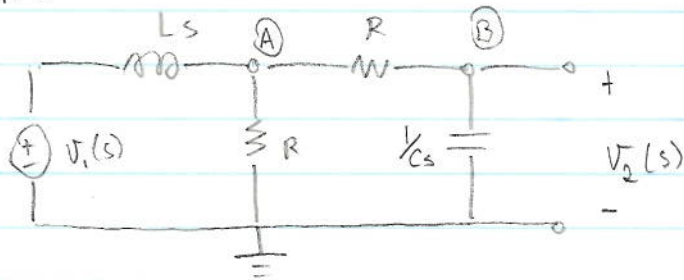


11/5



$$V_B(s) = V_2(s)$$

KCL A:

$$\frac{1}{Ls} (V_A(s) - V_1(s)) + \frac{1}{R} V_A(s) + \frac{1}{R} (V_A(s) - V_2(s)) = 0$$

KCL B:

$$\frac{1}{R} (V_2(s) - V_A(s)) + Cs V_2(s) = 0$$

$$\therefore V_2(s) \left( \frac{1}{R} + sC \right) = \frac{1}{R} V_A(s)$$

$$V_A(s) = V_2(s) (1 + sRC)$$

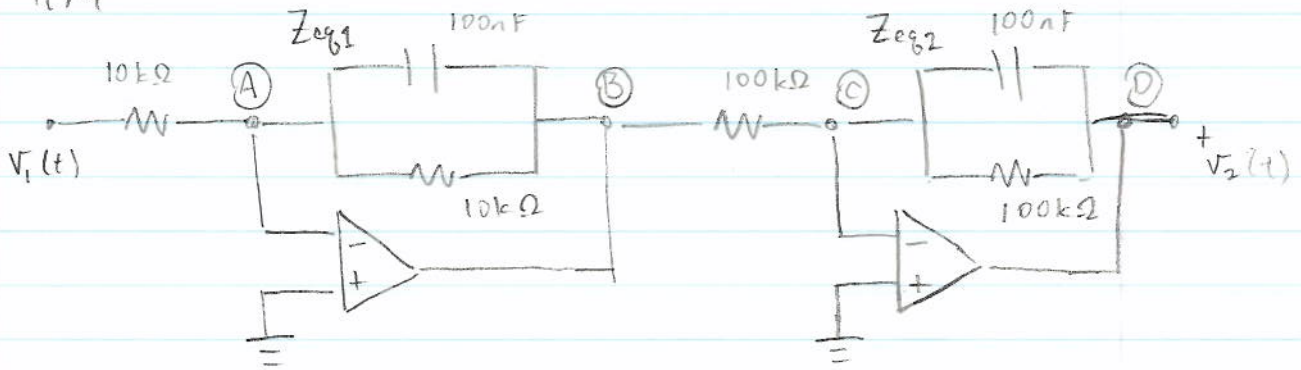
$$\text{KCL A} \Rightarrow V_A(s) \left( \frac{1}{Ls} + \frac{2}{R} \right) - \frac{V_2(s)}{R} = \frac{V_1(s)}{Ls}$$

$$V_2(s) \left[ (1 + sRC) \left( \frac{1}{Ls} + \frac{2}{R} \right) - \frac{1}{R} \right] = \frac{V_1(s)}{Ls}$$

$$V_2(s) \left[ \frac{R(1 + sRC) + 2Ls(1 + sRC) - Ls}{sRL} \right] = \frac{V_1(s)}{sL}$$

$$V_2(s) = V_1(s) \cdot \frac{R}{\underbrace{s^2(2RLC) + s(R^2C + L) + R}} = T(s)$$

11/9



$$10 \text{ k}\Omega \parallel 100 \text{ nF}$$

$$\Rightarrow Z_{eq1} = \frac{10^4 \times \frac{1}{10^{-7} s}}{10^4 + \frac{1}{10^{-7} s}} = \frac{10^{11}}{10^4 s + 10^7} = \frac{10^7}{s + 1000}$$

$$100 \text{ k}\Omega \parallel 100 \text{ nF}$$

$$\Rightarrow Z_{eq2} = \frac{10^7}{s + 100}$$

$$V_A = V_C = 0$$

KCL @ A :

$$V_1(s) \times \frac{1}{10^4} + V_B(s) \times \left( \frac{10^7}{s + 1000} \right)^{-1} = 0$$

$$\therefore V_B(s) = -V_1(s) \times \frac{10^3}{s + 1000}$$

KCL @ C :

$$\frac{1}{10^5} V_B(s) + V_2(s) \left( \frac{10^7}{s + 100} \right)^{-1} = 0$$

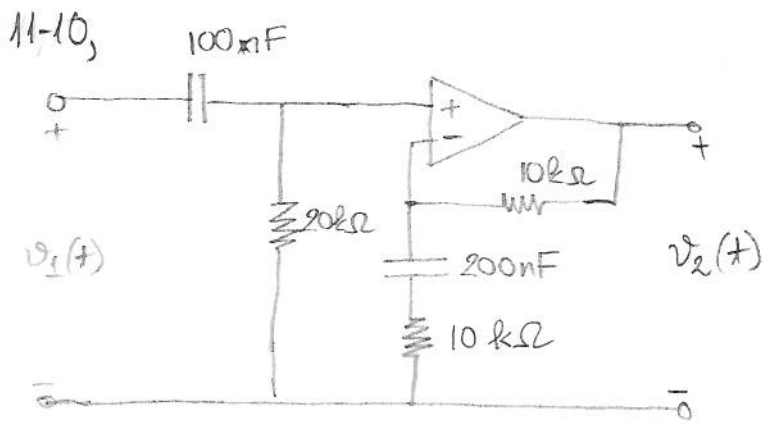
$$\Rightarrow V_2(s) = -\frac{V_B(s)}{10^5} \times \frac{10^7}{s + 100}$$

$$= V_1(s) \times \frac{10^5}{(s + 100)(s + 1000)}$$

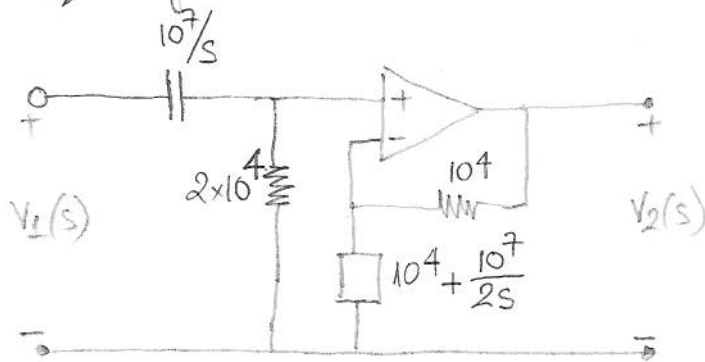
$$\underbrace{\hspace{10em}}_{T(s)}$$

Zeros :  $\phi$

Poles : -100, -1000



→ transform into s-domain and simplify circuit



$$\rightarrow V_N = \frac{10^4 + \frac{10^7}{2s}}{10^4 + 10^4 + \frac{10^7}{2s}} V_2(s) = \frac{2s + 10^3}{4s + 10^3} V_2(s)$$

$$\rightarrow V_P = \frac{2 \times 10^4}{2 \times 10^4 + \frac{10^7}{s}} V_1(s) = \frac{2s}{2s + 10^3} V_1(s)$$

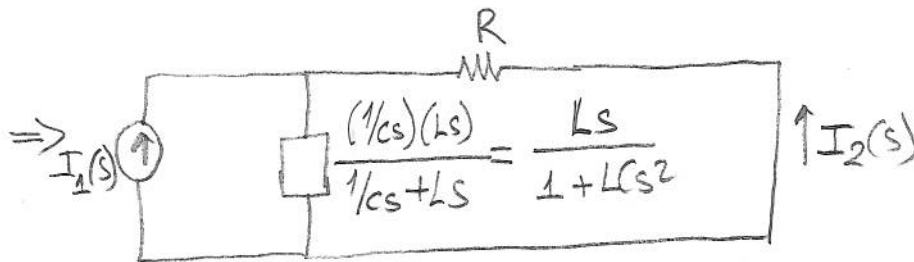
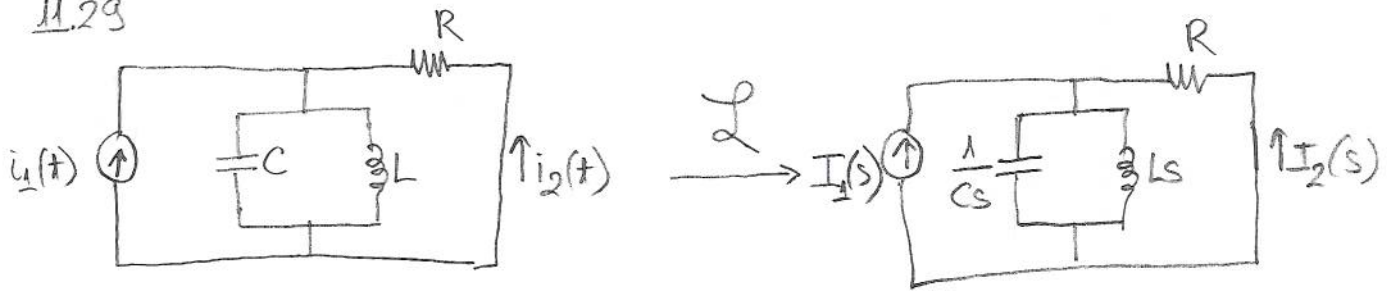
$$\rightarrow V_N = V_P \Rightarrow \frac{V_2(s)}{V_1(s)} = \frac{2s}{(2s + 10^3)} \times \frac{4s + 10^3}{(2s + 10^3)} = \boxed{\frac{2s(4s + 10^3)}{(2s + 10^3)^2}}$$

$$\rightarrow \text{poles: } 2s + 10^3 = 0 \Rightarrow \boxed{s = -500 \text{ (double)}}$$

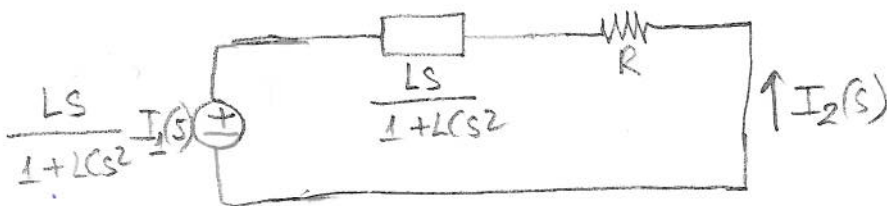
$$\rightarrow \text{zeros: } \rightarrow \boxed{s = 0}$$

$$\rightarrow 4s + 10^3 = 0 \Rightarrow \boxed{s = -250}$$

11.29



$\rightarrow i_1(t) = 5 \cos(1000t) \text{ mA}$   
 $\rightarrow R = 10^3 \Omega$   
 $\rightarrow L = 2 \text{ H}$   
 $\rightarrow C = 5 \times 10^{-7} \text{ F}$



$$\Rightarrow I_2(s) = \frac{-\frac{Ls}{1+LCs^2} I_1(s)}{R + \frac{Ls}{1+LCs^2}} \Rightarrow T(s) = \frac{I_2(s)}{I_1(s)} = \frac{-s/RC}{s^2 + s/RC + 1/LC}$$

$$\Rightarrow T(j\omega) = \frac{-j\omega/RC}{-\omega^2 + j\omega/RC + 1/LC} \Rightarrow |T(j\omega)| = \frac{\omega/RC}{\sqrt{(\frac{1}{LC} - \omega^2)^2 + (\frac{\omega}{RC})^2}}$$

$\rightarrow$  for  $i_1(t) = 5 \cos(1000t) \Rightarrow \omega = 1000 \text{ rad/s}$

$$\Rightarrow |T(1000j)| = 1.0 \text{ and } \angle T(1000j) = \frac{-\pi}{2} - \tan^{-1} \left[ \frac{\omega/RC}{1/LC - \omega^2} \right] = -\pi \text{ rad}$$

$$\Rightarrow i_2(t) = (5)(1) \cos[1000t - 1.5708] = \boxed{1.5 \cos[1000t + \pi]} \text{ mA}$$

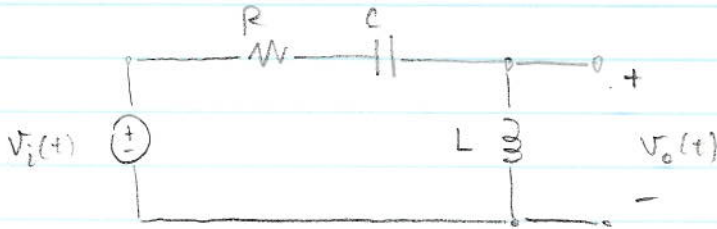
$\rightarrow$  for  $i_1(t) = 10 \cos(2000t) \Rightarrow \omega = 2000 \text{ rad/s}$

$$\Rightarrow |T(2000j)| = 0.8 \text{ and } \angle T(2000j) = -3.79 \text{ rad}$$

$$\Rightarrow i_2(t) = (10)(0.8) \cos[2000t - 1.7296] = \boxed{8 \cos[2000t + 3.79]} \text{ mA}$$

11/54

$$T(s) = \frac{s^2}{(s+1000)(s+4000)} = \frac{s^2}{s^2 + 5000s + 4 \times 10^6}$$



$$V_o(s) = \frac{sL}{R + \frac{1}{sC} + sL} V_i(s)$$

$$V_o(s) = \frac{s^2 LC}{s^2 LC + sRC + 1} = \frac{s^2}{s^2 + s\left(\frac{R}{L}\right) + \frac{1}{LC}}$$

$$\Rightarrow \frac{R}{L} = 5000$$

$$\frac{1}{LC} = 4 \times 10^6$$

$L = 5 \times 10^{-2} \text{ H}$ $\Rightarrow C = 5 \times 10^{-6} \text{ F}$ $\Rightarrow R = 250 \text{ } \Omega$
---



11/59 a) In the first circuit:

This is an inverting amp

$$\therefore T(s) = - \frac{Z_2}{Z_1}$$

$$\text{where } Z_2 = \frac{10^4 \times 10^7 s^{-1}}{10^4 + 10^7 s^{-1}} = \frac{10^7}{s + 10^3}$$

$$Z_1 = 10^4 + \frac{1}{25} \times 10^9 s^{-1} = 10^4 + 4 \times 10^7 s^{-1}$$

$$\begin{aligned} - \frac{Z_2}{Z_1} = T(s) &= \frac{-10^3}{s + 10^3} \times \frac{1}{1 + 4000 s^{-1}} \\ &= \frac{-1000 s}{(s + 1000)(s + 4000)} \end{aligned}$$

In the 2<sup>nd</sup> circuit, voltage division

$$\begin{aligned} V_2(s) = V_1(s) &= \frac{10^4 \times \frac{1}{5} \times 10^8 s^{-1}}{10^4 + \frac{1}{5} \times 10^8 s^{-1}} \\ &= \frac{2 \cdot 10^7}{s + 2 \cdot 10^7} + 2 \cdot 10^4 + 4 \cdot 10^7 s^{-1} \end{aligned}$$

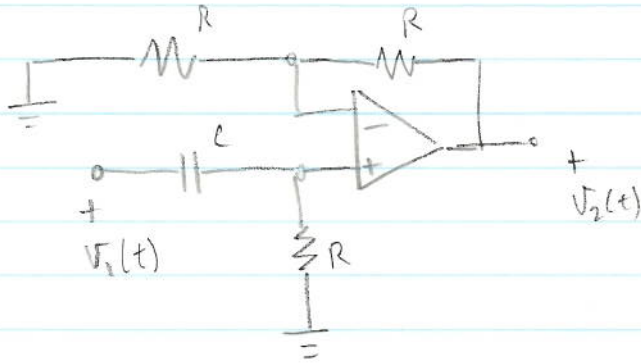
part b and c: cct 1 is better for both cases. For load resistance ~ 1000 kOhms, the transfer function of cct 2 will alter.

$$\begin{aligned} &= V_1(s) \frac{10^3}{s + 2 \cdot 10^3} \\ &= \frac{10^3}{s + 2 \cdot 10^3} + \frac{2000}{s} + 1 \end{aligned}$$

part d: disagree. Cct 2 is analyzed assuming no current drawn from the load. If cct 2 is put in front of cct 1, a current will be drawn.

$$\begin{aligned} &= V_1(s) \frac{10^3}{10^3 s + 2 \cdot 10^3 s + 4 \cdot 10^6 + s^2 + s(2 \cdot 10^3)} \\ &= V_1(s) \frac{10^3}{s^2 + 5000s + 4 \cdot 10^6} = \frac{1000}{(s + 1000)(s + 4000)} \end{aligned}$$

12/6



$$V_p(s) = V_1(s) \times \frac{R}{R + \frac{1}{sC}} = V_1(s) \times \frac{s}{s + \frac{1}{RC}}$$

$$V_2(s) = \frac{2R}{R} V_p(s)$$

$$= \frac{2s}{s + \frac{1}{RC}} V_1(s) \quad \boxed{\text{High-pass filter}}$$

pass band gain  $\Leftrightarrow s \rightarrow \infty$

$$\boxed{T(s) \rightarrow 2}$$

choose RC s.t.  $f_c = 5 \text{ kHz}$   
 $\Leftrightarrow \omega_c = 10^4 \pi$

$$|T(j\omega_c)| = \frac{2\omega_c}{\sqrt{\omega_c^2 + \left(\frac{1}{RC}\right)^2}} \Rightarrow \omega_c = \frac{1}{RC}$$

$$\therefore RC = \pi^{-1} \times 10^{-4}$$

Choose whatever  
values you like.

12-9,

$$T(s) = \frac{2}{10^2 + 20/s} = \frac{200s}{s+2000} \Rightarrow T(j\omega) = \frac{200j\omega}{j\omega + 2000}$$

→ high pass filter

$$\Rightarrow |T(j\omega)| = \frac{200\omega}{\sqrt{\omega^2 + 4 \times 10^6}}$$

→ passband gain = 200

→ cut off frequency  $\omega_c$  occurs when  $|T(j\omega_c)| = \frac{200}{\sqrt{2}}$

$$\Rightarrow \frac{\omega_c}{\sqrt{\omega_c^2 + 4 \times 10^6}} = \frac{1}{\sqrt{2}} \Rightarrow \omega_c = 2000 \text{ rad/s}$$

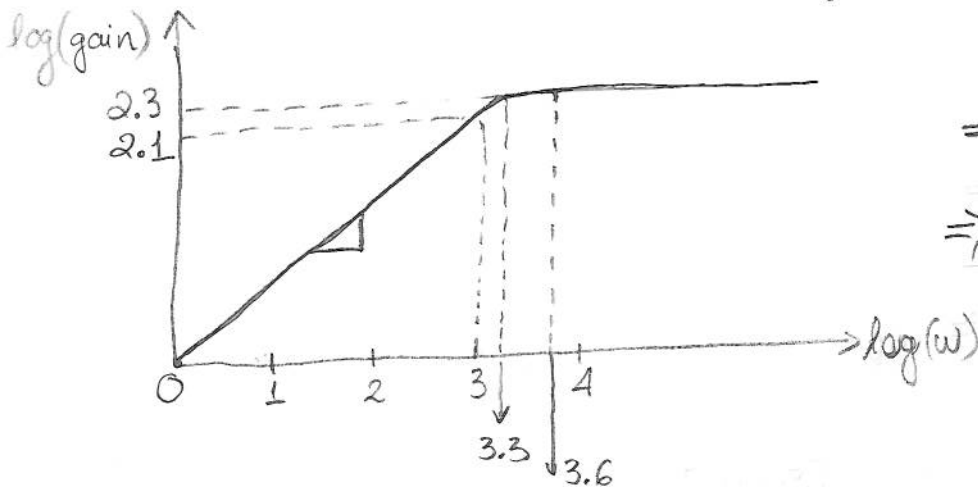
→ Bode-plot in log-log coordinate:

$$\rightarrow \omega_c = 2000 \text{ rad/s} \Rightarrow \log_{10}(\omega_c) = 3.3$$

$$\rightarrow \text{gain} = 200 \Rightarrow \log_{10}(\text{gain}) = 2.3$$

$$\rightarrow \omega = 0.5\omega_c = 1000 \text{ rad/s} \Rightarrow \log_{10}(\omega) = 3$$

$$\rightarrow \omega = 2\omega_c = 4000 \text{ rad/s} \Rightarrow \log_{10}(\omega) = 3.6$$



$$\Rightarrow \text{slope} = \frac{2.3}{3.3} = 0.7$$

$$\Rightarrow \log(\text{gain}) = 0.7 \times \log(\omega)$$

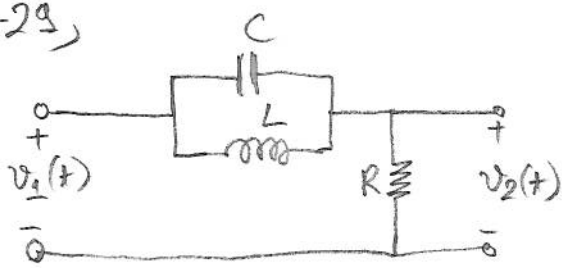
$$\rightarrow \text{for } \omega = 0.5\omega_c, \log(\text{gain}) = 0.7 \times 3 = 2.1 \Rightarrow \text{gain} = 125.9$$

$$\rightarrow \text{for } \omega = \omega_c, \log(\text{gain}) = 2.3 \Rightarrow \text{gain} = 200$$

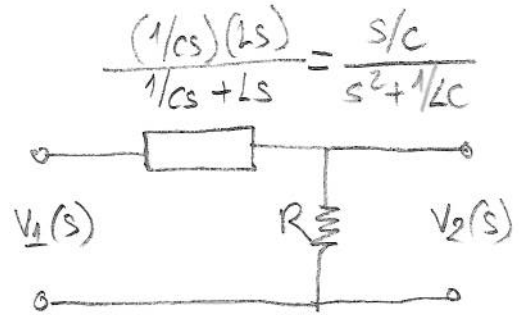
$$\rightarrow \text{for } \omega = 2\omega_c, \log(\text{gain}) = 2.3 \Rightarrow \text{gain} = 200$$



12-29,



$\mathcal{L}$



$$\rightarrow V_2(s) = \frac{R}{R + \frac{s/c}{s^2 + 1/LC}} V_1(s) \Rightarrow T(s) = \frac{s^2 + 1/LC}{s^2 + s/RC + 1/LC}$$

$$\Rightarrow T(j\omega) = \frac{-\omega^2 + 1/LC}{-\omega^2 + j\omega/RC + 1/LC} \Rightarrow |T(j\omega)| = \frac{1/LC - \omega^2}{[(1/LC - \omega^2)^2 + (\omega/RC)^2]^{1/2}}$$

$\rightarrow$  as  $\omega \rightarrow \frac{1}{\sqrt{LC}}$ ,  $|T(j\omega)| \rightarrow 0 \Rightarrow$  bandstop filter

$\rightarrow$  as  $\omega \rightarrow 0$ ,  $|T(j\omega)| \rightarrow 1$   
 $\rightarrow$  as  $\omega \rightarrow \infty$ ,  $|T(j\omega)| \rightarrow 1$  }  $\Rightarrow$  gain = 1

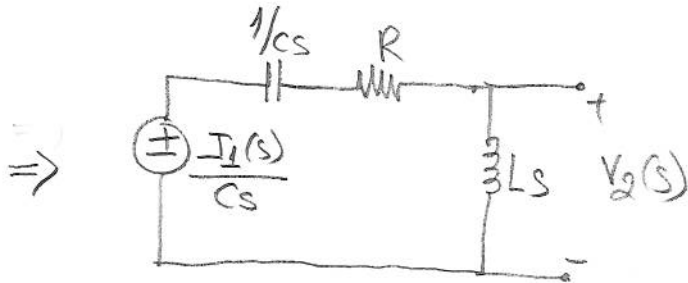
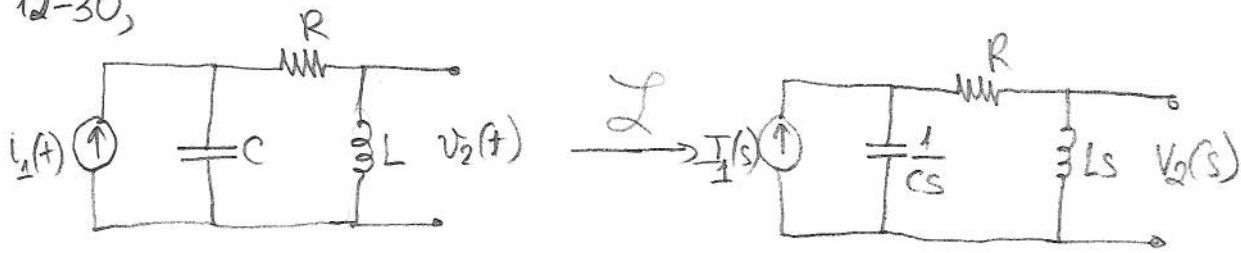
$\rightarrow$  notch frequency =  $1/\sqrt{LC}$  rad/s

$\rightarrow$  cutoff frequency  $\omega_c$  occurs when  $|T(j\omega)| = \frac{1}{\sqrt{2}}$

$$\Rightarrow \frac{1/LC - \omega_c^2}{[(1/LC - \omega_c^2)^2 + (\omega_c/RC)^2]^{1/2}} = \frac{1}{\sqrt{2}} \Rightarrow \omega_c^2 + \frac{\omega_c}{RC} - \frac{1}{LC} = 0$$

$$\Rightarrow \omega_c = \frac{1}{2RC} [-1 \pm \sqrt{1 + 4RC/L}]$$

12-30,



$$\rightarrow V_2(s) = \frac{Ls}{\frac{1}{Cs} + R + Ls} \frac{I_1(s)}{Cs} = \frac{s/c}{s^2 + \frac{R}{L}s + \frac{1}{LC}} I_1(s)$$

$$\Rightarrow V_2(j\omega) = \frac{j\omega/c}{-\omega^2 + \frac{R}{L}j\omega + \frac{1}{LC}} I_1(j\omega)$$

$$\rightarrow \text{as } \omega \rightarrow 0, |V_2(j\omega)| \rightarrow 0$$

$$\rightarrow \text{as } \omega \rightarrow \infty, |V_2(j\omega)| \rightarrow 0$$

$$\rightarrow \text{when } \omega = \frac{1}{\sqrt{LC}}, |V_2(j\omega)| = \frac{L}{CR} |I_1(j\omega)|$$

$\Rightarrow$  this is a bandpass filter