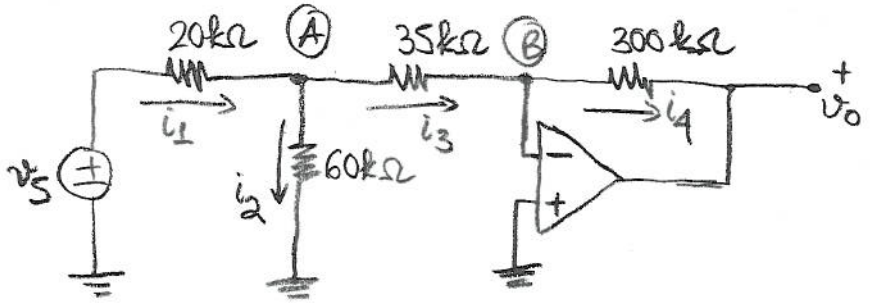
Homework 5: 4.21, 22, 23, 25, 26
30, 32, 33, 36, ~~37~~

4.21,



→ Op-Amp: $v_p = v_N = 0$ (ground) $\Rightarrow v_B = 0$

$i_p = i_N = 0$

→ KCL @ B: $i_3 - i_4 = 0 \Rightarrow \frac{v_A - \overrightarrow{v_B}^{0}}{35k\Omega} = \frac{\overrightarrow{v_B}^{0} - v_o}{300k\Omega} \Rightarrow v_A = \frac{-35}{300} v_o$ (1)

→ KCL @ A: $i_1 - i_2 - i_3 = 0 \Rightarrow \frac{v_s - v_A}{20k\Omega} - \frac{v_A - 0}{60k\Omega} - \frac{v_A - \overrightarrow{v_B}^{0}}{35k\Omega} = 0$

$$\Rightarrow \frac{v_s}{20k\Omega} - \left(\frac{1}{20k\Omega} + \frac{1}{60k\Omega} + \frac{1}{35k\Omega}\right)(v_A) = 0$$

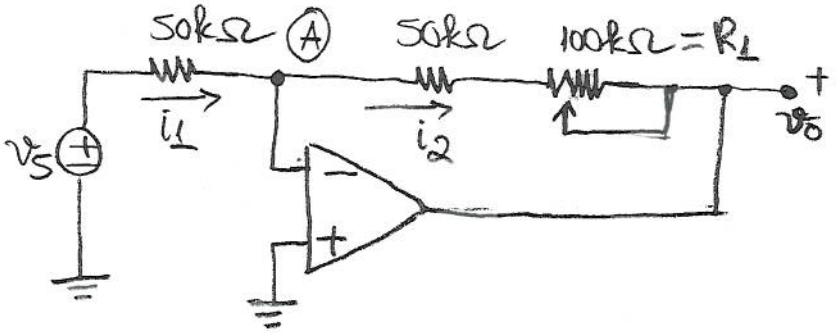
→ substitute (1) into above,

$$\Rightarrow \frac{v_s}{20k\Omega} - \left(\frac{1}{20k\Omega} + \frac{1}{60k\Omega} + \frac{1}{35k\Omega}\right)\left(\frac{-35}{300}\right) v_o = 0$$

$$\Rightarrow v_s + \frac{2}{9} v_o = 0$$

$$\Rightarrow \boxed{v_o = -4.5\, v_s}$$

4.22)



→ Resistor R_1 varies from $0 \rightarrow 100\,k\Omega$

→ Op-Amp : $v_p = v_N = 0$ (ground) $\Rightarrow v_A = 0$

$i_p = i_N = 0$

→ KCL @ A: $i_1 - i_2 = 0 \Rightarrow \dfrac{v_s - \overset{\rightarrow 0}{v_A}}{50k\Omega} - \dfrac{\overset{\rightarrow 0}{v_A} - v_o}{50k\Omega + R_1} = 0$

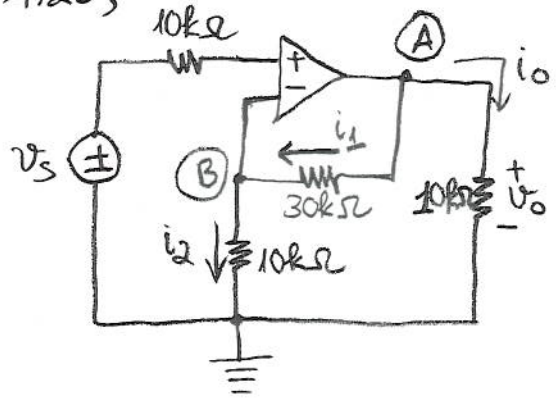
$$\Rightarrow \frac{v_o}{v_s} = \frac{-(50k\Omega + R_1)}{50k\Omega}$$

→ $R_1 = 0 \Rightarrow \dfrac{v_o}{v_s} = -1$

→ $R_1 = 100\,k\Omega \Rightarrow \dfrac{v_o}{v_s} = -3$

$$\Rightarrow \boxed{-1 > \frac{v_o}{v_s} > -3}$$

4.23,



(a) → Op Amp: $v_p = v_N \Rightarrow v_B = v_s$

$i_p = i_N = 0$

→ $v_A = v_o$

→ KCL @ B: $i_1 - i_2 = 0 \Rightarrow \dfrac{v_A - v_B}{30k\Omega} - \dfrac{v_B - 0}{10k\Omega} = 0$ ($v_A = v_o$, $v_B = v_s$, v_s)

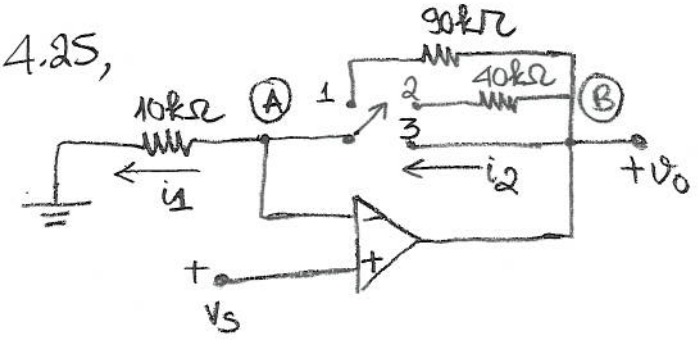
$\Rightarrow \dfrac{v_o - v_s}{30k\Omega} - \dfrac{v_s}{10k\Omega} = 0 \Rightarrow \boxed{v_o = 4v_s}$

(b) → $v_s = 1.5V$

$\Rightarrow v_o = (4)(1.5V) = 6V$

→ $i_o = \dfrac{v_o}{10k\Omega} = \dfrac{6V}{10k\Omega} \Rightarrow \boxed{i_o = 0.6\,mA}$

4.25,



→ i_2 = current running from node (B) to node (A) with resistor R_L in between. $R_L = 90k\Omega$ (position 1); $R_L = 0$ (position 3)

→ Op-Amp: $v_p = v_N \Rightarrow v_A = v_S$

$i_p = i_N = 0$

→ $v_B = v_o$

→ KCL @ (A): $i_2 - i_1 = 0 \Rightarrow \dfrac{v_B - v_A}{R_L} - \dfrac{v_A - 0}{10k\Omega} = 0$ ($v_B \to v_o$, $v_A \to v_S$, $v_A \to v_S$)

$\Rightarrow \dfrac{v_o}{R_L} - \left(\dfrac{1}{R_L} + \dfrac{1}{10k\Omega}\right) v_S = 0$

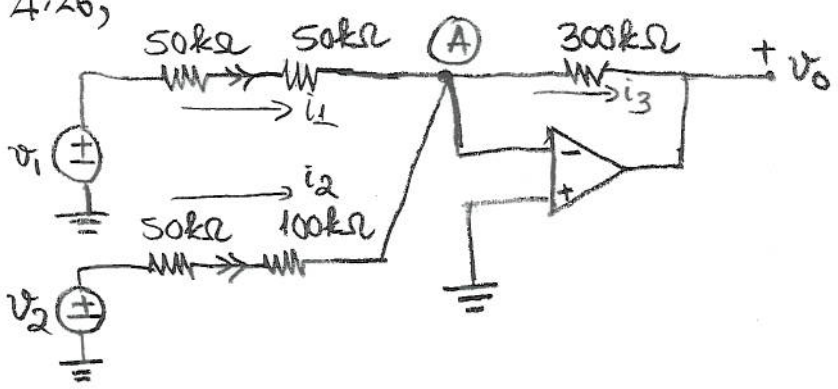
$\Rightarrow \dfrac{v_o}{v_S} = 1 + \dfrac{R_L}{10k\Omega}$

→ position 1: $R_L = 90k\Omega \Rightarrow \boxed{v_o/v_S = 10}$

→ position 2: $R_L = 40k\Omega \Rightarrow \boxed{v_o/v_S = 5}$

→ position 3: $R_L = 0\Omega \Rightarrow \boxed{v_o/v_S = 1}$

4.26,



→ Op-Amp: $v_N = v_p \Rightarrow v_A = 0$ (ground)

$i_N = i_p = 0$

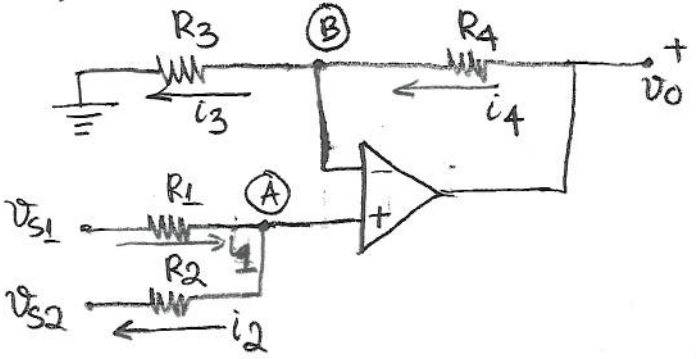
→ KCL @ A: $i_1 + i_2 - i_3 = 0$

$$\Rightarrow \frac{v_{s1} - \overset{\to 0}{v_A}}{100k\Omega} + \frac{v_{s2} - \overset{\to 0}{v_A}}{150k\Omega} - \frac{\overset{\to 0}{v_A} - v_o}{300k\Omega} = 0$$

$$\Rightarrow 3v_{s1} + 2v_{s2} + v_o = 0$$

$$\Rightarrow \boxed{v_o = -3v_{s1} - 2v_{s2}}$$

4.30,



→ Op-Amp: $v_p = v_N \Rightarrow v_A = v_B$

$i_p = i_N = 0$

→ KCL @ B: $i_4 - i_3 = 0 \Rightarrow \dfrac{v_o - v_B}{R_4} - \dfrac{v_B - 0}{R_3} = 0$ ($v_B \to v_A$)

$$\Rightarrow \frac{v_o}{R_4} - \left(\frac{1}{R_3} + \frac{1}{R_4}\right) v_A = 0 \Rightarrow \underline{v_A = \frac{R_3}{R_3 + R_4} v_o}$$

→ KCL @ A: $i_1 - i_2 = 0 \Rightarrow \dfrac{v_{S1} - v_A}{R_1} - \dfrac{v_A - v_{S2}}{R_2} = 0$

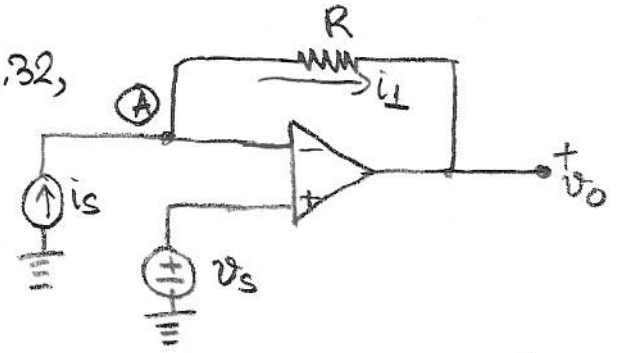
$$\Rightarrow \frac{v_{S1}}{R_1} + \frac{v_{S2}}{R_2} - \left(\frac{R_1 + R_2}{R_1 R_2}\right)(v_A) = 0$$

→ substitute v_A in,

$$\Rightarrow \frac{v_{S1}}{R_1} + \frac{v_{S2}}{R_2} - \left(\frac{R_1 + R_2}{R_1 R_2}\right)\left(\frac{R_3}{R_3 + R_4}\right) v_o = 0$$

$$\Rightarrow \boxed{v_o = \left(1 + \frac{R_4}{R_3}\right)\left(\frac{R_2 v_{S1} + R_1 v_{S2}}{R_1 + R_2}\right)}$$

4.32,



→ Op-Amp : $v_p = v_N \Rightarrow v_A = v_S$

$i_p = i_N = 0$

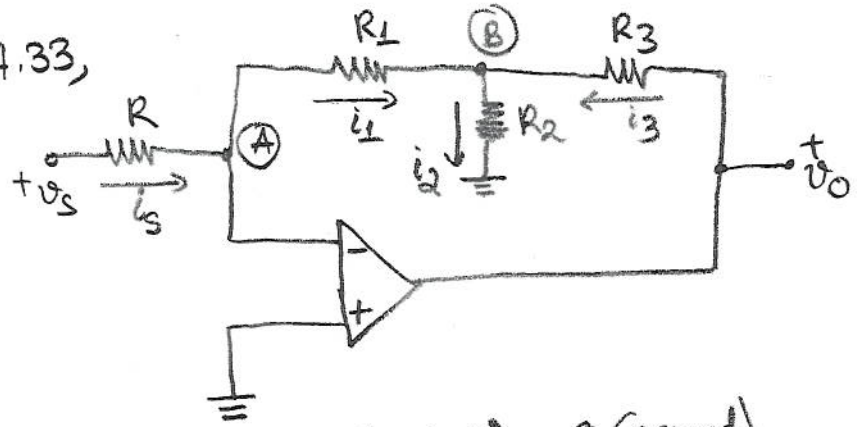
→ KCL @ A : $i_S - i_1 = 0 \Rightarrow i_S - \dfrac{v_A - v_O}{R} = 0$ ($v_A \to v_S$)

$\Rightarrow R i_S - v_S + v_O = 0$

$\Rightarrow v_O = v_S - R i_S$ in term of $v_O = K_1 v_S + K_2 i_S$

$\Rightarrow \boxed{K_1 = 1 \text{ and } K_2 = -R}$

4.33,



→ Op-Amp : $v_N = v_p \Rightarrow v_A = 0$ (ground)

$i_N = i_p = 0$

→ KCL @ A : $i_s - i_1 = 0 \Rightarrow \frac{v_s - \overrightarrow{v_A}^{0}}{R} - \frac{\overrightarrow{v_A}^{0} - v_B}{R_1} = 0 \Rightarrow \underline{\underline{v_B = \frac{-R_1}{R} v_s}}$

→ KCL @ B : $i_1 + i_3 - i_2 = 0 \Rightarrow \frac{\overrightarrow{v_A}^{0} - v_B}{R_1} + \frac{v_o - v_B}{R_3} - \frac{v_B - 0}{R_2} = 0$

$$\Rightarrow \frac{v_o}{R_3} - \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) v_B = 0$$

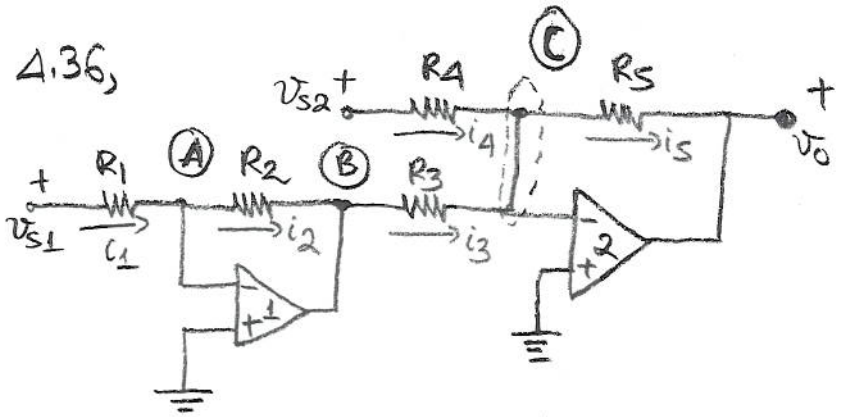
→ substitute v_B in,

$$\Rightarrow \frac{v_o}{R_3} - \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)\left(\frac{-R_1}{R}\right) v_s = 0$$

$$\Rightarrow v_o = -\left(\frac{R_3}{R} + \frac{R_1 R_3}{R R_2} + \frac{R_1}{R}\right) v_s$$

$$\Rightarrow \boxed{v_o = -\left(\frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R R_2}\right) v_s}$$

4.36,



→ Op-Amp 1: $v_p = v_N = 0$ (ground) $\Rightarrow v_A = 0$

$i_p = i_N = 0$

→ KCL @ A: $i_1 - i_2 = 0 \Rightarrow \frac{v_{s1} - 0}{R_1} - \frac{0 - v_B}{R_2} = 0 \Rightarrow \underline{\underline{v_B = \frac{-R_2}{R_1} v_{s1}}}$

→ Op-Amp 2: $v_p = v_N = 0$ (ground) $\Rightarrow v_C = 0$

$i_p = i_N = 0$

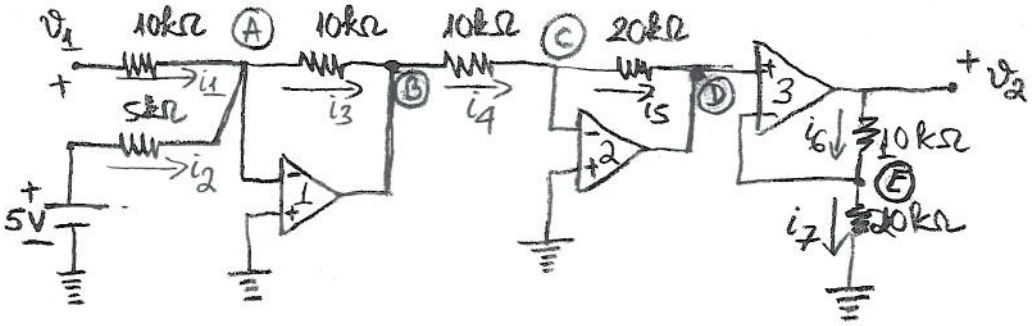
→ KCL @ C: $i_3 + i_4 - i_5 = 0 \Rightarrow \frac{v_B - 0}{R_3} + \frac{v_{s2} - 0}{R_4} - \frac{0 - v_o}{R_5} = 0$

$\Rightarrow \frac{v_B}{R_3} + \frac{v_{s2}}{R_4} + \frac{v_o}{R_5} = 0 \Rightarrow v_o = \frac{-R_5}{R_3} v_B - \frac{R_5}{R_4} v_{s2}$

→ substitute v_B in,

$\Rightarrow \boxed{v_o = \frac{R_2 R_5}{R_1 R_3} v_{s1} - \frac{R_5}{R_4} v_{s2}}$

4.37,



→ Op-Amp 1: $v_p = v_N = 0$ (ground) $\Rightarrow v_A = 0$

→ KCL @ A: $i_1 + i_2 - i_3 = 0 \Rightarrow \frac{v_1 - 0}{10k\Omega} + \frac{5V - 0}{5k\Omega} - \frac{0 - v_B}{10k\Omega} = 0 \Rightarrow \underline{v_B = -10V - v_1}$

$i_p = i_N = 0$

→ Op-Amp 2: $v_p = v_N = 0$ (ground) $\Rightarrow v_C = 0$

$i_p = i_N = 0$

→ KCL @ C: $i_4 - i_5 = 0 \Rightarrow \frac{v_B - 0}{10k\Omega} - \frac{0 - v_D}{20k\Omega} = 0 \Rightarrow v_D = -2v_B$

→ substitute v_B in $\Rightarrow \underline{v_D = 2(10V + v_1)}$

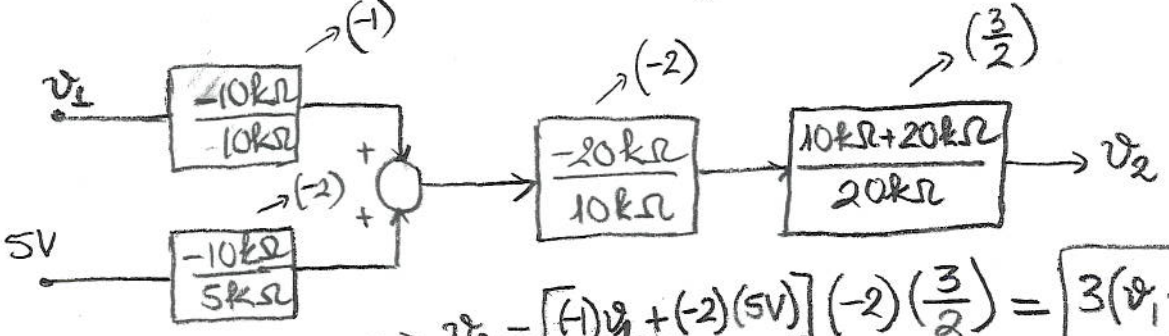
→ Op-Amp 3: $v_p = v_N \Rightarrow v_D = v_E$

$i_p = i_N = 0$

→ KCL @ D: $i_6 - i_7 = 0 \Rightarrow \frac{v_2 - v_E}{10k\Omega} - \frac{v_E - 0}{20k\Omega} = 0 \Rightarrow v_2 = \frac{3}{2} v_D$ ($v_E \to v_D$, $v_E \to v_D$)

→ substitute v_D in $\Rightarrow \boxed{v_2 = 3(10 + v_1)}$

* Block diagram: the circuit above consists of a summer, an inverter and a non-inverter connecting in series.



$\Rightarrow v_2 = [(-1)v_1 + (-2)(5V)](-2)(\frac{3}{2}) = \boxed{3(v_1 + 10)}$