MAE 140 – Linear Circuits – Fall 2007
Final

Instructions

1) This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities.

2) You have 170 minutes.

3) On the questions for which we have given the answers, please provide detailed derivations.

Question 1 — Equivalent Circuits

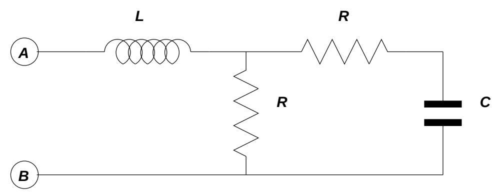

Figure 1: Circuit for Question 1 (i)

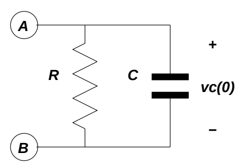

Figure 2: Circuit for Question 1 (ii)

Part (i) [5 marks] Assuming zero initial conditions, find the impedance equivalent to the circuit in Figure 1 as seen from terminals A and B. The answer should be given as a ratio of two polynomials.

Part (ii) [5 marks] Assuming that the initial condition of the capacitor is as indicated in the diagram, redraw the circuit shown in Figure 2 in s-domain. Then use source transformations to find the s-domain Thévenin equivalent to the circuit as seen from terminals A and B.

Question 2 — Laplace domain circuit analysis

Part (i) [3 marks] Consider the circuit depicted in Figure 3. The current source i_s is a constant current supply, which is kept in place for a very long time until the switch is opened at time $t = 0$. Show that the initial capacitor voltage is given by

$$v_C(0^-) = i_s R_2.$$

[Show your working.]

Part (ii) [2 mark] Use this initial condition to transform the circuit into the s-domain for time $t \geq 0$.
[The symbolic formulæ are shown in the text on page 449.]
[Show your working.]

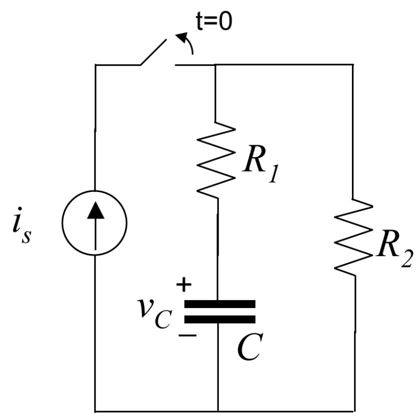

Figure 3: RC circuit for Laplace analysis.

Part (iii) [5 marks] Use s-domain circuit analysis and inverse Laplace transforms to show that the capacitor voltage satisfies,

$$v_C(t) = i_s R_2 \exp\left(\frac{-t}{(R_1 + R_2)C}\right) u(t).$$

[Show your working.]

Question 3 — Active Filter Analysis and Design

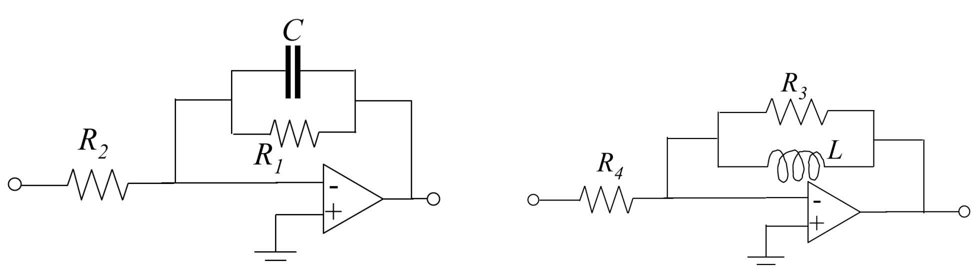

Figure 4: (a) RC parallel op-amp circuit. (b) RL parallel op-amp circuit.

Part (i) [3 marks] Show that the the transfer functions of the op-amp circuits in Figure 4 are given by

$$T_{RC}(s) = \frac{-1}{R_2} \times \frac{\frac{1}{C}}{s + \frac{1}{R_1 C}}, \qquad T_{RL}(s) = \frac{-1}{R_4} \times \frac{sR_3}{s + \frac{R_3}{L}}.$$

Part (ii) [5 marks] Showing your reasoning, determine the nature of these two filters' frequency responses. Further, determine the gain of the filters and their cut-off frequencies.

If $C = 100\text{nF}$, find R values so that the RC filter has cutoff frequency 5KHz and gain 5.

If $L = 10\text{mH}$, find R values so that the RL filter has cutoff frequency 1KHz and gain 5.

Part (iii) [2 marks] You are required to build a bandpass filter with;

- lower cut-off frequency of 1KHz,
- upper cut-off frequency of 5KHz, and
- passband gain of 25.

Explain whether the two filters above can be used to achieve this design specification. If so, then show how. If not, then suggest how the above calculations might be changed to yield such a design.

Question 4 — Op-Amp Analysis and Application

The figure below shows a circuit known as a *gyrator* or *positive impedance inverter*. It is primarily used in active circuit design to implement inductors, which are difficult to manufacture to specification and within a small volume.

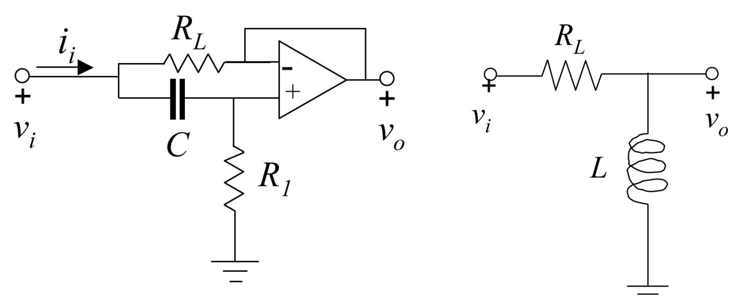

Figure 5: (a) Gyrator circuit. (b) RL-circuit.

Part (i) [5 marks] Using the fundamental op-amp relationships show that the voltage transfer function in Figure 5(a) is given by.

$$T_v(s) = \frac{V_o(s)}{V_i(s)} = \frac{s}{s + \frac{1}{R_1 C}}.$$

Part (ii) [2 marks] Perform the same calculation for the RL-circuit shown in Figure 5(b) to show that its transfer function is

$$T_v(s) = \frac{sL}{sL + R_L}.$$

Thus, the gyrator circuit effectively implements a circuit involving an inductor by using a capacitor and an op-amp. Determine the equivalent inductor value L in the RL-circuit in terms of the element values, R_1, R_L, C, in the gyrator circuit

Part (iii) [3 marks] Show that the equivalent impedance of the gyrator circuit seen from the input, v_i, is given by

$$Z_{eq}(s) = \frac{sR_1CR_L + R_L}{sR_LC + 1},$$

while, for the RL-circuit it is given by

$$Z_{eq}(s) = sL + R_L.$$

Part (iv) [3 mark, Bonus] Describe the limitations in using this substitute circuit to realize an inductor in an application.

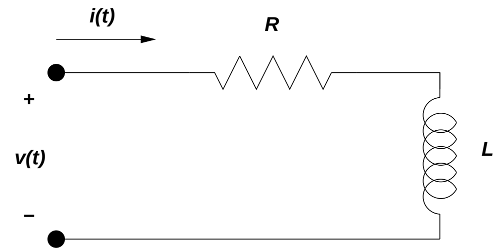

Figure 6: Motor equivalent RL circuit

Question 5 — Power Factor Compensation

Part (i) [3 marks] A large single phase alternating current (AC) induction electric motor can be represented by the RL circuit in Figure 6. Assuming zero initial conditions, find the equivalent impedance of the motor, $Z(s)$, and the equivalent admitance, $Y(s) = Z(s)^{-1}$. Compute $Y(j\omega)$, its real and imaginary parts. This is the admittance of the motor at the AC supply frequency of ω radians per second.

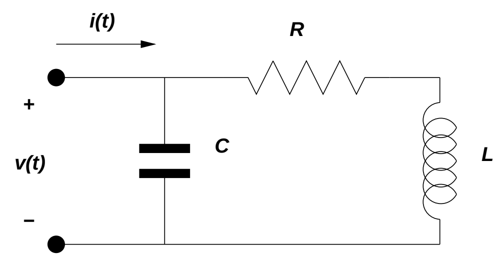

Figure 7: Motor equivalent RL circuit with capacitor C in parallel

Part (ii) [3 marks] Now consider the circuit in Figure 7, in which a capacitor is added in parallel to the motor. Assuming zero initial conditions, find the new equivalent admitance $Y_C(s)$. Compute $Y_C(j\omega)$, its real and imaginary parts. Use this to show that C may be chosen to make the imaginary part of the admittance zero at the supply frequency ω.

Part (iii) [4 marks] If the supply voltage to the motor is $v(t) = V\cos(\omega t)$, write the corresponding sinusoidal steady state current $i(t)$ drawn by the machine with admittance $Y(s)$. Do this in the time domain as a $\cos$ function.

Compute the power $p(t) = v(t)i(t)$ and use the relationship $\cos(A)\cos(B) = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$, to derive an expression for the power absorbed by the motor as a function of time. Show that the average value of this power contains a term in $\mathrm{Re}(Y(j\omega)) = |Y(j\omega)|\cos[\angle(Y(j\omega))]$.

If a capacitor is chosen as in the previous item, does the introduction of this capacitor in parallel with the motor change the average power absorbed by the motor? Why?

Part (iv) [3 mark, Bonus] Your power supplier (SDG&E and the like) will likely *demand* that you insert a capacitor C as above. They will enforce this by measuring $\cos[\angle(Y(j\omega))]$, aka *power factor*, and gently asking you to keep this number as close to one as possible, like you did by choosing C above. Why do you think these nice people would do that?