MAE 140 HW #4

3.10, 3.11, 3.13, 3.20, 3.26, 3.28, 3.31, 4.1, 4.7, 4.13

3.10) a)
$$\begin{bmatrix} 2+2+4\ \text{k}\Omega & -4\ \text{k}\Omega \\ -4\ \text{k}\Omega & 4+2+4\ \text{k}\Omega \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} 5-15\ \text{V} \\ 15\ \text{V} \end{bmatrix}$$

→
$$\boxed{\begin{bmatrix} 8 & -4 \\ -4 & 10 \end{bmatrix} \text{k}\Omega \begin{bmatrix} i_A \\ i_B \end{bmatrix} \text{mA} = \begin{bmatrix} -10 \\ 15 \end{bmatrix} \text{V}}$$

b) $i_A = -0.625\ \text{mA}$, $i_B = 1.25\ \text{mA}$

$i_x = -i_A + i_B = \boxed{1.875\ \text{mA}}$

$V_x = i_B \cdot (4\text{k}\Omega) = (1.25)(4)\ \text{V} = \boxed{5\ \text{V}}$

3.11) a)
$$\begin{bmatrix} 30+10 & -10 & -30 \\ -10 & 10+30+30 & -30 \\ -30 & -30 & 30+90+30 \end{bmatrix} \Omega \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} A = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} V$$

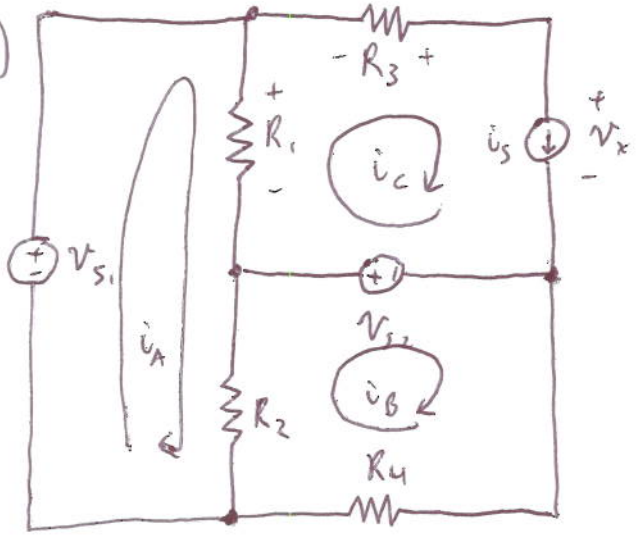
$$\rightarrow \boxed{\begin{bmatrix} 40 & -10 & -30 \\ -10 & 70 & -30 \\ -30 & -30 & 150 \end{bmatrix} \Omega \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} A = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} V}$$

b) $i_A = 0.333$, $i_B = 0.0833$, $i_C = 0.0833$

$V_x = (i_A - i_B)(10\Omega) = \boxed{2.5\ V}$

$i_x = i_A - i_C = \boxed{0.25\ A}$

3.13) a)



$$\begin{bmatrix} R_1+R_2 & -R_2 & -R_1 \\ -R_2 & R_2+R_4 & 0 \\ -R_1 & 0 & R_1+R_3 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} v_{S1} \\ -v_{S2} \\ v_{S2}-v_x \end{bmatrix} \begin{matrix} +R_1 i_S \\ +0\cdot i_S \\ \end{matrix}$$

$\hookrightarrow i_C = i_S$

$$\rightarrow \boxed{\begin{bmatrix} R_1+R_2 & -R_2 \\ -R_2 & R_2+R_4 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} v_{S1}+R_1 i_S \\ -v_{S2} \end{bmatrix}, \quad i_C = i_S}$$

b) $\begin{bmatrix} 20 & -10 \\ -10 & 11 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} 12+10(2.5) \\ -0.5 \end{bmatrix} \rightarrow \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} 3.35 \\ 3.00 \end{bmatrix} mA$

$v_x = v_{S2} - v_{R_1} - v_{R_3} = 0.5\ V - (i_C - i_A)R_1 - (i_S)R_3$

$= 0.5 - (2.5 - 3.35)10 - (2.5)2$

$\rightarrow \boxed{v_x = 4V}$

$i_x = i_A \rightarrow \boxed{i_x = 3.35\ mA}$

c) (power supplied by v_{S1}) $= v_{S1} \cdot i_A = (12\ V)(3.35\ mA)$

$= \boxed{40.2\ mW}$, or according to book's convention, $\boxed{-40.2\ mW}$

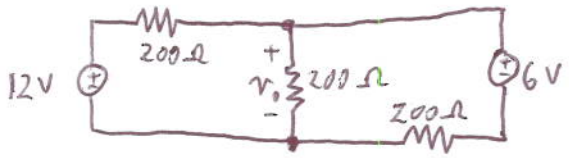
3.20) $$\begin{bmatrix} 6 & -6 & 0 \\ -6 & 6+2+8 & -2 \\ 0 & -2 & 4+2 \end{bmatrix} k\Omega \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} mA = \begin{bmatrix} 5+10 \\ 0 \\ -10 \end{bmatrix} V$$

→ $i_A = 3.75$ mA

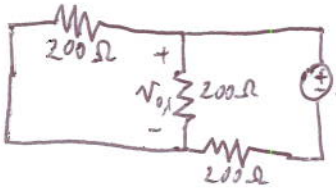
$i_B = 1.25$ mA

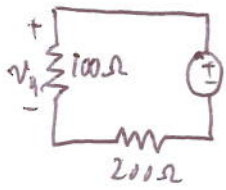
$i_C = -1.25$ mA

3.26) Find v_o by superposition:



set 12V to zero ...

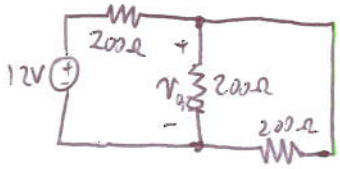


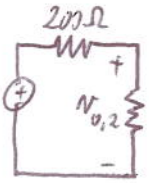


$\rightarrow v_{o,1} = 6\left(\frac{100}{300}\right)$

$\rightarrow v_{o,1} = 2V$

now set 6V to zero ...



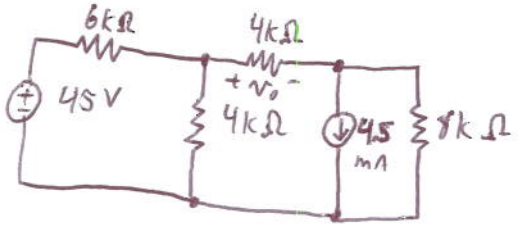


$\rightarrow v_{o,2} = 12\left(\frac{100}{300}\right)$

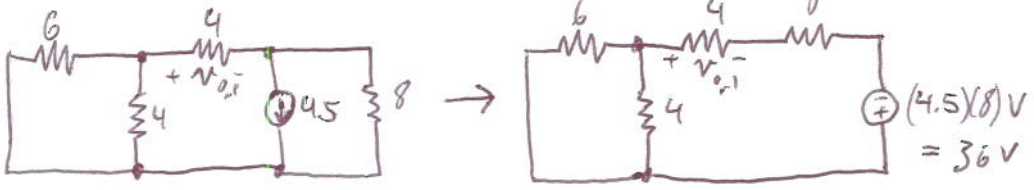
$\rightarrow v_{o,2} = 4V$

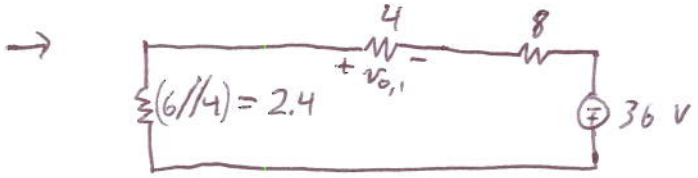
$v_o = v_{o,1} + v_{o,2} = \boxed{6V}$

3.28) use superposition to find v_o



set 45 V to zero...

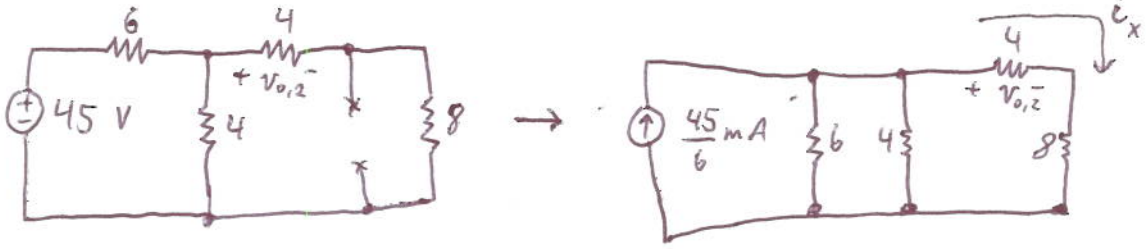




$$v_{o,1} = 36\left(\frac{4}{2.4+4+8}\right)$$

$$\rightarrow v_{o,1} = 10\ V$$

now set 4.5 mA to zero...

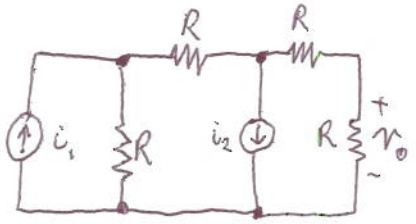


$$i_x = \frac{45}{6}\,mA\left(\frac{\left(\frac{1}{4+8}\right)}{\frac{1}{6}+\frac{1}{4}+\frac{1}{4+8}}\right) = 1.25\ mA$$

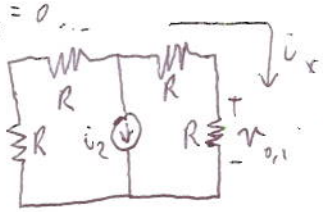
$$v_{o,2} = (i_x)(4k\Omega) = 5\ V$$

$$v_o = v_{o,1} + v_{o,2} = \boxed{15\ V}$$

3.31) use superposition to find v_o

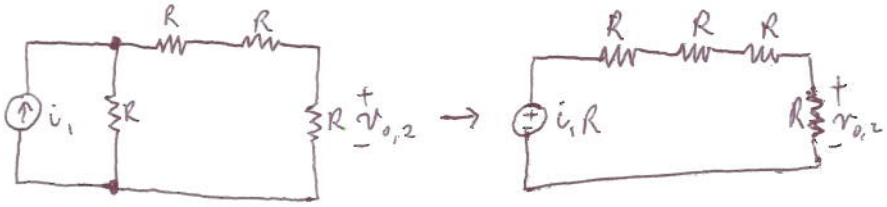


set $i_1 = 0$ ...



$\rightarrow i_x = \frac{-i_2}{2}$, $v_{o,1} = -i_x R = \frac{-1}{2} i_2 R$

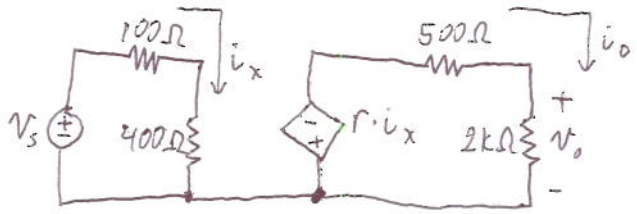
now set $i_2 = 0$ ...



$$v_{o,2} = i_1 R \left(\frac{R}{4R}\right) = \frac{1}{4} i_1 R$$

$$v_o = v_{o,1} + v_{o,2} = \frac{-1}{2} i_2 R + \frac{1}{4} i_1 R = \boxed{\frac{R(i_1 - 2i_2)}{4}}$$

4.1) Find $\frac{v_o}{v_s}$ and $\frac{i_o}{i_x}$ For $r = 4\,k\Omega$



$$i_o = \frac{-r\,i_x}{2.5\,k\Omega} \rightarrow \frac{i_o}{i_x} = \frac{-r}{2.5\,k\Omega} = \frac{-4}{2.5} = -1.6$$

$$\boxed{\frac{i_o}{i_x} \triangleq K_I = -1.6}$$

$$i_x = \frac{v_s}{(100+400)\,\Omega} \rightarrow v_s = 500\,\Omega\,(i_x)$$

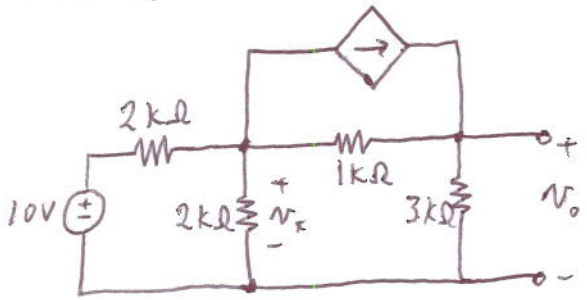
$$v_o = i_o\,(2\,k\Omega)$$

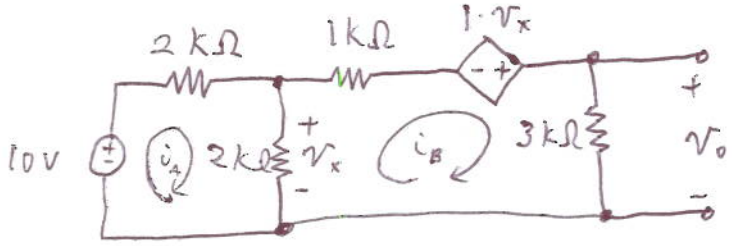
$$\left.\begin{array}{l} i_x = \frac{v_s}{(100+400)\,\Omega} \rightarrow v_s = 500\,\Omega\,(i_x) \\ v_o = i_o\,(2\,k\Omega) \end{array}\right\} \frac{v_o}{v_s} = \frac{i_o\,(2\,k\Omega)}{i_x\,(500\,\Omega)}$$

$$\frac{v_o}{v_s} = 4\,\frac{i_o}{i_x} = -6.4$$

$$\boxed{\frac{v_o}{v_s} \triangleq K_v = -6.4}$$

4.7) Find v_o:





$$\begin{bmatrix} 2+2 & -2 \\ -2 & 2+1+3 \end{bmatrix} k\Omega \begin{bmatrix} i_A \\ i_B \end{bmatrix} mA = \begin{bmatrix} 10 \\ v_x \end{bmatrix} V$$

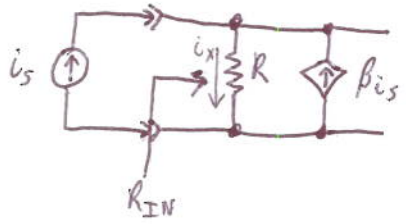
$v_x = (i_A)(2k\Omega)$

$$\begin{bmatrix} 4 & -2 \\ (-2-2) & 6 \end{bmatrix} k\Omega \begin{bmatrix} i_A \\ i_B \end{bmatrix} mA = \begin{bmatrix} 10 \\ 0 \end{bmatrix} V$$

$\rightarrow \quad i_A = 3.333\ mA, \quad i_B = 1.667\ mA$

$v_o = (i_B)(3k\Omega) = \boxed{5V}$

4.13) Find R_{IN}



$$R_{IN} = \frac{V_{IN}}{i_{IN}} = \frac{(i_x)R}{i_s}, \qquad i_x = i_s + \beta i_s = i_s(1+\beta)$$

$$R_{IN} = \frac{i_s(1+\beta)R}{i_s} = (1+\beta)R$$

$$\boxed{R_{IN} = R(\beta+1)}$$