Homework #6: 9.7, 9.9, 9.20, 9.22
10.6, 10.7, 10.13, 10.15, 10.19, 10.30

9.7 a) $f_1(t) = [5e^{-5t} - 10e^{-20t}]u(t)$

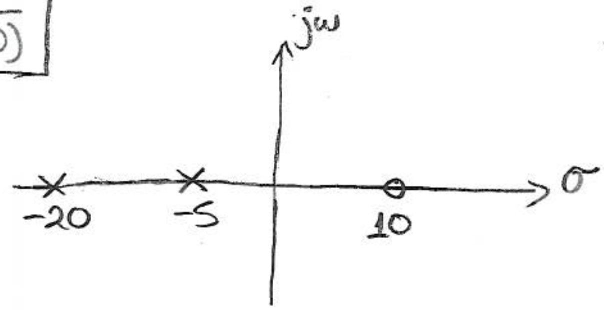
$\Rightarrow F_1(s) = \frac{5}{s+5} - \frac{10}{s+20}$

$\Rightarrow \boxed{F_1(s) = \frac{-5(s-10)}{(s+5)(s+20)}}$

$\rightarrow$ zeros : $s = 10$

$\rightarrow$ poles : $\rightarrow s = -5$

$\rightarrow s = -20$



b) $f_2(t) = [5\cos(10t) + 7\cos(20t)]u(t)$

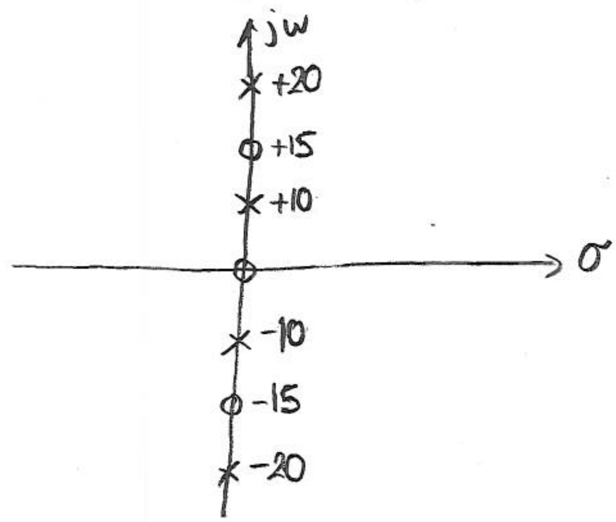
$\Rightarrow F_2(s) = \frac{5s}{s^2+100} + \frac{7s}{s^2+400} = \boxed{\frac{12s(s^2+225)}{(s^2+100)(s^2+400)}}$

$\rightarrow$ zeros : $\rightarrow s = 0$

$\rightarrow s^2 + 225 = 0 \Rightarrow s = \pm 15j$

$\rightarrow$ poles : $\rightarrow s^2+100 = 0 \Rightarrow s = \pm 10j$

$\rightarrow s^2+400 = 0 \Rightarrow s = \pm 20j$



9.9, a) $f_1(t) = \delta(t) - (625te^{-50t})u(t)$

$$\Rightarrow F_1(s) = 1 - \frac{625}{(s+50)^2} = \boxed{\frac{(s+25)(s+75)}{(s+50)^2}}$$

$\rightarrow$ zeros : $\rightarrow$ $s+25=0 \Rightarrow \boxed{s=-25}$

$\rightarrow$ $s+75=0 \Rightarrow \boxed{s=-75}$

$\rightarrow$ poles : $\rightarrow$ $(s+50)^2=0 \Rightarrow \boxed{s=-50 \text{ (double poles)}}$

b) $f_2(t) = [5 + e^{-20t} - 6\cos(10t) + 2\sin(10t)]u(t)$

$$\Rightarrow F_2(s) = \frac{5}{s} + \frac{1}{s+20} - \frac{6s}{s^2+10^2} + \frac{2(10)}{s^2+10^2}$$

$$\Rightarrow \boxed{F_2(s) = \frac{1000(s+10)}{s(s+20)(s^2+100)}}$$

$\rightarrow$ zeros: $\rightarrow$ $s+10=0 \Rightarrow \boxed{s=-10}$

$\rightarrow$ poles: $\rightarrow$ $\boxed{s=0}$

$\rightarrow$ $s+20=0 \Rightarrow \boxed{s=-20}$

$\rightarrow$ $s^2+100=0 \Rightarrow \boxed{s=\pm 10j}$

9.20, a) $F_1(s) = \dfrac{\alpha^2}{s^2(s+\alpha)} = \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{C}{s+\alpha}$

$\Rightarrow As(s+\alpha) + B(s+\alpha) + Cs^2 = \alpha^2$

$\Rightarrow As^2 + A\alpha s + Bs + B\alpha + Cs^2 = \alpha^2$

$\Rightarrow (A+C)s^2 + (A\alpha + B)s + B\alpha = \alpha^2$

$$\Rightarrow \begin{cases} A + C = 0 \\ A\alpha + B = 0 \\ B\alpha = \alpha^2 \end{cases} \Rightarrow \begin{cases} A = -1 \\ B = \alpha \\ C = 1 \end{cases}$$

$\Rightarrow F_1(s) = \dfrac{-1}{s} + \dfrac{\alpha}{s^2} + \dfrac{1}{s+\alpha}$

$\Rightarrow f_1(t) = -u(t) + \alpha t u(t) + e^{-\alpha t}u(t)$

$\Rightarrow \boxed{f_1(t) = [-1 + \alpha t + e^{-\alpha t}]u(t)}$

b) $F_2(s) = \dfrac{\alpha^2}{s(s+\alpha)^2} = \dfrac{A}{s} + \dfrac{B}{s+\alpha} + \dfrac{C}{(s+\alpha)^2}$

$\Rightarrow A(s+\alpha)^2 + Bs(s+\alpha) + Cs = \alpha^2$

$\Rightarrow As^2 + 2A\alpha s + A\alpha^2 + Bs^2 + B\alpha s + Cs = \alpha^2$

$\Rightarrow (A+B)s^2 + (2A\alpha + B\alpha + C)s + A\alpha^2 = \alpha^2$

$$\Rightarrow \begin{cases} A + B = 0 \\ 2A\alpha + B\alpha + C = 0 \\ A\alpha^2 = \alpha^2 \end{cases} \Rightarrow \begin{cases} A = 1 \\ B = -1 \\ C = -\alpha \end{cases}$$

$\Rightarrow F_2(s) = \dfrac{1}{s} - \dfrac{1}{s+\alpha} - \dfrac{\alpha}{(s+\alpha)^2}$

$\Rightarrow f_2(t) = u(t) - e^{-\alpha t}u(t) - \alpha t e^{-\alpha t}u(t)$

$\Rightarrow \boxed{f_2(t) = [1 - (1+\alpha t)e^{-\alpha t}]u(t)}$

9.22, a) $F_1(s) = \dfrac{16}{(s+2)(s^2+12s+36)} = \dfrac{16}{(s+2)(s+6)^2} = \dfrac{A}{s+2} + \dfrac{B}{s+6} + \dfrac{C}{(s+6)^2}$

$\Rightarrow A(s+6)^2 + B(s+2)(s+6) + C(s+2) = 16$

$\Rightarrow As^2 + 12As + 36 + Bs^2 + 8Bs + 12B + Cs + 2C = 16$

$\Rightarrow (A+B)s^2 + (12A+8B+C)s + (36+12B+2C) = 16$

$\Rightarrow \begin{cases} A+B=0 \\ 12A+8B+C=0 \\ 36+12B+2C=16 \end{cases} \Rightarrow \begin{cases} A=1 \\ B=-1 \\ C=-4 \end{cases}$

$\Rightarrow F_1(s) = \dfrac{1}{s+2} + \dfrac{-1}{s+6} + \dfrac{-4}{(s+6)^2}$

$\Rightarrow f_1(t) = e^{-2t}u(t) - e^{-6t}u(t) - 4te^{-6t}u(t)$

$\Rightarrow \boxed{f_1(t) = \left[e^{-2t} - (1+4t)e^{-6t}\right]u(t)}$

b) $F_2(s) = \dfrac{2(s^2+2)}{s(s^2+4)} = \dfrac{A}{s} + \dfrac{Bs+C}{s^2+4}$

$\Rightarrow A(s^2+4) + s(Bs+C) = 2s^2+4$

$\Rightarrow As^2 + 4A + Bs^2 + Cs = 2s^2+4$

$\Rightarrow (A+B)s^2 + Cs + 4A = 2s^2+4$

$\Rightarrow \begin{cases} A+B=2 \\ C=0 \\ 4A=4 \end{cases} \Rightarrow \begin{cases} A=1 \\ B=1 \\ C=0 \end{cases}$

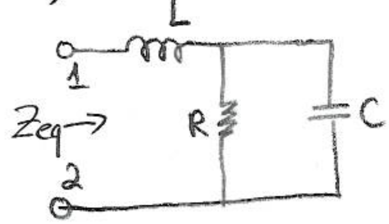
$\Rightarrow F_2(s) = \dfrac{1}{s} + \dfrac{s}{s^2+4}$

$\Rightarrow f_2(t) = u(t) + \cos(2t)u(t)$

$\Rightarrow \boxed{f_2(t) = [1+\cos(2t)]u(t)}$

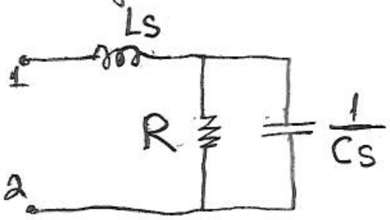
OR In complex form: $f_2(t) = \left[1 + \frac{1}{2}e^{2jt} + \frac{1}{2}e^{-2jt}\right]u(t)$

10.6,



$R = 1k\Omega$
$L = 1H$
$C = 500nF$

→ transform cct into s-domain:



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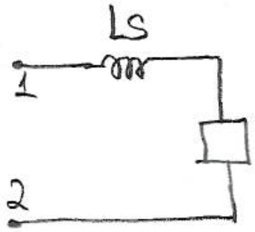
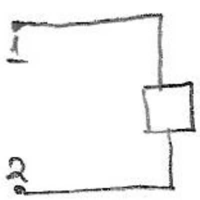


$$\frac{(R)\left(\frac{1}{Cs}\right)}{R+\frac{1}{Cs}} = \frac{R}{RCs+1}$$

⟹



$$Z_{eq} = Ls + \frac{R}{RCs+1} = \boxed{\frac{RLCs^2+Ls+R}{RCs+1}}$$

→ sub in numbers $\Rightarrow Z_{eq} = \dfrac{s^2+2\times10^3 s+2\times10^6}{s+2\times10^3} = \dfrac{(s+10^3)^2+10^6}{s+2\times10^3}$

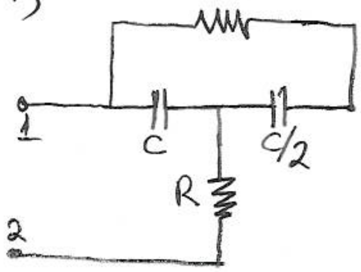
→ zeros: → $(s+10^3)^2+10^6 = 0 \Rightarrow (s+10^3)^2 = -10^6$

$\Rightarrow s+10^3 = \pm 10^3 j$

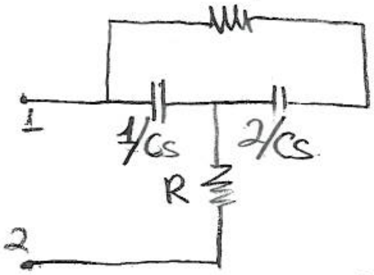
$\Rightarrow \boxed{s = -10^3 \pm 10^3 j}$

→ poles: → $s+2\times10^3 = 0 \Rightarrow \boxed{s = -2\times10^3}$

10.7,

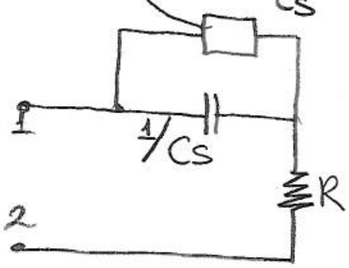


$\xrightarrow{\mathcal{L}}$



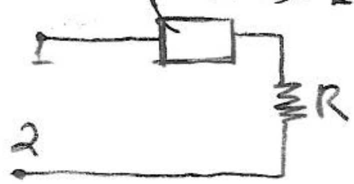
$2R + \frac{2}{Cs} = \frac{2(RCs+1)}{Cs}$

$\Rightarrow$

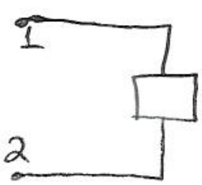


$\Longrightarrow$

$$\frac{(1/Cs)\left[\frac{2(RCs+1)}{Cs}\right]}{(1/Cs)+\left[\frac{2(RCs+1)}{Cs}\right]} = \frac{2(RCs+1)}{Cs(2RCs+3)}$$



$\Rightarrow$



$$Z_{eq} = R + \frac{2(RCs+1)}{Cs\,(2RCs+3)} = \frac{2R^2C^2s^2 + 5RCs + 2}{Cs(2RCs+3)}$$

$$\Rightarrow \boxed{Z_{eq} = \frac{(2RCs+1)(RCs+2)}{Cs(2RCs+3)}}$$

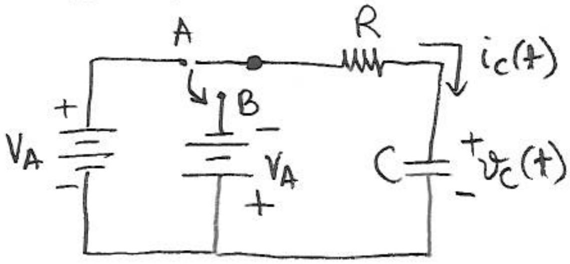
$\rightarrow$ zeros: $\rightarrow 2RCs + 1 = 0 \Rightarrow \boxed{s = \frac{-1}{2RC}}$

$\rightarrow RCs + 2 = 0 \Rightarrow \boxed{s = \frac{-2}{RC}}$

$\rightarrow$ poles: $\rightarrow 2RCs + 3 = 0 \Rightarrow \boxed{s = \frac{-3}{2RC}}$

$\rightarrow \boxed{s = 0}$

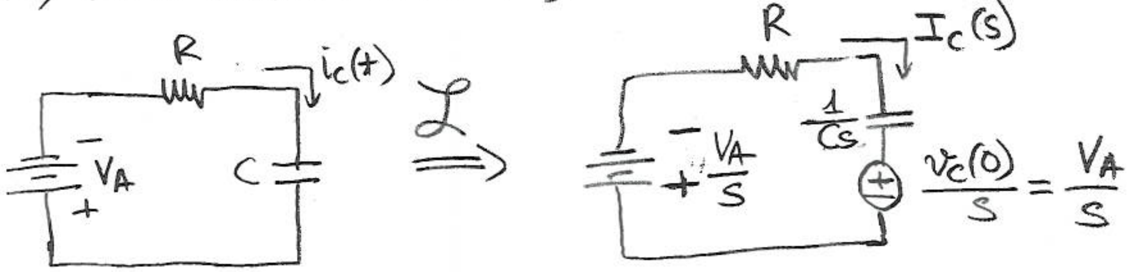
10.13,



→ switch is at A for a long time

$\Rightarrow v_C(0) = V_A$

→ when switch is at B, cct becomes:



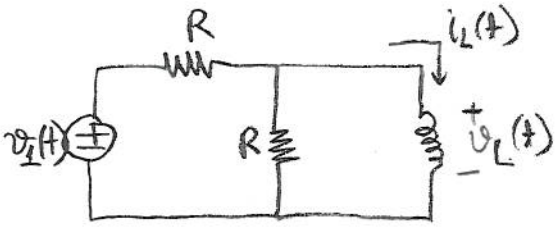
→ KVL around the loop: $-\frac{V_A}{s} - RI_C(s) - \frac{1}{Cs}I_C(s) - \frac{V_A}{s} = 0$

$$\Rightarrow \left(R + \frac{1}{Cs}\right) I_C(s) = \frac{-2V_A}{s}$$

$$\Rightarrow I_C(s) = \frac{-2V_A/s}{\left(R + \frac{1}{Cs}\right)} = \frac{-2CV_A}{RCs+1} = \boxed{\frac{-2V_A/R}{s + 1/RC}}$$

$$\Rightarrow \boxed{i_c(t) = \frac{-2V_A}{R} e^{-t/RC} u(t)}$$

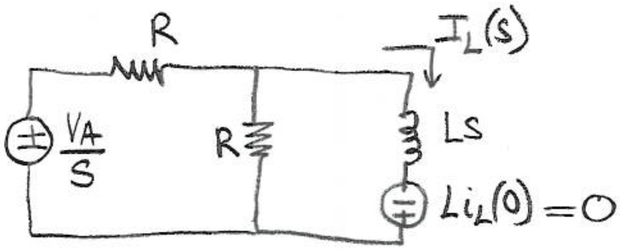
10.15,



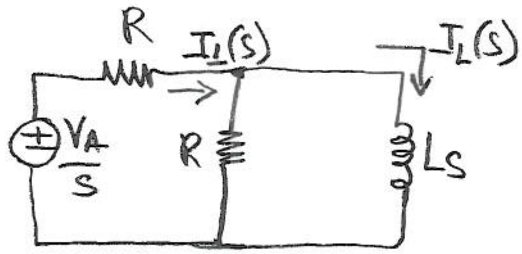
$\rightarrow$ no initial energy

$\rightarrow v_1(t) = V_A\, u(t)$

$\overset{\mathcal{L}}{\Longrightarrow}$



$\Rightarrow$



$$\rightarrow I_1(s) = \frac{V_A/s}{Z_{eq}} = \frac{V_A/s}{R + \left(\frac{RLs}{R+Ls}\right)} = \frac{V_A(R+Ls)}{s(R^2+2RLs)}$$

$\rightarrow$ use current divider to find $I_L(s)$

$$\Rightarrow I_L(s) = \frac{R}{R+Ls} I_1(s) = \frac{R}{R+Ls} \frac{V_A(R+Ls)}{s(R^2+2RLs)}$$

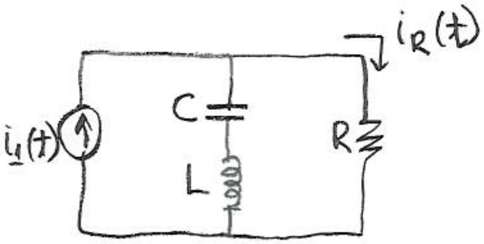
$$\Rightarrow \boxed{I_L(s) = \frac{V_A}{s(R+2Ls)}}$$

$$\Rightarrow I_L(s) = \frac{V_A/R}{s} + \frac{-V_A/R}{s + R/2L}$$

$$\Rightarrow i_L(t) = \frac{V_A}{R} u(t) - \frac{V_A}{R} e^{-\frac{R}{2L}t} u(t)$$

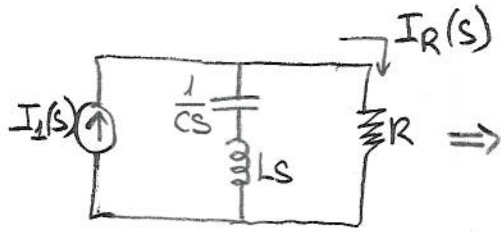
$$\Rightarrow \boxed{i_L(t) = \frac{V_A}{R}\left[1 - e^{-\frac{R}{2L}t}\right] u(t)}$$

10.19,

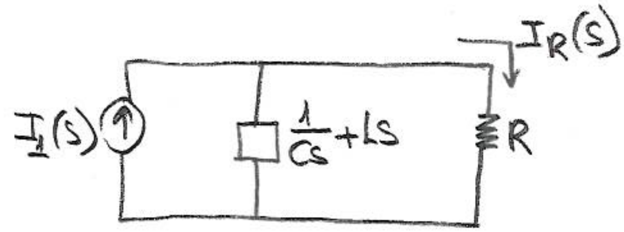


$\rightarrow$ zero state $\Rightarrow$ Initial condition $\rightarrow 0$.

$\mathcal{L}$
$\Rightarrow$



$\Rightarrow$

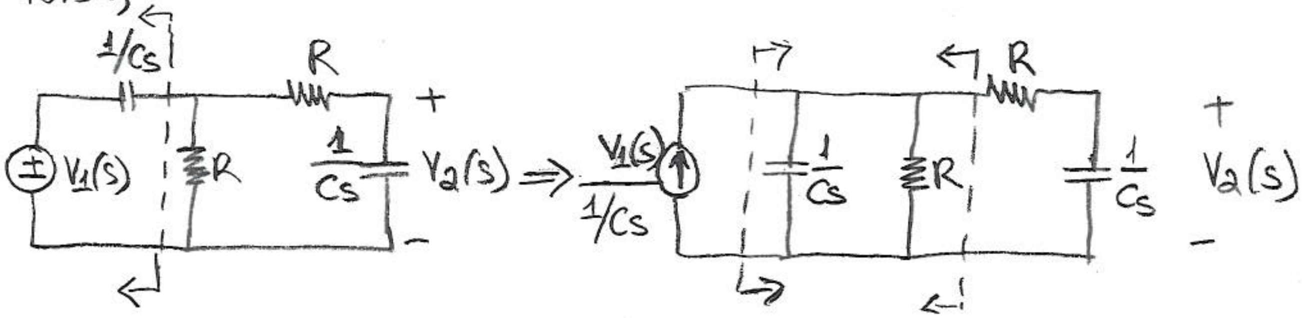


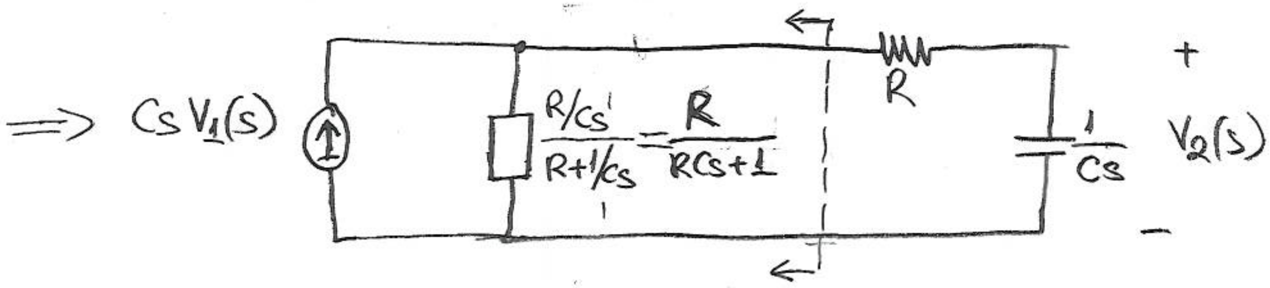
$\rightarrow$ use current divider:

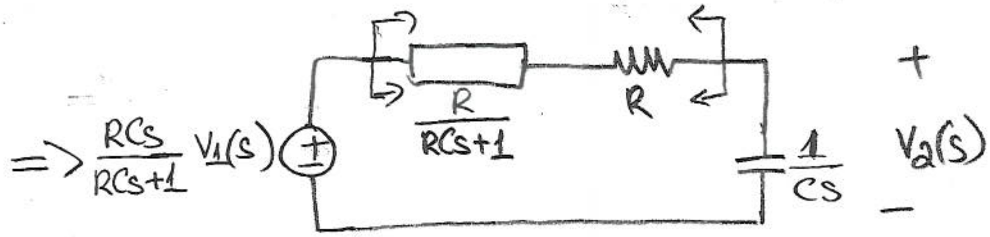
$$\Rightarrow I_R(s) = \frac{\left(\frac{1}{Cs}+Ls\right)}{R+\left(\frac{1}{Cs}+Ls\right)} I_1(s)$$

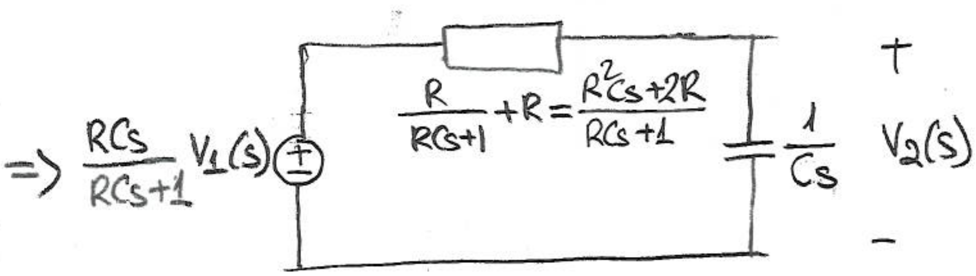
$$\Rightarrow \boxed{I_R(s) = \frac{LCs^2+1}{LCs^2+RCs+1} I_1(s)}$$

10.30,









$\rightarrow$ voltage divider

$$\Rightarrow V_2(s) = \frac{1/Cs}{\frac{1}{Cs} + \frac{R^2Cs+2R}{RCs+1}} \frac{RCs}{RCs+1} V_1(s)$$

$$\Rightarrow \boxed{V_2(s) = \frac{RCs}{R^2C^2s^2+3RCs+1} V_1(s)}$$