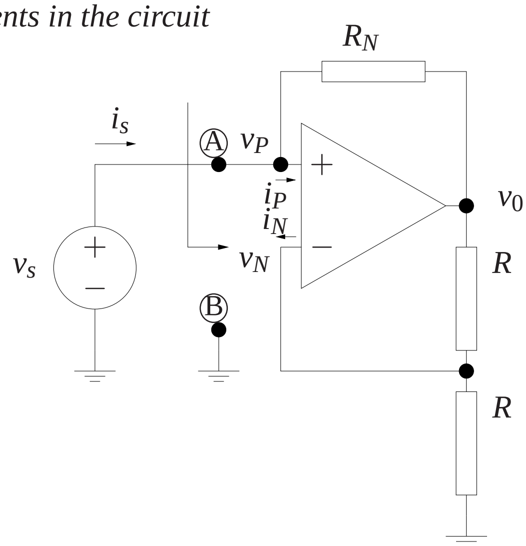
Question 3 [OpAmp Circuit Analysis]

Regarding the following resistive OpAmp circuit:



a) [4 marks] Show that the current $i_s = -v_s/R_N$.

Label voltages and currents in the circuit



Because $i_N = 0$, the resistors R form a voltage divider

$$v_N = \frac{R}{R+R}v_0 = \frac{1}{2}v_0.$$

KCL at node Ⓐ implies

$$i_s = i_P + \frac{(v_P - v_0)}{R_N}$$

These two equations combined with $v_P = v_N = v_s$ and $i_P = 0$. yield

$$i_s = \frac{(v_P - v_0)}{R_N} = \frac{(v_s - 2v_s)}{R_N} = -\frac{v_s}{R_N}$$

as desired.

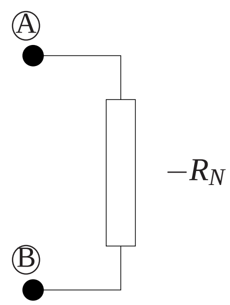
REMARK: You get one mark for stating v_N correctly. One more for stating KCL @ node Ⓐ. One for using $v_P = v_N = v_s$, $i_p = i_n = 0$. One for getting the right answer.

b) [2 marks] Draw the equivalent circuit as seen from terminals Ⓐ and Ⓑ in the direction indicated by the arrow. Explain why this circuit is called a "negative resistance"?
Hint: Recall that an equivalent circuit is one which has the same *v*-*i* characteristic.

From part a)

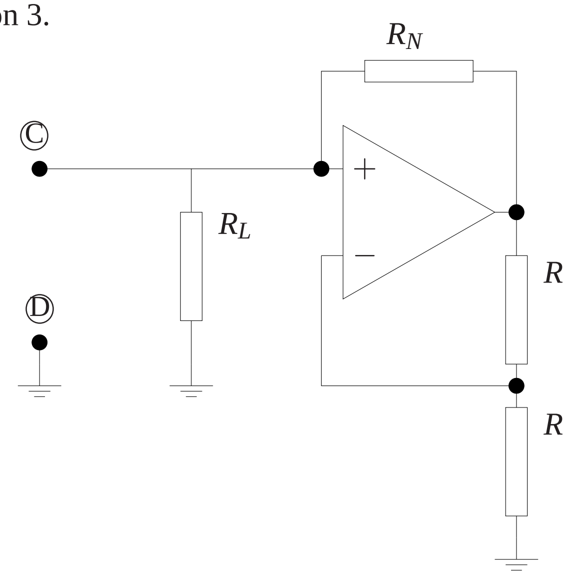
$$v_s = R_s\, i_s, \qquad R_s = -R_N.$$

The above is Ohm's Law for a "negative resistance" R_N. Hence the name of the circuit. The equivalent is simply

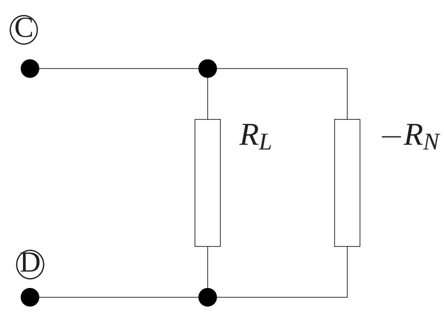


Question 4 [OpAmp Circuit Analysis (bonus)]

[2 marks] Consider the connection of the "negative resistance" circuit of Question 3 in parallel with a load R_L as seen in the next circuit. What is the equivalent circuit as seen from terminals Ⓒ and Ⓓ? Draw the circuit. What happens when $R_L = R_N$?
Hint: Make use of Question 3.



Use the equivalent obtained in Question 3



from where an equivalent can be obtained after associating R_L and $-R_N$ in parallel

$$R_{\text{EQ}} = R_L // - R_N = \frac{1}{\frac{1}{R_L} - \frac{1}{R_N}} = \frac{R_L R_N}{R_N - R_L}$$

When $R_N \rightarrow R_L$ then $R_{\text{EQ}} \rightarrow \infty$, that is, the equivalent becomes an open circuit.

REMARK: One mark for R_{EQ} and one for the limit case.